

Citation:

Voigt, W., Remarks on the Leyden observations of the Zeeman-effect at low temperatures, in: KNAW, Proceedings, 11, 1908-1909, Amsterdam, 1909, pp. 360-366

will consist of a

Cf. $\{(2p-1)(p-1)_p\}$,

whose missing pairs of elements can be united to sets of $(p-1)$, so that a

Cf. $\{2p(p-1)_p, p(2p-1)_{p-1}\}$

is formed which breaks up into p principal- $(2p-1)$ -sides.

As in the series $p=4$, just as for $p=3$, the two extensions of the theorems of NETTO can be applied, we can form, in the first set of hundred, systems $S(4, 2)$ for:

$$n = 13, 25, 49; 4, 16, 40, 52, 64, 76, 100.$$

The still missing values are:

$$n = 37, 61, 73, 85, 97; 28, 88,$$

of which 85 might be acquired by 28.

Physics. — “Remarks on the Leyden observations of the Zeeman-Effect at low temperatures.” By W. VOIGT. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the Meeting of October 31, 1908).

The observations of KAMERLINGH ONNES and JEAN BEQUEREL¹⁾ on the ZEEEMAN-Effect at exceedingly low temperatures have led to some surprising results, among which are two, very interesting from the theoretical standpoint. These I will try to throw light upon in the following.

I. It has been found, when the observation was made along the optic axis of an uniaxial crystal (where such a body acts as if isotropic) a longitudinal magnetic field being used, that the components of the ZEEEMAN-doublet have different intensities. The component on the side of the shorter wave-length had generally (though not always) the greater intensity.

This result seems to show, that in the crystal one sense of rotation is preferred to the other. Therefore it might be interesting, to consider firstly the effect of a magnetic field on a naturally active crystal. I may at this point remark that, contrary to what should be expected, the effects of the natural activity and of the magnetic field do not superpose each other, but rather singular combined effects appear in the neighbourhood of an absorption band.

¹⁾ J. BEQUEREL and KAMERLINGH ONNES. These Proceedings February 29th 1908.

The following notation will be made use of:

A, B, C components of magnetic force.

X, Y, Z " " electric " "

$\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$ " " magnetic polarization.

$\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}$ " " electric " "

f_h, v_h, δ_h " " electric partial polarizations.

$a_h, b_h, c_h, \sigma_h, \epsilon_h$ parameters.

P external magnetic force, supposed // Z .

v velocity of light in vacuum.

Then the equations of the theory based on the suppositions I have made for an isotropic active body become:

$$\mathfrak{A}' = v \left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y} \right), \dots \dots \dots (1)$$

$$\mathfrak{X}' = v \left(\frac{\partial C}{\partial y} - \frac{\partial B}{\partial z} \right) \dots \dots \dots (2)$$

$$p_h + a_h x'_h + b_h x''_h + c_h P y'_h = \epsilon_h X - \sigma_h A', \dots \dots \dots (3)$$

$$\mathfrak{X} = X + \sum f_h, \dots \dots \dots (4)$$

$$\mathfrak{A} = A + \sum \sigma_h x'_h / \epsilon_h, \dots \dots \dots (5)$$

By a simple calculation we obtain for the so-called complex refractive index n in the direction of the magnetic field the value

$$n = \sqrt{(1 + E)(1 + \Theta)} \pm \Delta, \dots \dots \dots (6)$$

where

$$E = \sum \frac{\epsilon_h}{p_h \pm c_h P v}, \quad \Theta = v^2 \sum \frac{\sigma_h^2}{\epsilon_h (p_h \pm c_h P v)},$$

$$\Delta = v \sum \frac{\sigma_h}{p_h \pm c_h P v};$$

v = frequency,

$$p_h = 1 + i v a_h - v^2 b_h.$$

The double sign \pm corresponds to the two waves of circular vibrations and of opposite sense of rotation propagated parallel to the field.

The complex index of refraction n is connected with the real index n and with the absorption index κ by the formula $n = n(1 - i\kappa)$. The constants σ_h and c_h measure the natural and the magnetic activity of the crystal. It is seen that even if these two effects are small in general, in the neighbourhood of an absorption band, where p_h is of the same order as $c_h P v$, they do not superpose at all.

If the natural activity vanishes, then also Θ and Δ vanish, and we have

$$n^2 = 1 + E,$$

which contains the theory of the longitudinal ZEE MAN effect.

Among the terms arising from the natural activity that of most interest is $\pm \Delta$, which is generally much larger than θ . This term which has opposite signs for the $+$ and the $-$ rotating waves, but which reaches its maximum at the same place as E , expresses a dissymmetry of the ZEE MAN doublet somewhat similar to that obtained in the experiments of ONNES and BECQUEREL. In order that this dissymmetry should have an appreciable magnitude it is only necessary that $v\sigma_h$ is commensurable with ϵ_h . Experience has shown that at the absorption bands of the crystals under consideration, ϵ_h is very small and so no difficulty is in the way.

Notwithstanding there is lack of agreement between the above formula and observation in two quite different directions. In the first case the observers did not notice any natural activity in the crystals they used, and from the symmetry of these crystals such was not to be expected, except in the case, that at low temperatures the constitution of the molecules changes. On the other hand the observed behaviour by a reversed magnetic field is not in agreement with the above formula.

Both objections disappear if in the formulae (3) we substitute for the terms $\sigma_h A'$, $\sigma_h B'$, $\sigma_h C'$ on the right hand side the terms O , $\sigma_h P A'$, $\sigma_h P B'$ and in the formulae (5) for the right hand terms resp. A , $B + P \sum \sigma_h \nu_h' / \epsilon_h$, $C + P \sum \sigma_h \delta_h' / \epsilon_h$. Such a series of terms corresponds exactly to the symmetry of the magnetic field and leads to the same formula (6) for n as that given above; only we have $P\sigma_h$ in place of σ_h . By this step both the above mentioned difficulties are removed; however the substitution leads from the sound basis of experience into the region of hypothesis. Observation shows that σ_h increases as the temperature diminishes.

11. In some crystals of rhombic symmetry it has appeared that in each of the three chief spectra certain absorption lines correspond to nearly the same wave length. If we observe in the direction of an axis of symmetry each of these lines is seen broken up into a doublet when the magnetic field is excited. In the present special case of equal wave length of the lines the distance of the doublet components is the same in some cases, in other cases differs. If we let the axes X , Y , Z , be parallel to the axes of the crystal and call R the direction of magnetic force, ω the direction of propagation of the ray, σ the direction of vibration, then the following table gives the result of observation:

$$\begin{array}{l}
R//X, \omega//X \begin{cases} \sigma//Y & \alpha_1 \\ \sigma//Z & \alpha_1 \end{cases} \\
\omega//Y \begin{cases} \sigma//Z & \alpha_1 \\ \sigma//X & \alpha'_1 \end{cases} \\
\omega//Z \begin{cases} \sigma//X & \alpha'_1 \\ \sigma//Y & \alpha_1 \end{cases} \\
R//Y, \omega//Y \begin{cases} \sigma//Y & \alpha'_2 \\ \sigma//Z & \alpha_2 \end{cases} \\
\omega//Y \begin{cases} \sigma//Z & \alpha_2 \\ \sigma//X & \alpha_2 \end{cases} \\
\omega//Z \begin{cases} \sigma//X & \alpha_2 \\ \sigma//Y & \alpha'_2 \end{cases} \\
R//Z, \omega//X \begin{cases} \sigma//Y & \alpha_3 \\ \sigma//Z & \alpha'_3 \end{cases} \\
\omega//Y \begin{cases} \sigma//Z & \alpha'_3 \\ \sigma//X & \alpha_3 \end{cases} \\
\omega//Z \begin{cases} \sigma//X & \alpha_3 \\ \sigma//Y & \alpha_3 \end{cases}
\end{array}$$

The theory I have developed¹⁾ makes the laws of the phenomena under consideration depend on 4×3 parameters $p_h, \overline{p}_h, g_h, f_h$.

p_h and \overline{p}_h ($h = 1, 2, 3$) are determined by the quasi elastic and damping forces acting on the electron in the crystal, when no magnetic field is excited. g_h ($h = 1, 2, 3$) measure the direct action of the magnetic field on the electron parallel to the axes X, Y, Z ; f_h ($h = 1, 2, 3$) determine certain couplings experienced by the electron parallel to these axes.

If we put

$$\begin{aligned}
N_1 &= p_1 \overline{p}_1 - f_1^2 v^2, & N_{2,3} &= p_2 p_3 - g_1^2 v^2, \\
N_2 &= p_2 \overline{p}_2 - f_2^2 v^2, & N_{3,1} &= p_3 p_1 - g_2^2 v^2, \\
N_3 &= p_3 \overline{p}_3 - f_3^2 v^2, & N_{1,2} &= p_1 p_2 - g_3^2 v^2,
\end{aligned}$$

where as before v represents the frequency, generally the magnetic effect on an absorption line in the above mentioned cases is given by the following functions:

¹⁾ W. VOIGT. Gött. Nachr. 28 Juli 1906; Magneto- und Electrooptik. Leipzig 1908, p. 235 u. f.

$$\begin{aligned}
R//X, \omega//X & \begin{cases} \sigma//Y & p_2/N_{23} \\ \sigma//Z & p_3/N_{23} \end{cases} \\
\omega//Y & \begin{cases} \sigma//Z & p_3/N_{23} \\ \sigma//X & (p_1 + \bar{p}_1)/N_1 \end{cases} \\
\omega//Z & \begin{cases} \sigma//X & (p_1 + \bar{p}_1)/N_1 \\ \sigma//Y & p_2/N_{23} \end{cases} \\
R//Y, \omega//X & \begin{cases} \sigma//Y & (p_2 + \bar{p}_2)/N_2 \\ \sigma//Z & p_3/N_{31} \end{cases} \\
\omega//Y & \begin{cases} \sigma//Z & p_3/N_{31} \\ \sigma//X & p_1/N_{31} \end{cases} \\
\omega//Z & \begin{cases} \sigma//X & p_1/N_{31} \\ \sigma//Y & (p_2 + \bar{p}_2)/N_2 \end{cases} \\
R//Z, \omega//X & \begin{cases} \sigma//Y & p_2/N_{12} \\ \sigma//Z & (p_3 + \bar{p}_3)/N_3 \end{cases} \\
\omega//Y & \begin{cases} \sigma//Z & (p_3 + \bar{p}_3)/N_3 \\ \sigma//X & p_1/N_{12} \end{cases} \\
\omega//Z & \begin{cases} \sigma//X & p_1/N_{12} \\ \sigma//Y & p_2/N_{12} \end{cases}
\end{aligned}$$

Supposing now the absorption lines to have the same *positions* in the three chief spectra, then the *real* parts of p_1, p_2, p_3 and also of $\bar{p}_1, \bar{p}_2, \bar{p}_3$ are equal; supposing the *intensities* of the absorption lines to be equal, then the *imaginary* parts are the same. If on the other hand g_1, g_2, g_3 and f_1, f_2, f_3 retain different values from each other, then also N_1, N_2, N_3 and N_{23}, N_{31}, N_{12} have different values from each other. In this case the system of distances between the doublet components obtained from theory agrees exactly with that observed. If the three absorptions are slightly different from each other, then the theoretical distances show small deviations from the mentioned law, which however will scarcely be within the range of perception.

What is interesting in the observations is that if we assume $g_1 = g_2 = g_3$, and therefore $N_{23} = N_{31} = N_{12}$, then theory is not in agreement with experience.

The parameters g_h give the direct influence of the external magnetic field on the vibrating electron. This influence appears then to be different when the field acts along the X -, the Y -, the Z -axis of the crystal. This result seems to me to verify the view I have deduced from other considerations, that the magnetic field inside the molecule, where an electron is in motion, can be very different from that in outside space. Reasoning in this way it is quite natural to imagine

that the field inside a molecule of a *crystal* gains different intensities if an external field of constant intensity acts successively parallel to the axes of the crystal.

It is true that in my former publications I have put $g_1 = g_2 = g_3$, because at that time there was no necessity to introduce more complicated suppositions. But the abovementioned observations show that generally g_1 is different from g_2 and g_3 .

If we do not accept this difference between the inner and outer field, then we must fall back on the very complicated assumptions of BECQUEREL and ONNES, that the apparent, probably electromagnetic mass of the electron is different parallel to the three axes of the crystal, and that the quasielastic forces in these directions are proportional to these masses. This would lead to the hypothesis of an immense number of different kinds of electrons, say of ellipsoidal form, which during their vibration remain parallel to themselves. The authors do not set forth how the law of the quasielastic forces would be explained. As opposed to the difficulties of this hypothesis I maintain that the above assumption of differences between the inside and outside magnetic fields is much simpler.

III. I should like to mention a general consideration arising from the preceding. It seems to me that by the irresponsible introduction of electrons which have forms masses and signs different from each other, we should lose what has been up to the present one of the chief advantages which characterise the electrontheory; the simplicity of the fundamental conception, and thus make the whole hypothesis of less value. We would have to be content with this depreciated value of the hypothesis, if we were compelled by undeniable results of experience to do so. But up to the present I cannot recognise any such evidence.

The chief objection of J. BECQUEREL to the hypothesis of inner-molecular magnetic fields lies in the fact that the ZEEMAN-effect is notably independent of temperature. But as we do not know anything definite about the cause of the inner field, it appears to me, that one cannot assert anything about the sensibility to temperature with certainty.

In this place I shall mention a consideration which seems to me to carry some weight against the precipitate assumption especially of *positive* electrons.

We learn from the theory of light that in bodies electrons oscillate about positions of equilibrium. Further the electron-theory permits only of forces of electromagnetic nature: the quasielastic forces

which bind the electrons to their equilibrium positions must therefore also be of electromagnetic nature. Thus here the only force to be considered, is electrostatic, we know from the theory of potential that an electrostatic field permits of a point-charge moving about a stable position of rest *only inside of a charge of opposite sign distributed over space*. Thus the assumption of *negative electrons* made up to the present time leads to the conception of *positive* electric charges distributed over a definite space. These two hypotheses contain no contradiction.

The assumption of *positive electrons* (of parallel properties to those of the negative) compels then to the conception of *negative* charges extended through space, and thus, as it seems to me, leads to the nullification of the whole theory. For of what use is an atomical conception of electricity which cannot be subsequently worked out?

Geophysics. — “*On Frequencies of the mean daily cloudiness at Batavia.*” By Dr. J. P. VAN DER STOK.

1. Since 1880 hourly observations of the cloudiness of the sky have been published by the Observatory at Batavia; if the daily means calculated from these records are arranged in groups, a frequency-table (Table I) is obtained which enables us to form a clear idea of the way in which the climate is affected by this highly important climatological factor.

From this Table it appears that, whilst northerly climates are characterized by a great number of cases in which the sky is entirely overcast or quite free from clouds (principally in April and September), these extreme values rarely occur at Batavia.

Only once in 26 years or in 9500 cases a serene sky lasting during 24 hours has been recorded and, taken over the whole year, the number of days during which the sky was entirely covered only amount to 1.4 % and, even in full West-Monsoon, to hardly more than $\frac{1}{2}$ %.

Furthermore Table I exhibits the fact that, notwithstanding the great number of records, irregularities still occur to a considerable extent and the sums taken over the whole year clearly demonstrate that extreme care must be taken in adding together frequency-series of different kinds, which may lead to irregularities of the most peculiar description in the curve of distribution; and these irregularities are by no means eliminated by a greater number of data.

In order to eliminate these irregularities three natural groups have