Geophysics. — *"On the duration of showers at Batavia."* By Dr. J. P. van der Stok.

Since January 1st 1866 hourly observations of different meteorological quantities have been made at the Batavia Observatory and also of the rainfall so that, at present, a series of 40 years is available comprising some 25000 rainy hours distributed over about 8200 showers.

In less favourable climates it is not practicable to have such observations made continuously night and day and, as self-registering instruments are subject to frequent interruptions, this series may be regarded as a unique material for investigation.

The purpose of this inquiry is to investigate the distribution of showers of variable duration in different seasons, and to apply to these frequencies suitable frequency-formulae.

A rainy hour is defined as every hour during which rain fell, if it were only 0.1 m.m.; the duration of a shower is defined as the number of subsequent hours in which rain was observed: e.g. by a shower of 10 hours' duration we do not assume incessant rain during this period, but that no hour has passed without some rain having fallen.

As during the first decade no observations were made on Sundays, the total amount of hours e.g. for January is not equal to:

$$40.31.24 = 29760$$

but to:

$$(40.31 - 44)\,24 = 28704.$$

The results of this inquiry have been summarized in Table I.

Average values of quantities so divergent and intermittent as rainfall are hardly sufficient to convey an adequate idea of the way in which this phenomenon affects the climate and it is questionable whether excessive quantities or durations are to be included in the computation, because even the mean values deduced from a long series of observations may be affected to an important degree by one shower and thus the meaning which we attach to averages, loses its value.

Consequently the frequencies given in Table II give not only a more complete, but also a more accurate idea of this climatological factor than the average values of Table I.

From this summary it appears that in April, almost suddenly, the condition of rainfall shows an alteration such, that the probability of showers of long duration is considerably reduced; in the next months this probability again increases whilst in August the distribution

TABLE I. Showers at Batavia, 1866—1905.

	Number of hours	Number of rainy hours	Percent. of rainy hours	Number of showers	Mean dur. of showers in hours	Mean numb. of showers per diem
January.. .	28704	4456	15.5	1367	3.26	1.14
February....	26136	4565	17.5	1251	3.65	1.15
March.. ..	28680	3026	10.5	939	3.22	0.79
April	27792	1704	6.1	664	2.57	0.57
May.... ..	28680	1138	4.0	428	2.66	0.36
June.	27768	1222	4.4	405	3.02	0.34
July... . .	28728	924	3.2	341	2.71	0.28
Augustus ..	28680	516	1.8	212	2.43	0.18
September..	27768	897	3.2	324	2.77	0.28
October ...	28704	1219	4.2	480	2.54	0.49
November ..	27768	2039	7.3	709	2.88	0.61
December .	28704	3372	11.7	1071	3.15	0.90
Year.... ...	338112	25078	7.42	8190	3.06	0.58

shows some resemblance to that of April. The phenomenon of rainfall, therefore, is subject to an annual variation consisting of a single period as shewn by the percentage of the showers and a semi-annual period with minima in April and August. Further it appears from Table II that showers of 24 consecutive hours rarely occur (14 cases in 8190), but that, although very rarely, showers of 100, 109 and even of 147 hours have been observed in February and March.

2. Besides the numerical representation as given in Table II, also an analytical representation is of some importance as, in the constants calculated, peculiarities of the curve of distribution often occur which appear neither in the numerical, nor in a graphic representation and, at the same time, these constants can be considered as a quantitative measure of these peculiarities.

In treating frequencies of this kind, the number of constants to be used must remain restricted to a few, as every constant necessitates the computation of an average of higher order and constants based on high moments will, in this case, be practically idle.

It seems, therefore, desirable not to introduce more than two

TABLE II.

Frequencies of duration of showers at Batavia, 1866—1905.

Dur. in hours	Jan.	Febr.	Mrch	Apr.	May	June	July	Aug	Sept.	Oct.	Nov.	Dec.
1	417	357	277	220	137	112	115	80	89	152	221	316
2	321	263	254	185	121	115	89	55	96	154	193	273
3	206	203	138	110	71	55	62	36	65	86	126	175
4	130	126	89	59	38	46	26	16	32	30	52	111
5	81	77	59	43	26	33	13	10	12	27	33	64
6	51	60	45	17	10	14	13	7	11	13	34	33
7	51	43	23	14	12	9	7	4	6	10	12	25
8	23	29	13	12	6	7	10	3	5	—	13	20
9	17	22	13	3	2	6	2	—	3	2	9	14
10	16	20	5	1	2	2	1	1	2	—	3	8
11	10	14	5	—	2	2	—	—	1	3	5	9
12	9	6	5	—	1	—	—	—	—	1	2	4
13	4	9	2	—	—	2	1	—	1	1	—	4
14	5	6	2	—	—	1	2	—	1	—	1	—
15	4	2	1	—	—	—	—	—	—	—	3	5
16	6	1	—	—	—	—	—	—	—	—	—	3
17	1	3	1	—	—	—	—	—	—	—	1	—
18	—	4	2	—	—	—	—	—	—	—	—	1
19	1	1	1	—	—	—	—	—	—	—	—	1
20	1	1	—	—	—	—	—	—	—	—	—	1
21	—	—	—	—	—	—	—	—	—	—	—	1
22	—	—	1	—	—	—	—	—	—	—	—	—
23	—	—	—	—	—	—	—	—	—	—	1	—
24	1	1	—	—	—	—	—	—	—	—	—	1
25	1	1	1	—	—	—	—	—	—	—	—	—
26	—	—	—	—	—	—	—	—	—	—	—	1
27	—	—	—	—	—	1	—	—	—	—	—	—
28	—	—	1	—	—	—	—	—	—	—	—	—
29	—	—	—	—	—	—	—	—	—	—	—	—
30	—	—	—	—	—	—	—	—	—	—	—	—
31	—	—	—	—	—	—	—	—	—	—	—	—
32	—	—	—	—	—	—	—	—	—	—	—	1
33	1	—	—	—	—	—	—	—	—	—	—	—
100	—	1	—	—	—	—	—	—	—	—	—	—
109	—	1	—	—	—	—	—	—	—	—	—	—
147	—	—	1	—	—	—	—	—	—	—	—	—
Total	1367	1251	939	664	428	405	341	212	324	480	709	1071

constants in the formulae, so that no average values higher than those of the second order need to be calculated and, as the phenomenon presents an annual and a semi-annual variation, it is reasonable to expect that this number will be sufficient to arrive at a suitable expression for the law of distribution.

Then, in the first place, the frequency-formula known as Type III of Prof. PEARSON's formulae finds an application.

As the quantities under consideration are rainy hours, with the exclusion of hours in which no rain has fallen, the function must vanish for $x = 0$ if we take the duration zero for origin of coordinates, then for increasing values of x the function will rapidly rise to a maximum-value and decrease in a continuous way without any definite limit.

In this case PEARSON's Type III assumes the form:

$$u = \mathfrak{A}\, e^{-mx}\, x^{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If we put

$$mx = z$$

the expression for the mean of the n^{th} order becomes:

$$\mu_n = \frac{\mathfrak{A}}{m^{p+n+1}} \int_0^{\infty} e^{-z}\, z^{p+n}\, dz$$

and

$$\mu_0 = \frac{\mathfrak{A}}{m^{p+1}} \int_0^{\infty} e^{-z}\, z^{p}\, dz = \frac{\mathfrak{A}\, \Gamma(p+1)}{m^{p+1}} = 1$$

from which we find for the determination of the two constants m and p the expressions.

$$m = \frac{\mu_1}{\mu_2 - \mu_1^2} \text{ en } p + 1 = \frac{\mu_1^2}{\mu_2 - \mu_1^2} . \quad . \quad . \quad . \quad . \quad (2)$$

$\mathfrak{A}$ being defined so that the area of the curve becomes equal to unity, this quantity must not be regarded as a characteristic of the curve.

Applying these formulae to the frequencies of Table II, we find the values given in Table III.

In computing the means it is to be noted that the duration of a shower of say 3 hours is not to be regarded as a duration between 2.5 and 3.5 hours, but as a mean duration of 2.5 hours, because any duration beyond 3 hours would transfer the quantity into the 4 hours group. Further it must be noted that for February and March the excessive durations of 100 and more hours have been excluded from the calculation.

TABLE III. Constants of PEARSON's formula, Type III.

	$\mathfrak{A}$	m	p
January	0.3249	0.357	0.163
February	0.3073	0.365	0 273
March	0.3483	0.400	0.227
April	0.6441	0.854	1.194
May	0.5575	0.731	0 941
June	0.3921	0.486	0.466
July	0 4863	0.600	0.625
August	0.6829	0.832	1.022
September	0.5044	0.677	0.876
October	0.5443	0.648	0.646
November	0.4129	0.491	0.413
December	0.3344	0 359	0.131

3. In the second place we have to consider the formula in series form given by the author in a former publication [1]) for the case that the function must vanish for $x = 0$:

$$u = \Sigma A'_n \psi'_{n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and, if the mean of the n^{th} order is represented by:

$$\mu_n = \int_0 u x^n \, dx$$

$$A'_n \pm \frac{\mu_n}{0!(n+1)!n!} - \frac{\mu_{n-1}}{1!n!(n-1)!} + \frac{\mu_{n-2}}{2!(n-1)!(n-2)!} \cdots \frac{(-1)^n}{n!}. \quad (4)$$

We may now introduce a suitable alteration of the scale value c, multiplying form. (3) by the factor h and further writing everywhere xh for x and $\mu_n h^n$ for μ_n.

$$A'_0 = \mu_0 = 1$$

$$4'_1 = \frac{\mu_1 h}{2!1!} - \mu_0 = \frac{\mu_1 h}{2} - 1$$

$$A'_2 = \frac{\mu_2 h^2}{3!2!} - \frac{\mu h}{2!1!} + \frac{1}{2!} \text{ etc.}$$

1) These proceedings Vol X (799—817).

As by the use of these formulae the function and its integral assume rather large and the A-coefficients small values, it is desirable to divide the function by $(n+1)!$ and to multiply the A-coefficients with the same factor, then:

$$u = \Sigma A_n \psi_{n+1}$$
$$A_n = A'_n (n+1)!$$
$$(n+1)! \psi_{n+1} = \psi'_{n+1}$$
$$A_0 = 1$$
$$A_1 = \mu_1 h - 2$$
$$A_2 = \frac{\mu_2 h^2}{2!} - \frac{3\mu_1 h}{1!} + 3$$
$$A_3 = \frac{\mu_3 h^3}{3!} - \frac{4\mu_2 h^2}{2!} + \frac{6\mu_1 h}{1!} - 4 \text{ etc.}$$

The constant h may then be determined by putting:

$$A_1 = 0$$

from which:

$$h = \frac{2}{\mu_1}$$

and

$$A_2 = \frac{2\mu_2}{\mu_1^2} - 3 = \frac{2\mu_2 - 3\mu_1^2}{\mu_1^2}.$$

TABLE IV. Constants of the series-formula.

	h	A_2
January	0.613	0.7206
February	0 573	0 5714
March	0.651	0.6292
April	0.778	—0.0893
May	0.752	0.0297
June	0.662	0.3640
July	0 738	0.2301
August	0.823	—0 0119
September	0.722	0.0652
October	0.787	0.2154
November	0.694	0.4148
December	0.635	0.7669

Table IV exhibits the values of the constants h and A_2 as calculated by means of these formulae.

Either formula gives a good image of the characteristic differences of the curve in the course of the year, viz. the sudden transition in April, the congruity between the curves for April and August and the single and double annual variations in the phenomenon.

If we define as normal the curve obtained by putting $p = 1$ in PEARSON's formula and $A_2 = 0$ in the series-formula, both formulae assume the form:

$$u = Ae^{-xh} x$$

and we see from Tables III and IV that in the East-monsoon the curve approaches the normal form; in April, May, August and September its shape is almost fully normal, but in June and July the curve clearly shows considerable deviations from this simple type.

Peculiarities of this kind are not perceptible in the *rainfall* during these months at Batavia, but in some other places, more directly exposed to the influence of the South-East-monsoon as Tjilatjap, a rather large increase of rainfall obtains after April, thus giving rise to a secondary maximum. In the West-monsoon the deviations from the normal type are greatest.

4. Owing, to the rather large irregularities in the data, the monthly frequencies as given in Table II are not very suitable to serve as material for putting frequency-formulae to the test of a comparison between observed and calculated aggregate values; for this purpose, therefore, all frequencies for the period April to November have been taken together and reduced to a total of 1000. (Table V).

In the last column of this Table frequencies are given between the limits x and 0, which frequencies are to be compared to the corresponding values calculated from the formula:

$$B = \int_0^x u dx$$

For the constants of the formulae we find:

$$h = 0.9141 \qquad \mathfrak{A} = 0.5366$$

$$A_2 = 0.6429 \qquad m = 0.5576$$

$$p = 0.2222$$

The integration of the series-formulae between the limits x and

TABLE V.
Frequencies of duration of showers, April – November.

Durat. in hours	I	II	III
1	1126	316	316
2	1008	283	599
3	611	172	771
4	299	84	855
5	197	55	910
6	119	34	944
7	74	21	965
8	56	16	981
9	27	8	989
10	12	3	992
11	13	4	996
12	4	1	997
13	5	1	998
14	5	1	999
15	3	1	1000
16	—		
17	1		
18	—		
19	—		
20	—		
21	—		
22	1		
23	1		
Total	3562	1000	

o offers no difficulties, from the differential equation of the ψ function it follows that:

$$B_{n+1} = \int_0^x \psi_{n+1}\, dx = - \frac{x^2 e^{-x}}{(n+1)!} \cdot \frac{1}{n} \frac{dT''_n}{dx}$$

Introducing the factor h we find:

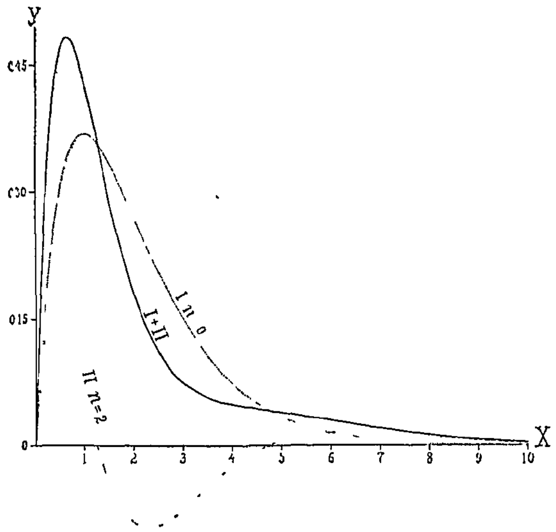
$$B_1 = 1 - (1 + xh)\, e^{-xh}$$

$$B_2 = -\frac{x^2 h^2}{2!}\, e^{-xh}$$

$$B_3 = -\frac{x^3 h^3}{3!}\, e^{-xh} - B_2$$

$$B_4 = -\frac{x^4 h^4}{4!}\, e^{-xh} - 2\,B_3 - B_2 \quad \text{etc.}$$

Values of these integrals are given in Table VIII and corresponding values of the function in Table VII. The figure shows how a frequency-curve (full line), as represented by the ψ functions, is constructed out of its components (broken line) for the case that both h and A_2 are equal to unity, which does not differ much from the conditions which obtain at Batavia during the dry season.



The integration between fixed limits of formula (1), Type III, is very laborious; substituting:

$$m\,x = z$$

we find by development:

$$\int_0^z u\,dz = \frac{\mathfrak{A}}{m^{p+1}}\,\frac{z^{p+1}}{p+1}\left[1 - \frac{p+1}{p+2}\,z + \frac{p+1}{p+3}\,\frac{z^2}{2!} - \text{etc.}\right]$$

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Owing to its feeble convergency, this expression is of no practical use for values of z greater than 3.

In SCHLÖMILCH's Compendium, 2te Aufl. p. 261 in the chapter "Unvollständige Gammafunctionen" the way is indicated to transform this expression into a much stronger converging series:

$$\mathfrak{A}\int_0^x e^{-mx}\,x^p\,dx = 1 - \frac{\mathfrak{A}e^{-x}}{m^{p+1}}\left[x^p - \frac{p}{\lambda}\left\{1 - \frac{a_1}{x+1} + \frac{a_2}{(x+1)(x+2)} - \text{etc.}\right\}\right.$$

where, in our case, $\lambda = 1 - p = 0.7778$ and

$a_1 = \lambda = 0.7778$ $\qquad$ $a_3 = \lambda^3 + \lambda = 1.2484$

$a_2 = \lambda^2 = 0\ 6050$ $\qquad$ $a_4 = \lambda^4 + 4\lambda^2 - \lambda = 2.0082$

$a_5 = \lambda^5 + 10\lambda^3 - 5\lambda^2 + 8\lambda = 8.1881.$

The results of this investigation are summarized in Table VI.

TABLE VI.

	Observ.	Calc. series			Calc. Type III	Observ. — Calc.	
		A_0B_0	A_2B_2	Total		Series	Type III
1	316	232.7	74.9	307 6	321 9	8	— 6
2	599	545.4	67.5	612 9	581.3	—14	18
3	771	758 9	13.4	772.3	746.5	— 1	24
4	855	879.7	—24.3	855.4	848.2	0	7
5	910	941.0	—36 4	904.6	909.8	5	0
6	944	973.1	—33.2	939.9	946.7	4	— 3
7	965	987.7	—24.8	962 9	968.6	2	— 4
8	981	994.5	—16.5	978 0	981.6	3	— 1
9	989	997.5	—10 1	987.4	989.2	2	0
10	992	998.9	— 5.9	993.0	993.7	— 1	— 2
11	996	999.5	— 3.3	996.2	996 3	0	0
12	997	999.8	— 1.8	998.0	997.9	— 1	— 1
13	998	999.9	— 0.93	999.0	998 8	— 1	— 1
14	999	999 96	— 0.48	999.48	999.28	0	0
15	1000	999.98	— 0.24	999.74	999.58	0	0

TABLE VII. Values of the function ψ_{n+1}.

x	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0905	−0.0860	0.0816	−0.0774	0.0732
0.2	0.1638	−0.1474	0.1321	−0.1097	0.1046
0.3	0.2223	−0.1889	0.1589	**−0.1320**	**0.1079**
0.4	0.2681	−0.2145	**0.1680**	−0.1280	0.0937
0.5	0.3033	−0.2275	0.1643	−0.1121	0.0697
0.6	0.3293	**−0.2305**	0.1515	−0.0892	0.0412
0.7	0.3476	−0.2260	0.1327	−0.0628	0.0121
0.8	0.3595	−0.2157	0.1102	−0.0355	−0.0151
0.9	0.3659	−0.2013	0.0860	−0.0090	−0.0388
1.0	**0.3679**	−0.1839	0.0613	0.0153	−0.0583
1.1	0.3662	−0.1648	0.0372	0.0368	−0.0731
1.2	0.3614	−0.1446	0.0145	0.0549	−0.0834
1.3	0.3543	−0.1240	−0.0065	0.0696	−0.0894
1.4	0.3452	−0.1036	−0.0253	0.0809	**−0.0916**
1.5	0.3347	−0.0837	−0.0418	0.0889	−0.0905
1.6	0.3230	−0.0646	−0.0560	0.0939	−0.0866
1.7	0.3106	−0.0466	−0.0678	**0.0962**	−0.0805
1.8	0.2975	−0.0298	−0.0774	0.0961	−0.0728
1.9	0.2842	−0.0142	−0.0848	0.0940	−0.0638
2.0	0.2707	0.0000	−0.0902	0.0902	−0.0541
2.1	0.2572	0.0129	−0.0939	0.0851	−0.0429
2.2	0.2438	0.0244	−0.0959	0.0789	−0.0340
2.3	0.2306	0.0344	**−0.0965**	0.0719	−0.0241
2.4	0.2178	0.0436	−0.0958	0.0645	−0.0147
2.5	0.2052	0.0513	−0.0941	0.0566	−0.0059
2.6	0.1931	0.0579	−0.0914	0.0487	0.0022
2.7	0.1815	0.0635	−0.0880	0.0409	0.0100
2.8	0.1703	0.0681	−0.0840	0.0332	0.0159
2.9	0.1596	0.0718	−0.0795	0.0257	0.0215
3.0	0.1494	0.0747	−0.0747	0.0187	0.0261
3.1	0.1397	0.0768	−0.0696	0.0121	0.0300
3.2	0.1304	0.0783	−0.0644	0.0059	0.0329
3.3	0.1217	0.0791	−0.0590	0.0003	0.0352
3.4	0.1135	**0.0794**	−0.0537	−0.0048	0.0366
3.5	0.1057	0.0793	−0.0484	−0.0094	0.0375
3.6	0.0984	0.0787	−0.0433	−0.0134	**0.0377**
3.7	0.0915	0.0778	−0.0383	−0.0169	0.0375
3.8	0.0850	0.0765	−0.0334	−0.0199	0.0368
3.9	0.0789	0.0750	−0.0288	−0.0217	0.0356
4.0	0.0733	0.0733	−0.0244	−0.0244	0.0342
4.1	0.0680	0.0714	−0.0203	−0.0260	0.0325
4.2	0.0630	0.0693	−0.0164	−0.0273	0.0306
4.3	0.0584	0.0671	−0.0127	−0.0281	0.0285
4.4	0.0540	0.0648	−0.0094	−0.0287	0.0263
4.5	0.0500	0.0625	−0.0063	**−0.0289**	0.0242
4.6	0.0462	0.0601	−0.0034	−0.0289	0.0217
4.7	0.0428	0.0577	−0.0008	−0.0286	0.0194
4.8	0.0395	0.0553	0.0016	−0.0281	0.0171
4.9	0.0365	0.0529	0.0037	−0.0275	0.0148

TABLE VII (continued).

x	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
5.0	0.0337	0.0506	0.0056	−0.0266	0.0126
5.1	0.0311	0.0482	0.0073	−0.0257	0.0106
5.2	0.0287	0.0459	0.0088	−0.0247	0.0086
5.3	0.0265	0.0437	0.0101	−0.0236	0.0067
5.4	0.0244	0.0415	0.0112	−0.0224	0.0049
5.5	0.0225	0.0393	0.0122	−0.0212	0.0033
5.6	0.0207	0.0373	0.0130	−0.0199	0.0018
5.7	0.0191	0.0353	0.0136	−0.0187	0.0004
5.8	0.0176	0.0334	0.0142	−0.0174	−0.0009
5.9	0.0162	0.0315	0.0146	−0.0161	−0.0020
6.0	0.0149	0.0298	0.0149	−0.0149	−0.0030
6.5	0.0098	0.0220	**0.0151**	−0.0091	−0.0063
7.0	0.0064	0.0160	0.0138	−0.0045	**−0.0074**
7.5	0.0042	0.0114	0.0119	−0.0012	−0.0070
8.0	0.0027	0.0081	0.00[illegible]8	0.0008	−0.0059
8.5	0.0017	0.0056	0.0079	0.0022	−0.0045
9.0	0.0011	0.0039	0.0061	0.0026	−0.0031
9.5	0.0007	0.0027	0.0047	**0.0027**	−0.0020
10.0	0.0005	0.0018	0.0035	0.0026	−0.0011
10.5	0.0003	0.0012	0.0026	0.0023	−0.0004
11.0	0.0002	0.0008	0.0019	0.0019	0.0000
11.5	0.0001	0.0006	0.0013	0.0016	0.0003
12.0	0.0001	0.0004	0.0010	0.0013	0.0004
12.5	0.0000	0.0002	0.0007	0.0010	**0.0005**
13.0	—	0.0002	0.0005	0.0008	0.0005
13.5	—	0.0001	0.0003	0.0006	0.0004
14.0	—	0.0001	0.0002	0.0004	0.0004
14.5	—	0.0001	0.0002	0.0003	0.0003
15.0	—	0.0000	0.0001	0.0002	0.0003
15.5	—	—	0.0001	0.0002	0.0002
16.0	—	—	0.0001	0.0001	0.0002
16.5	—	—	0.0000	0.0001	0.0001
17.0	—	—	—	0.0001	0.0001
17.5	—	—	—	0.0000	0.0001
18.0	—	—	—	—	0.0001
18.5	—	—	—	—	0.0000

TABLE VIII. Values of the Integral B_{n+1}.

x	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0047	−0.0045	0.0044	−0.0042	0.0041
0.2	0.0175	−0.0164	0.0153	−0.0143	0.0133
0.3	0.0369	−0.0333	0.0300	−0.0269	0.0241
0.4	0.0616	−0.0536	0.0465	−0.0400	0.0343
0.5	0.0902	−0.0758	0.0632	−0.0521	0.0425
0.6	0.1219	−0.0988	0.0790	−0.0622	0.0481
0.7	0.1558	−0.1217	0.0933	−0.0699	**0.0507**
0.8	0.1912	−0.1438	0.1054	−0.0748	0.0505
0.9	0.2275	−0.1647	0.1153	**−0.0770**	0.0478
1.0	0.2642	−0.1839	0.1226	−0.0766	0.0429
1.1	0.3010	−0.2014	0.1276	−0.0740	0.0363
1.2	0.3374	−0.2169	0.1301	−0.0694	0.0285
1.3	0.3732	−0.2303	**0.1305**	−0.0631	0.0198
1.4	0.4082	−0.2417	0.1289	−0.0556	0.0107
1.5	0.4422	−0.2510	0.1255	−0.0471	0.0016
1.6	0.4751	−0.2584	0.1206	−0.0379	−0.0073
1.7	0.5068	−0.2640	0.1144	−0.0284	−0.0157
1.8	0.5372	−0.2678	0.1071	−0.0188	−0.0234
1.9	0.5663	−0.2700	0.0990	−0.0092	−0.0302
2.0	0.5940	**−0.2707**	0.0902	0.0000	−0.0361
2.1	0.6204	−0.2700	0.0810	0.0088	−0.0410
2.2	0.6454	−0.2681	0.0715	0.0170	−0.0449
2.3	0.6692	−0.2652	0.0619	0.0245	−0.0478
2.4	0.6915	−0.2613	0.0523	0.0314	−0.0498
2.5	0.7127	−0.2565	0.0428	0.0374	−0.0508
2.6	0.7326	−0.2511	0.0335	0.0427	**−0.0509**
2.7	0.7513	−0.2450	0.0245	0.0472	−0.0504
2.8	0.7689	−0.2384	0.0159	0.0509	−0.0491
2.9	0.7854	−0.2314	0.0077	0.0538	−0.0472
3.0	0.8009	−0.2240	0.0000	0.0560	−0.0448
3.1	0.8153	−0.2165	−0.0072	0.0575	−0.0420
3.2	0.8288	−0.2087	−0.0139	0.0584	−0.0389
3.3	0.8414	−0.2008	−0.0201	**0.0587**	−0.0354
3.4	0.8532	−0.1929	−0.0257	0.0585	−0.0318
3.5	0.8641	−0.1850	−0.0308	0.0578	−0.0281
3.6	0.8743	−0.1771	−0.0354	0.0567	−0.0244
3.7	0.8838	−0.1692	−0.0395	0.0551	−0.0206
3.8	0.8926	−0.1615	−0.0431	0.0533	−0.0169
3.9	0.9008	−0.1539	−0.0462	0.0512	−0.0133
4.0	0.9084	−0.1465	−0.0488	0.0488	−0.0098
4.1	0.9155	−0.1393	−0.0511	0.0463	−0.0064
4.2	0.9220	−0.1323	−0.0529	0.0437	−0.0033
4.3	0.9281	−0.1255	−0.0544	0.0409	−0.0003
4.4	0.9337	−0.1188	−0.0555	0.0380	0.0024
4.5	0.9389	−0.1125	−0.0563	0.0352	0.0049
4.6	0.9437	−0.1064	−0.0567	0.0323	0.0072
4.7	0.9482	−0.1005	−0.0569	0.0294	0.0093
4.8	0.9523	−0.0948	**−0.0569**	0.0266	0.0111
4.9	0.9561	−0.0894	−0.0566	0.0238	0.0127

TABLE VIII (continued).

x	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
5.0	0.9596	—0.0812	—0.0562	0.0211	0.0140
5.1	0.9628	—0.0793	—0.0555	0.0184	0.0151
5.2	0.9658	—0.0746	—0.0547	0.0159	0.0162
5.3	0.9686	—0.0701	—0.0538	0.0135	0.0169
5.4	0.9711	—0.0659	—0.0527	0.0112	0.0175
5.5	0.9734	—0.0618	—0.0515	0.0090	0.0179
5.6	0.9756	—0.0580	—0.0503	0.0070	0.0182
5.7	0.9776	—0.0544	—0.0489	0.0050	**0.0183**
5.8	0.9794	—0.0509	—0.0475	0.0032	0.0182
5.9	0.9811	—0.0477	—0.0461	0.0016	0.0181
6.0	0.9827	—0.0446	—0.0446	0.0000	0.0178
6.5	0.9887	—0.0318	—0.0371	—0.0060	0.0154
7.0	0.9927	—0.0223	—0.0298	—0.0093	0.0119
7.5	0.9953	—0.0156	—0.0233	—0.0107	0.0083
8.0	0.9970	—0.0107	—0.0179	**—0.0107**	0.0050
8.5	0.9981	—0.0074	—0.0135	—0.0100	0.0024
9.0	0.9988	—0.0050	—0.0100	—0.0088	0.0005
9.5	0.9992	—0.0034	—0.0073	—0.0074	—0.0008
10.0	0.9995	—0.0023	—0.0053	—0.0061	—0.0015
10.5	0.9997	—0.0015	—0.0038	—0.0048	—0.0019
11.0	0.9998	—0.0010	—0.0027	—0.0038	**—0.0020**
11.5	0.9999	—0.0007	—0.0019	—0.0029	—0.0019
12.0	0.9999	—0.0004	—0.0013	—0.0022	—0.0017
12.5	1.0000	—0.0003	—0.0009	—0.0017	—0.0015
13.0	—	—0.0002	—0.0006	—0.0012	—0.0012
13.5	—	—0.0001	—0.0004	—0.0009	—0.0010
14.0	—	—0.0001	—0.0003	—0.0007	—0.0008
14.5	—	—0.0001	—0.0002	—0.0005	—0.0007
15.0	—	0.0000	—0.0001	—0.0003	—0.0005
15.5	—	—	—0.0001	—0.0002	—0.0004
16.0	—	—	—0.0001	—0.0002	—0.0003
16.5	—	—	0.0000	—0.0001	—0.0002
17.0	—	—	—	—0.0001	—0.0002
17.5	—	—	—	—0.0001	—0.0001
18.0	—	—	—	0.0000	—0.0001
18.5	—	—	—	—	—0.0001
19.0	—	—	—	—	0.0000

Mathematics. — *"On curves of order four with two flecnodal points or with two biflecnodal points."* By Prof. JAN DE VRIES.

1. The points of a binodal curve of order four, C_4, are projected out of the two double points O_1 and O_2 by two pencils in correspondence (2 , 2).

So such a C_4 is determined by the relation

$$a_{22}\lambda^2\mu^2+a_{21}\lambda^2\mu+a_{20}\lambda^2+a_{12}\lambda\mu^2+a_{11}\lambda\mu+a_{10}\lambda+a_{02}\mu^2+a_{01}\mu+a_{00}=0,$$

where

$$\lambda = x_1 : x_3 \text{ and } \mu = x_2 : x_3 .$$

According to a well-known property the eight singular rays (λ)