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which bind the electrons to their equilibrium positions must therefore also be of electromagnetic nature. Thus here the only force to be considered, is electrostatic, we know from the theory of potential that an electrostatic field permits of a point-charge moving about a stable position of rest *only inside of a charge of opposite sign distributed over space*. Thus the assumption of *negative electrons* made up to the present time leads to the conception of *positive* electric charges distributed over a definite space. These two hypotheses contain no contradiction.

The assumption of *positive electrons* (of parallel properties to those of the negative) compels then to the conception of *negative* charges extended through space, and thus, as it seems to me, leads to the nullification of the whole theory. For of what use is an atomical conception of electricity which cannot be subsequently worked out?

**Geophysics.** — “*On Frequencies of the mean daily cloudiness at Batavia.*” By Dr. J. P. VAN DER STOK.

1. Since 1880 hourly observations of the cloudiness of the sky have been published by the Observatory at Batavia; if the daily means calculated from these records are arranged in groups, a frequency-table (Table I) is obtained which enables us to form a clear idea of the way in which the climate is affected by this highly important climatological factor.

From this Table it appears that, whilst northerly climates are characterized by a great number of cases in which the sky is entirely overcast or quite free from clouds (principally in April and September), these extreme values rarely occur at Batavia.

Only once in 26 years or in 9500 cases a serene sky lasting during 24 hours has been recorded and, taken over the whole year, the number of days during which the sky was entirely covered only amount to 1.4 % and, even in full West-Monsoon, to hardly more than  $\frac{1}{2}$  %.

Furthermore Table I exhibits the fact that, notwithstanding the great number of records, irregularities still occur to a considerable extent and the sums taken over the whole year clearly demonstrate that extreme care must be taken in adding together frequency-series of different kinds, which may lead to irregularities of the most peculiar description in the curve of distribution; and these irregularities are by no means eliminated by a greater number of data.

In order to eliminate these irregularities three natural groups have

TABLE 1. Frequencies of the mean daily cloudiness as recorded at Batavia 1880—1905.

Cloud- iness	Jan.	Febr.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
0	—	—	—	—	—	—	—	1	—	—	—	—	1
0.5	—	—	—	—	3	10	4	4	2	2	—	—	25
1	1	—	1	2	10	11	18	19	9	9	1	—	81
1.5	2	2	2	6	10	26	39	33	21	14	4	—	164
2	5	4	1	7	36	39	52	59	37	31	9	4	284
2.5	2	7	13	25	34	52	52	75	49	36	13	4	362
3	7	7	15	35	53	52	69	84	69	44	21	5	461
3.5	5	10	30	49	70	54	<b>78</b>	<b>85</b>	88	52	30	13	564
4	9	10	33	45	70	66	71	76	<b>89</b>	58	33	24	584
4.5	29	24	40	59	<b>80</b>	<b>68</b>	62	74	78	<b>84</b>	37	31	666
5	30	23	39	64	80	55	80	58	57	63	48	33	650
5.5	39	25	70	76	60	58	56	42	62	60	62	53	663
6	51	34	73	79	58	49	46	52	38	58	61	74	674
6.5	68	69	68	<b>88</b>	62	44	45	32	47	60	<b>79</b>	53	715
7	61	77	<b>93</b>	67	43	43	29	42	38	56	64	64	677
7.5	78	86	84	50	50	45	24	33	33	56	76	93	708
8	100	81	77	51	39	42	23	12	19	31	78	<b>102</b>	655
8.5	<b>101</b>	90	66	36	24	33	28	13	16	40	62	94	603
9	90	<b>94</b>	51	29	10	16	18	7	21	32	52	67	487
9.5	93	66	39	10	8	14	6	3	6	14	<b>38</b>	62	359
10	35	25	11	2	1	3	6	2	1	6	11	30	133
Total	806	734	806	780	806	780	806	806	780	806	780	806	9496

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been formed as shown in Table II; in these series irregularities still occur, but by far the greater part have disappeared and they furnish good material for the application and the testing of frequency-formulae. For this purpose two alterations have to be made: in the first place the scale value has to be altered so that the extreme limits are denoted, not by 0 and 10, but by  $\pm 1$ , and consequently the origin of coordinates must be chosen in the middle between the limits;

TABLE II. Frequencies of the mean daily cloudiness, Batavia, 1880—1905

Cloud- iness	I East Monsoon	II Trans- ition	III West- Monsoon	I East Monsoon	II Trans- ition	III West- Monsoon	New scale values
0	1	—	—	—	—	—	—1.0
0.5	23	2	—	6	1	—	—0.9
1	67	13	1	17	4	—	—0.8
1.5	134	26	4	34	8	2	—0.7
2	223	48	13	56	15	6	—0.6
2.5	262	87	13	66	27	6	—0.5
3	327	115	19	82	36	8	—0.4
3.5	375	161	28	94	51	12	—0.3
4	372	169	43	93	53	18	—0.2
4.5	362	220	84	91	69	36	—0.1
5	330	214	86	83	68	37	0.0
5.5	278	268	117	70	84	50	0.1
6	243	272	159	61	86	68	0.2
6.5	230	295	190	58	93	81	0.3
7	195	280	202	49	88	86	0.4
7.5	185	266	257	47	84	110	0.5
8	135	237	283	34	75	120	0.6
8.5	114	204	285	29	64	121	0.7
9	72	164	251	18	52	107	0.8
9.5	37	101	221	9	32	94	0.9
10	13	30	9	3	10	38	1.0
Total	3978	3172	2346	1000	1000	1000	

this may be done by simply assuming the denotation as given in the last column of Table II.

In the second place the frequencies corresponding to the extreme limits must be excluded from the calculation because they must be considered as representing peculiar meteorological conditions and also because they cannot be taken as representing average values between  $\mp 1.05$  and  $\mp 0.95$  in the same sense as any other group.

Therefore, in calculating the constants of frequency-formulae, these extreme values are omitted and the remaining frequencies again reduced to a total of 1000.

2. In the first place the frequency-formula known as Type I of Prof. PEARSON's formulæ finds an application.

$$u = \mathfrak{A} \left(1 + \frac{x}{p}\right)^a \left(1 - \frac{x}{q}\right)^b$$

which, for the assumed conditions and choice of origin, takes the simple form :

$$u = \mathfrak{A} (1 + x)^a (1 - x)^b \dots \dots \dots (1)$$

and can be regarded as a generalization of the condition that the function vanishes for  $x = \pm 1$ .

Its constants may be calculated from the following relations.

$$\mathfrak{A} = \frac{1}{2^{a+b+1}} \frac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)} \dots \dots \dots (2)$$

$$\frac{(\mu+1)^{(n)}}{(\mu+1)^{(n-1)}} = \frac{2(a+n)}{a+b+n+1} \dots \dots \dots (3)$$

where

$$(\mu+1)^{(n)} \mu_n + n \mu_{n-1} + \frac{n(n-1)}{2!} \mu_{n-2} + \text{etc.}$$

and  $\mu_n$  represents the mean of the  $n^{\text{th}}$  order.

As, besides  $\mathfrak{A}$ , by which the area of the curve is defined as equal to unity, only two constants appear in formula (1) as characteristics of the curve, it is sufficient to calculate the means of the first and second order  $\mu_1$  and  $\mu_2$ .

Putting

$$p = 1 - \mu_1, \quad q = \frac{1 - \mu_2}{1 + \mu_1},$$

we find:

$$a + 1 = \frac{(2-p)q}{2(p-q)}, \quad b + 1 = \frac{pq}{2(p-q)} \dots \dots \dots (4)$$

This formula offers the advantage that, the constants  $a$  and  $b$  being known, a simple expression can be given for the situation of the maximum-value.

$$x_m = \frac{a-b}{a+b}.$$

In the second place we have to consider the expression in series-form proposed by the author in an earlier publication<sup>1)</sup>, which may be regarded as a generalized zonal function, modified according to the condition :

<sup>1)</sup> These Proceedings Vol. X. (799-817).

$u = 0$  for  $x = \pm 1$ .

$$u = \Sigma A'_n R_{n+2} \dots \dots \dots (5)$$

$$R_{n+2} = (x^2 - 1) R'_n = (x^2 - 1) \left[ x^n - \frac{n(n-1)}{2 \cdot (2n+1)} x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2 \cdot 4 \cdot (2n+1)(2n-1)} x^{n-4} - \text{etc.} \right]$$

$$A'_n = \beta \left[ \mu_n - \frac{n(n-1)}{2 \cdot (2n+1)} \mu_{n-2} + \frac{n(n-1)(n-2)(n-3)}{2 \cdot 4 \cdot (2n+1)(2n-1)} \mu_{n-4} - \text{etc.} \right] \quad (6)$$

$$\beta = - \frac{(2n+3)(2n+1)!(2n+1)!}{2^{2n+1}(n+2)!n!n!} \dots \dots \dots (7)$$

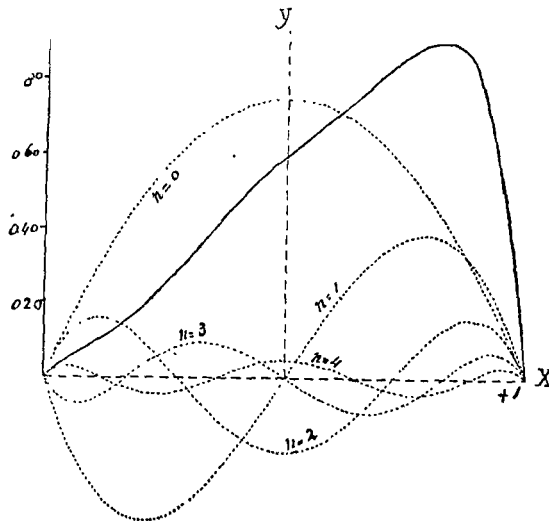
$$\mu_n = \int_{-1}^{+1} u x^n dx.$$

The use of the proposed series enables us to introduce more constants than two, which, in this case, is a decided advantage as the means of higher order necessarily decrease and, therefore, the convergency is assured.

For the position of the maximum-value however no definite expression can be derived from these formulae, and it has to be determined by approximative methods.

Values of the function  $R_{n+2}$  for  $n=0$  to  $n=4$  have been calculated and are given in Table X; for the first term, which remains the same for all curves,  $A_0 R_2$  has been given instead of  $R_2$ .

The figure represents the way in which a frequency-curve (full



line) according to this formula is constructed out of its constituents (broken line) for the case:

$$A_1 = A_2 = A_3 = A_4 = 1,$$

so that the ordinates of the curve are found simply by taking together the 5 columns of Table X.

3. The following constants of formulae (1) and (5) are deduced from the data given in Table II (2<sup>nd</sup> part), the frequencies for a cloudiness 0 and 10 being omitted and the total reduced to 1000. For reasons to be given furtheron, the values of the  $A_n$  constants are not quite in conformity with the expression (6), the sign being inverted and the values divided by  $n + 3$ , so that;

$$(n + 3) A_n = - A'_n$$

I. East-Monsoon.

$\mu_1 = -0.0554$	$A_1 = -0.0519$	$\mathfrak{A} = 0.8416$
$\mu_2 = +0.1690$	$A_2 = -0.1017$	$a = 1.3654$
$\mu_3 = -0.0095$	$A_3 = +0.1631$	$b = 1.6428$
$\mu_4 = +0.0631$	$A_4 = -0.0650$	

II. Months of Transition.

$\mu_1 = 0.2136$	$A_1 = +0.2003$	$\mathfrak{A} = 0.7486$
$\mu_2 = 0.1999$	$A_2 = -0.0003$	$a = 2.1470$
$\mu_3 = 0.0921$	$A_3 = +0.0069$	$b = 1.0393$
$\mu_4 = 0.0859$	$A_4 = +0.0081$	

III. West-Monsoon.

$\mu_1 = 0.4545$	$A_1 = +0.4261$	$\mathfrak{A} = 0.4326$
$\mu_2 = 0.3191$	$A_2 = +0.3901$	$a = 3.4001$
$\mu_3 = 0.2173$	$A_3 = +0.2584$	$b = 0.6502$
$\mu_4 = 0.1683$	$A_4 = +0.1299$	

In the constants of either formula the differences characteristic for the different seasons are well marked.

4. In order to examine in how far the results of the computation by means of the frequency-formulae agree with the data, we have to integrate the expressions between the limits  $x$  and  $-1$ .

For the formula (5) in seriesform this offers no difficulty; from the differential equation:

$$(x^2 - 1) \frac{d^2 R_{n+2}}{dx^2} = (n + 2)(n + 1) R_{n+2}$$

we readily find:

$$I'_n = \int_{-1}^x R_{n+2} dx = \frac{(x^2-1)^2}{n+3} \cdot \frac{1}{n} \frac{dR'_n}{dx} \dots \dots (8)$$

from which, the  $R'_n$  function being known, we easily derive the expressions (10). Formula (8) holds good for all values of  $n$  except  $n = 0$ , in which case:

$$I'_0 = \frac{x^3-3x-2}{3} \dots \dots \dots (9)$$

The  $A_n$  coefficients, calculated by formula (6) being comparatively large and the values computed by (8) small, it is desirable to omit the factor  $(n+3)$  in (8) and (9) and to divide the expression for  $A_n$  by the same quantity, at the same time the sign of  $\beta$ , and therefore also the signs of (8) and (9), can be changed.

With these premises the integrals assume the form:

$$\left. \begin{aligned} I_0 &= x(1-x^2) + 2(1+x) \\ I_1 &= -(1-x)^2 \\ I_2 &= I_1 x \\ I_3 &= I_1 \left( x^2 - \frac{1}{7} \right) \\ \cdot \\ I_4 &= I_1 x \left( x^2 - \frac{1}{3} \right) = I_2 \left( x^2 - \frac{1}{3} \right) \end{aligned} \right\} \dots \dots (10)$$

Numerical values of these integrals calculated for values of  $x$  increasing by 0.05 are given in Table IX; in the first column, which remains the same for curves of different description, instead of  $I$ , the product  $A_0 I_0$  has been given.

Integrating formula (1) between the limits  $x$  and  $-1$ , we encounter the difficulty that, putting:

$$x = 2z - 1$$

and developing, we obtain the form:

$$-1 \int^x u dx = \frac{a+b+1}{2} \mathfrak{U} \left[ \frac{z^{a+1}}{a+1} - b \frac{z^{a+2}}{a+2} + \frac{b(b-1)}{2!} \frac{z^{a+3}}{a+3} - \text{etc.} \right]$$

which, evidently, for large values of  $z$  slowly converges, so that a considerable number of terms have to be taken into account if, as is necessary in our case, we desire to determine the values with an accuracy up to the third decimal.

Other forms of development are of course possible, but I have not succeeded in finding less laborious expressions.



5. As, owing to this difficulty, the computations necessary for the testing of the formulae had to be restricted, the frequencies of Table II have been aggregated between wider limits: Table III exhibits these frequencies reduced to a total of 1000; in the first part for all data, in the second part with the omission of frequencies for a cloudiness 0 and 10. It is this series which has to be compared with the results of the calculation.

TABLE III.

	I	II	III	I	II	III
-1.00 tot -0.75	23	5	0	23	5	0
-0.75 " -0.55	90	23	8	90	23	8
-0.55 " -0.35	148	63	14	149	64	15
-0.35 " -0.15	187	104	30	188	105	31
-0.15 " 0.05	174	137	73	175	138	76
0.05 " 0.25	131	170	118	131	172	123
0.25 " 0.45	107	181	167	107	183	174
0.45 " 0.65	81	159	230	81	161	239
0.65 " 0.85	47	116	228	47	117	237
0.85 " 1.00	12	42	132	9	32	97

The results of the computation according to formula (5) are given in Table IV.

TABLE IV. I. East-Monsoon.

	$A_1I_1$	$A_2I_2$	$A_3I_3$	$A_4I_4$	$A_0I_0$ + total
-1 00 tot -0.75	0.0099	-0.0146	-0.0131	-0.0021	0 0231
-0.75 " -0.55	0.0153	-0.0126	0 0004	0.0026	0.0917
-0.55 " -0 35	0.0147	-0.0002	0.0151	0 0032	0 1520
-0.35 " -0 15	0.0096	0.0128	0.0164	-0 0008	0 1781
-0 15 " +0.05	0 0021	0 0196	0 0040	-0.0040	0.1708
+0 05 " +0 25	-0 0060	0.0173	-0.0113	-0.0028	0.1433
+0 25 " +0.45	-0.0126	0.0068	-0.0177	0 0015	0.1091
+0 45 " +0.65	-0.0157	-0.0071	-0.0090	0.0037	0.0760
+0 65 " +0 85	-0 0133	-0 0154	0 0079	0.0004	0 0447
+0 85 " +1.00	-0.0040	-0.0067	0 0073	-0 0017	0.0109

TABLE IV. II. Transition.

	$A_1I_1$	$A_2I_2$	$A_3I_3$	$A_4I_4$	$A_1I_1$ + total
-1.00 tot -0.75	-0.0383	0.0000	-0.0006	0.0003	0.0044
-0.75 " -0.55	-0.0591	0.0000	0.0000	-0.0003	0.0267
-0.55 " -0.35	-0.0568	0.0000	0.0006	-0.0004	0.0625
-0.35 " -0.15	-0.0372	0.0000	0.0007	0.0001	0.1037
-0.15 " +0.05	-0.0079	0.0001	0.0002	0.0005	0.1420
+0.05 " +0.25	0.0233	0.0001	-0.0005	0.0003	0.1693
+0.25 " +0.45	0.0487	0.0000	-0.0007	-0.0002	0.1789
+0.45 " +0.65	0.0606	0.0000	-0.0004	-0.0005	0.1638
+0.65 " +0.85	0.0514	0.0000	0.0003	0.0000	0.1168
+0.85 " +1.00	0.0154	0.0000	0.0003	0.0002	0.0319

TABLE IV. III. West-Monsoon.

	$A_1I_1$	$A_2I_2$	$A_3I_3$	$A_4I_4$	$A_1I_1$ + total
-1.00 tot -0.75	-0.0816	0.0561	-0.0208	0.0043	0.0010
-0.75 " -0.55	-0.1257	0.0484	0.0007	-0.0053	0.0042
-0.55 " -0.35	-0.1208	0.0007	0.0241	-0.0063	0.0168
-0.35 " -0.15	-0.0790	-0.0493	0.0257	0.0016	0.0391
-0.15 " +0.05	-0.0168	-0.0754	0.0064	0.0079	0.0712
+0.05 " +0.25	0.0495	-0.0664	-0.0179	0.0056	0.1169
+0.25 " +0.45	0.1035	-0.0260	-0.0281	-0.0029	0.1776
+0.45 " +0.65	0.1289	0.0271	-0.0143	-0.0074	0.2384
+0.65 " +0.85	0.1093	0.0591	0.0126	-0.0008	0.2453
+0.85 " +1.00	0.0328	0.0256	0.0115	0.0033	0.0892

Table V shows the differences between the results of the observation  $O$  (Table III 2<sup>nd</sup> part) and of the computation  $C$  (Table IV last column).

TABLE V.  $\Delta = O - C$ , formula in seriesform.

	I. East-Monsoon	II. Transition	III. West-Monsoon.
-1 00 tot -0.75	0	1	-1
-0.75 „ -0 55	-2	-4	4
-0.55 „ -0.35	-3	1	-2
-0.35 „ -0 15	10	1	-8
-0.15 „ 0.05	4	-4	5
0.05 „ 0.25	-12	3	6
0 25 „ 0 45	-2	4	-4
0.45 „ 0.65	5	-3	1
0 65 „ 0 85	2	0	-8
0.85 „ 1 00	-2	0	8
$\sqrt{\frac{\Delta^2}{n}}$	5 57	2 63	5.39

The question whether or not PEARSON'S formula (1) leads to an equally satisfactory result can be readily answered by supposing formula (1) to be expanded according to formula (5). Then, of course, the coefficients  $A_1$  and  $A_2$  will be the same whether calculated directly or with the help of  $a$  and  $b$  and, as the fourth term plays an unimportant part, all depends upon the third term which is determined by the mean of the third order  $\mu_3$ .

If by means of (3) we calculate  $\mu_3$  from  $a$  and  $b$ , we find:

	observed	calculated	observed	calculated
		by (3)		by (3)
East-Monsoon	$\mu_3 = -0.0095$	-0.0225	$\mu_4 = 0.0631$	0.0641
Transition	„ = 0.0921	0.0902	„ = 0.0859	0.0856
West-Monsoon	„ = 0.2173	0.3219	„ = 0.1683	0.1917

From this we may conclude that, for the months of transition formula (1) will lead to excellent results; for the West-Monsoon the agreement will be satisfactory; but for the East-Monsoon rather large discrepancies are to be expected.

TABLE VI.  $\Delta = O - C$ , formula Pearson.

	I East-Monsoon	II Transition	III West-Monsoon
-1.00 tot -0.75	-13	- 1	0
- 0.75 " -0.55	- 1	- 4	4
-0.55 " -0.35	12	3	0
-0.35 " -0.15	25	3	7
-0.15 " 0.05	6	- 2	0
0.05 " 0.25	-25	2	- 2
0.25 " 0.45	-18	2	- 7
0.45 " 0.65	- 1	- 5	12
0.65 " 0.85	10	1	4
0.85 " 1.00	5	1	- 4
$\sqrt{\frac{\Delta^2}{n}}$	14.32	2.72	5.42

The differences given in Table VI confirm this expectation as well as the computation of the position of the maximum-value.

## Position of maximum-values.

	Observed	Calculated	
	Table II.	form. PEARSON	form. series
East-Monsoon	-0.25	-0.09	-0.20
Transition	0.30	0.35	0.35
West-Monsoon	0.65	0.90	0.65

The use of frequency-formulae with two constants will, therefore, give satisfactory results only in those cases where the curve is of a comparatively simple construction or if, by chance,  $\mu_3$  (or  $\Delta_3$ ) as calculated by means of  $a$  and  $b$ , are approximately equal to the observed values.

Moreover, as the constants appear in exponential form, it will be difficult to calculate Tables for the function and its integrals as  $p$  and  $q$  may assume all possible values and occur in an infinite number of combinations.

Constants for every month computed according to both formulae from the data of Table I with the omission of the extreme values are given in Tables VII and VIII.

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TABLE VII.  
Constants of formula (5).

	$A_1$	$A_2$	$A_3$
January	0 4293	0.4418	0.3514
February	0 4247	0 3957	0.2653
March	0.2962	0.0847	-0.0207
April	0 1481	-001385	-0.0792
May	0.0114	-0.1572	0.0609
June	-0.0003	-0 1939	0.0988
July	-0 0672	-0.0758	0 1769
August	-0.1330	-0.1093	0.2607
September	-0.0548	-0 1509	0.2363
October	0.0724	-0.0735	0.0885
November	0.2747	0.0961	0 0425
December	0.3795	0.2530	0.1355

TABLE VIII.  
Constants of formula Pearson, Type I.

	$\eta$	$a$	$b$
January	0.420	3.242	0.577
February	0 436	3 278	0 611
March	0.656	3.041	1.101
April	0.880	2 671	1 670
May	0.922	1.824	1.756
June	0 924	2 049	2.050
July	0.798	1.225	1 568
August	0.831	1 439	2.246
September	0 894	1 645	1.974
October	0.813	1.573	1.204
November	0.664	2.475	0.900
December	0.534	3.484	0.900

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TABLE IX. Values of  $I_n$ .

$x$	$A_0 I_0$	$n=1$	$n=2$	$n=3$	$n=4$
-1.00	0.0000	0.0000	0.0000	0.0000	0.0000
-0.95	0.0018	-0.0095	0.0090	-0.0072	0.0051
-0.90	0.0073	-0.0361	0.0325	-0.0241	0.0155
-0.85	0.0160	-0.0770	0.0655	-0.0446	0.0255
-0.80	0.0280	-0.1296	0.1037	-0.0644	0.0318
-0.75	0.0430	-0.1914	0.1436	-0.0803	0.0329
-0.70	0.0608	-0.2651	0.1821	-0.0903	0.0285
-0.65	0.0812	-0.3335	0.2168	-0.0933	0.0193
-0.60	0.1040	-0.4096	0.2458	-0.0889	0.0066
-0.55	0.1291	-0.4865	0.2676	-0.0777	-0.0082
-0.50	0.1563	-0.5625	0.2813	-0.0602	-0.0234
-0.45	0.1853	-0.6360	0.2862	-0.0379	-0.0374
-0.40	0.2160	-0.7056	0.2822	-0.0121	-0.0489
-0.35	0.2482	-0.7700	0.2695	0.0147	-0.0568
-0.30	0.2818	-0.8281	0.2484	0.0438	-0.0604
-0.25	0.3164	-0.8789	0.2197	0.0707	-0.0595
-0.20	0.3520	-0.9216	0.1843	0.0948	-0.0541
-0.15	0.3884	-0.9555	0.1433	0.1150	-0.0445
-0.10	0.4253	-0.9801	0.0980	0.1303	-0.0317
-0.05	0.4625	-0.9950	0.0498	0.1397	-0.0165
0.00	0.5000	-1.0000	0.0000	0.1429	0.0000
0.05	0.5375	-0.9950	-0.0498	0.1397	0.0165
0.10	0.5748	-0.9801	-0.0980	0.1303	0.0317
0.15	0.6117	-0.9555	-0.1433	0.1150	0.0445
0.20	0.6480	-0.9216	-0.1843	0.0948	0.0541
0.25	0.6836	-0.8789	-0.2167	0.0707	0.0595
0.30	0.7183	-0.8281	-0.2485	0.0438	0.0604
0.35	0.7518	-0.7700	-0.2695	0.0147	0.0568
0.40	0.7840	-0.7056	-0.2822	-0.0121	0.0489
0.45	0.8147	-0.6360	-0.2862	-0.0379	0.0374
0.50	0.8438	-0.5625	-0.2813	-0.0602	0.0234
0.55	0.8709	-0.4865	-0.2676	-0.0777	0.0082
0.60	0.8960	-0.4096	-0.2458	-0.0889	-0.0066
0.65	0.9189	-0.3335	-0.2168	-0.0933	-0.0193
0.70	0.9393	-0.2651	-0.1821	-0.0903	-0.0285
0.75	0.9570	-0.1914	-0.1436	-0.0803	-0.0329
0.80	0.9720	-0.1296	-0.1037	-0.0644	-0.0318
0.85	0.9840	-0.0770	-0.0655	-0.0446	-0.0255
0.90	0.9928	-0.0361	-0.0325	-0.0241	-0.0155
0.95	0.9982	-0.0095	-0.0090	-0.0072	-0.0051
1.00	1.0000	0.0000	0.0000	0.0000	0.0000

TABLE X. Values of the function  $R_{n+2}$ .

$x$	$A_n R_2$	$n=1$	$n=2$	$n=3$	$n=4$
1.00	0.0000	0.0000	0.0000	0.0000	0.0000
-0.95	0.0731	-0.0926	0.0685	-0.0439	0.0254
-0.90	0.1425	-0.1710	0.1159	-0.0652	0.0311
-0.85	0.2081	-0.2359	0.1450	-0.0693	0.0244
-0.80	0.2700	-0.2880	0.1584	-0.0609	0.0110
-0.75	0.3281	-0.3281	0.1586	-0.0439	-0.0048
-0.70	0.3825	-0.3570	0.1479	-0.0219	-0.0199
-0.65	0.4331	-0.3754	0.1285	0.0023	-0.0321
-0.60	0.4800	-0.3840	0.1024	0.0263	-0.0402
-0.55	0.5231	-0.3836	0.0715	0.0484	-0.0436
-0.50	0.5625	-0.3750	0.0375	0.0670	-0.0424
-0.45	0.5981	-0.3589	0.0020	0.0811	-0.0370
-0.40	0.6300	-0.3360	-0.036	0.0902	-0.0281
-0.35	0.6581	-0.3071	-0.0680	0.0940	-0.0167
-0.30	0.6825	-0.2730	-0.1001	0.0924	-0.0039
-0.25	0.7031	-0.2344	-0.1289	0.0858	0.0092
-0.20	0.7200	-0.1920	-0.1536	0.0746	0.0217
-0.15	0.7331	-0.1466	-0.1735	0.0595	0.0324
-0.10	0.7425	-0.0990	-0.1881	0.0414	0.0406
-0.05	0.7481	-0.0499	-0.1970	0.0213	0.0458
0.00	0.7500	0.0000	-0.2000	0.0000	0.0476
0.05	0.7481	0.0499	-0.1970	-0.0213	0.0458
0.10	0.7425	0.0990	-0.1881	-0.0414	0.0406
0.15	0.7331	0.1467	-0.1735	-0.0595	0.0324
0.20	0.7200	0.1920	-0.1536	-0.0746	0.0217
0.25	0.7031	0.2344	-0.1289	-0.0858	0.0092
0.30	0.6825	0.2730	-0.1001	-0.0924	-0.0039
0.35	0.6581	0.3071	-0.0680	-0.0940	-0.0167
0.40	0.6300	0.3360	-0.0336	-0.0902	-0.0281
0.45	0.5981	0.3589	0.0020	-0.0811	-0.0370
0.50	0.5625	0.3750	0.0375	-0.0670	-0.0424
0.55	0.5231	0.3836	0.0715	-0.0484	-0.0436
0.60	0.4800	0.3840	0.1024	-0.0263	-0.0402
0.65	0.4331	0.3754	0.1285	-0.0023	-0.0321
0.70	0.3825	0.3570	0.1479	0.0219	-0.0199
0.75	0.3281	0.3281	0.1586	0.0439	-0.0048
0.80	0.2700	0.2880	0.1584	0.0609	0.0110
0.85	0.2081	0.2359	0.1450	0.0693	0.0244
0.90	0.1425	0.1710	0.1159	0.0652	0.0311
0.95	0.0731	0.0926	0.0685	0.0439	0.0254
1.00	0.0000	0.0000	0.0000	0.0000	0.0000