where the elements of the last row are respectively in the four cases :

$$6(b_4 b_2'),\ 12 b_1 b_2,\ -(b_0 b_4 + 2 b_2 m),\ 2 b_1 b_4,\ b_4 m,\ -2 b_3 b_4,\ b_4^2,\ 0$$

$$6(b_3 b_2'),\ 3 b_0 b_2,\ -b_0 b_3,\ 2 b_1 b_3 - b_2 m,\ b_3 m,\ -2 b_3^2,\ b_3 b_4,\ 0$$

$$6(b_1 b_2'),\ 0,\ -b_0 b_1,\ 2 b_1^2,\ b_1(m - 6 b_2),\ b_2 m - 2 b_1 b_3,\ b_1 b_4,\ -3 b_2 b_4$$

$$6(b_0 b_2'),\ 0,\ -b_0^2,\ 2 b_0 b_1,\ b_0(m - 6 b_2),\ -2 b_0 b_3,\ b_0 b_4 + 2 b_2 m,\ -12 b_2 b_3$$

6. Following the same way in the general case, we obtain for μ_0' the quotient of two determinants each of order $2n+1$. If we reduce these as before, the denominator will be seen to be independent of λ and μ; and the numerator will only contain the quantities $\mu_{n-1}, \mu_{n-2} \cdots \mu_1, \mu_0$ in two columns. Now $\mu_{n-1}, \mu_{n-2}, \cdots \mu_1$ may be expressed as linear functions of μ_0, and this proves at once that the numerator must be a polynomial of the second degree in μ_0. If, therefore the necessary conditions are satisfied, the quantity μ_0 is an integral of an equation of RICCATI. The substitution which reduces the given differential equation to this equation of RICCATI will then be found from

$$y^n + \mu_{n-1}\, y^{n-1} + \ldots \mu_1\, y + \mu_0 = 0$$

by determining $\mu_{n-1}, \ldots \mu_1$ in function of μ_0 and expressing μ_0 in function of y.

Physics. — "*The law of shift of the central component of a triplet in a magnetic field.*" By Prof. P. ZEEMAN.

In two communications to this Academy [1]) on "Change of wavelength of the middle line of triplets" I gave conclusive evidence obtained by means of MICHELSON's echelon-spectroscope that the central line of some triplets is shifted. The fact of this displacement was established simultaneously with my own observations by GMELIN [2]) and JACK [3]). GMELIN first gave the law of shift in the case of the mercury line 5791. According to him the change of wavelength under consideration is proportional to the square of the magnetic force.

In the second part of a former paper on "Magnetic resolution of spectral lines and magnetic force" measurements concerning the asymmetrical resolution of the mercury line 5791 are given [4]).

1) P. ZEEMAN. These Procedings February 1908, April 1908.
2) GMELIN. Physikalische Zeitschrift. 9. Jahrgang S. 212—214. 1908.
3) JACK see VOIGT. Magneto-optik. S. 178.
4) ZEEMAN. These Proceedings November 1907.

Supposing that the asymmetry of the separation is entirely due to the shift of the central line towards the red, one should conclude from the communicated numbers that the displacement increases nearly linearly with the strength of field. This investigation was made with ROWLAND's grating, the principal object in view being to prove the existence of asymmetrical separations. I succeeded in this respect, but I think now I have overrated the accuracy of the extremely difficult determinations of the amount of the asymmetry. In fields of the order of 20000 gauss the asymmetry is 35 thousandth parts of an Ångström unit, while the ROWLAND grating used permits in the chosen, first order to resolve lines, the difference of whose wavelengths is 0.12 Å.U., hence with the field intensities mentioned we have to do with a quantity which is already four times smaller than the limit imposed by the resolving power.

It is only because we have to do in determining the asymmetry with a difference of two quantities which are above the limit set by the resolving power, that there may be question of measurement.

When we reach however the utmost limits of the method used then sources of error come to the front, which partly are caused by our mode of appreciation of the distance of two adjacent lines, partly are connected with particularities in the formation of images by gratings, not yet sufficiently understood.

It is therefore undoubtedly to be preferred to use for the further investigation of the shift of the central line a method warranting greater resolving power. GMELIN in his investigation has used MICHELSON's echelon grating, and it seems that he has largely succeeded by systematic procedure to interprete quantitatively the results given by this instrument. His result therefore possesses high probability and moreover is now supported by the theory given by VOIGT [1]) in order to explain the large asymmetrical separations, a theory which assumes the existence of couplings between the electrons.

I thought it however to be worth while to investigate the matter by a method independent of ROWLAND's and MICHELSON's apparatus. FABRY and PEROT's method seemed most appropriate. The greater part of the measurements communicated in this paper have been obtained with a 5 m.m. étalon, already used on a former occasion. Some determinations were made with an étalon with distance-pieces of *invar* as suggested by FABRY and PEROT in order to diminish the dependence upon temperature. It was constructed for me by JOBIN.

1) VOIGT. Magneto-optik. S. 261.

The thickness of the air-layer in this étalon was nearly 25 m.m. With this distance and using the light of the mercury line 5790 in the magnetic field the limit of the method is being rapidly approached. Hence the accuracy of the results obtained with the 25 m.m. étalon is in our case hardly superior to that to be reached with the 5 m.m. apparatus.

The arrangement of the apparatus was described with sufficient detail on a former occasion [1]). For the purpose now in view it was desirable to investigate exclusively the vibrations parallel to the magnetic force. A calcspar-rhomb therefore was placed between the source of light and the first lens. Two images of the radiating vacuum-tube are now obtained near together on the étalon, the non-desired one being screened off. A photograph was taken with the field on, and before and afterwards one with the field off.

Besides the inner ring, always also the second ring, in some cases also the third and fourth one, was measured and the result used in the wave-length calculation.

The formula for the calculation is the one first given by FABRY and PEROT, still remarkably simplified in our case [2]).

In the following table the results are given relating to the mercury line 5791. The first column contains the number of the experiment, the second one the reference-number of the spectrogram; $\Delta\lambda_0$ is the change of wavelength of the central component. The field intensities are given in the last column. Their *relative* values, which are only necessary for establishing the law connecting displacement and strength of field, are exact. These numbers must be increased with 1 or 2 °/₀ in order to reduce them to gausses.

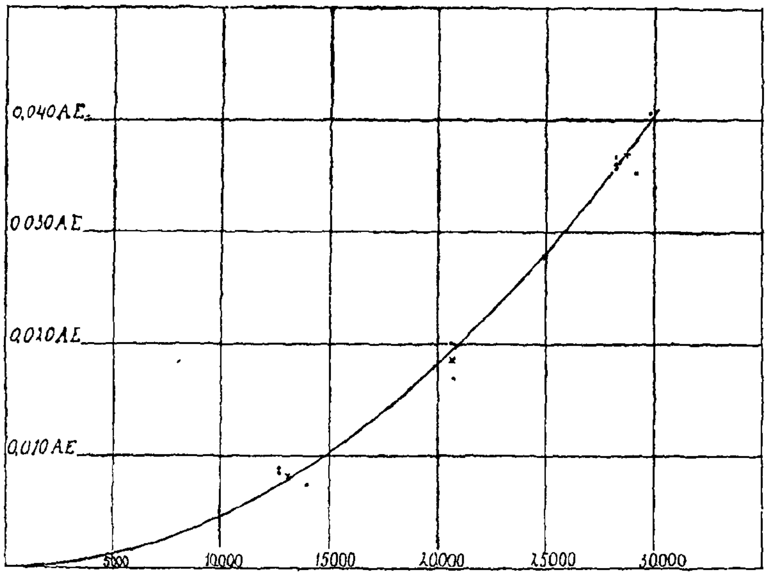
Experiment	Plate n°.	$\Delta\lambda_0$ in A. U.	H.
1	208^c	0.0085	12700
2	209^b	0.0088	12700
3	211	0.0169	20700
4	212^c	0.0074	13950
5	214^c	0.0201	20600
6	218^b	0.0367	28250
7	218^d	0.0358	28250
8	219^b	0.0360	28250
9	220^b	0.0353	29170
10	220^d	0.0406	29780

[1]) ZEEMAN. These Procee dings December 1907.

[2]) See These Proceedings December 1907, February 1908.

The experiments 4 and 5 are made with the 25 m.m. étalon, the other ones with the 5 mm. apparatus. In the figure the results are graphed. The smallness of the displacements may be illustrated by the statement, that the outer components of the triplet 5791 are separated 0.500 A.U. from the unmodified position in a field of 29750 Gauss. The ordinate measuring 0.500 A.U. would be 75 cm. in the figure.

The results 1, 2 and 4; 3 and 5; 6, 7, 8, 9, 10 were combined in each case by assigning simply to each mean displacement the mean magnetic intensity. The three principal values, thus obtained are indicated by crosses. These points and the origin lie very approximately on a parabola.



Inspection of the figure or a simple calculation easily shows that the quadratic law is obeyed within the limits of the errors of observation of the measured displacements. The magnitude of the displacement has been measured in the average in each of the ten points to within 0.002 or 0.003 Å.U.

In order to show how the values of $\Delta\lambda_0$ were obtained, I will give the calculation of one case in full.

$$\lambda_m - \lambda_0 = \frac{\lambda_0}{8R^2}(x_0^2 - x_m^2)$$

$$\lambda_0 = 5791 \text{ Å. E. } \quad H = 12700$$

$$\text{Etalon } 2\,d = 10\, m.m. \quad R = 120\, m.m.$$

x_0, x_m diameters of the rings in *m.m.*
x_0 mean of 2 diameters on plates taken before and after x_m.
First ring:

$x_0 = 3.662$ $\quad x_0^2 = 13.410$
$x_m = 3.640$ $\quad x_m^2 = 13\ 250$ $\quad$ 0.160

Second ring: $\quad$ 0.171

$x_0 = 2.608$ $\quad x_0^2 = 6.802$
$x_m = 2.573$ $\quad x_m^2 = 6\ 620$ $\quad$ 0.182

$$\Delta \lambda_0 = \frac{0.171 \lambda_0}{8 R^2} = 0.0086 \text{ Å. E.}$$

In the case of the triplet of the mercury line 5770 no displacement of the central line could be found. In a field of 28250 the following values of the diameters were obtained with the 5 m.m. étalon:

First ring	Second ring	
2.199	3.409	field off.
2.193	3.408	field on.
2.199	3.394	field off.

Hence the central line of 5770 remains within the limits of experimental error exactly in the position of the unmodified one.

Physics. — "*Contribution to the theory of binary mixtures,*" XII. (Continued). By Prof. J. D. VAN DER WAALS.

In the discussion in the preceding contribution on the question whether there is any possibility that values of $v > b_2$ might occur in the case that the locus of the points of intersection of the curves $\frac{d^2\psi}{dx^2} = 0$ and $\frac{d^2\psi}{dv^2} = 0$ is a closed curve, we have also discussed (p. 433) the case that (φ''') or:

$$n - 1 - n \sqrt{\left\{A - x\frac{dA}{dx}\right\}} \mp \sqrt{\left\{A + (1-x)\frac{dA}{dx}\right\}} = 0$$

would be imaginary over the full width from $x = 0$ to $x = 1$. We have reduced this equation there to the following form:

$$n - 1 - nx \sqrt{\frac{c}{a}\,\frac{a_2 - c(1-x)^2}{a}} \mp (1-x)\sqrt{\frac{c}{a}\,\frac{a_1 - cx^2}{a}} = 0$$

and shown that if $n > 2$, the value of $a_1 - cx^2$ may become negative for the high values of x. The limiting value of x is then