5. When each ray through a singular point determines a system of tangents with *index two*, then the equation is projectively reducible to an equation of the form

$$\frac{dy}{dx} = \frac{N(x)y^2 + P(x)y + Q(x)}{R(x)y + S(x)}.$$

For, this equation determines for $x = m$ the tangents of a conic and by the substitution

$$x = \frac{\alpha}{\gamma}, \qquad y = \frac{\beta}{\gamma}$$

(see § 1) it is transformed into an equation having in $\alpha = 0, \gamma = 0$ a singular point, whilst each ray of the pencil $\alpha = m\gamma$ possesses the above indicated property.

The equation

$$x\frac{dy}{dx} = \frac{x^3 + y^3 - 2x^2y^2 - xy}{y^2 - 2x^2y - x}$$

is in this case, for each ray $y = mx$ furnishes a system of tangents with index two. By the substitution

$$x = \frac{1}{v}, \qquad y = \frac{u}{v}$$

it passes into the equation (of RICCATI)

$$\frac{dv}{du} = 2u - u^2 v + v^2.$$

This can be reduced with the aid of the solution $v = u^2$ to the equation (of BERNOULLI)

$$\frac{dw}{du} = u^2 w + w^2,$$

where $w = v - u^2$. By $w = z^{-1}$ we then arrive at a linear differential equation.

Botany. — Mr. VAN DER STOK presents in behalf of S. H. KOORDERS a communication entitled: "*Polyporandra Junghuhnii, a hitherto undescribed species of the order of Icacinaceae, found in 's Rijks Herbarium at Leiden by* S. H. KOORDERS" *(Plantae Junghuhnianae ineditae II)* [1].

(Communicated in the meeting of February 27, 1909).

Polyporandra Junghuhnii, KDS *n. spec. Frutex? scandens, ramulis teretiusculis novellis pubescentibus. Folia opposita, oblonga, basi acuta vel obtusa, apice sensim acuminata;* 12—13 *cm. longa et* 4—5 *cm. lata, petiolo* 1—1½ *cm. longo, subcoriacea, supra praeter costam*

[1] Continuation of *Plantae Junghuhnianae ineditae I* in Proceedings of the Mathematical and Physical Section, of June 27 1908, p. 158—162.

sulcatam pubescentem glabra, subtus puberula et trinervia, nervis lateralibus utrinque 5—7 *adscentibus in margine exeuntibus, nervis secundariis inter primarios transversis atque venis reticulatis, subtus distincte prominentibus. Cirrhi in specimine Jungh. desunt. Flores dioici; masc. nondum aperti, cymoso-paniculati; feminei ignoti. Inflorescentiae axillares laxae folium subaequantes; pedunculi pedicellique pubescentes; bracteae caducissimae (?), in specimine Jungh. deficientes; pedicelli alabastris oblongis breviores; calyx sub-campanulatus 5-partitus,* 2 *mm. longus, laciniis lanceolatis scariosis, erectis, acuminatis* 1—1.2 *millim. longis, extus appresse pilosis. Petala* 5 *crassiuscula, calyce breviores, extus pilis longis appressis albis. Stamina* 5—6 *rarissime* 7 *filamentis brevissimis, teretiusculis, glabris; antheris oblongis vel linearibus* 8—10*-locularibus. Pollen globoso-tetraëdrum laeve* 10 *μ diam. Ovarium rudimentum subnullum. Fructus ignotus.*

SUMATRA: *"Hochangkola-Tobing" (leg.* JUNGHUHN *anno?* 1839. — *Plantae Junghuhnianae ineditae n.* 542 *in Herb. Lugd. Batav.).*

The species described above is the third representative of the genus *Polyporandra* BECC., belonging to the *Icacinaceae* and related to *Jodes* Bl. This species, *Polyporandra Junghuhnii*, was found by me in 1908 in 's Rijks Herbarium at Leiden (*Plantae Junghuhnianae ineditae* N°. 542) and had been collected by JUNGHUHN in the Battak country at "Hochangkola-Tobing". It differs *i. a.* from the two already known species of *Polyporandra* in the structure of the calyx; in *P. scandens* BECCARI (in Malesia I (1877) 125 tab. 7) and *P. Hansemanni* ENGLER (in ENGLER Botan. Jahrb. XVI, Beiblatt, N°. 39 (1893) 13) the calyx is cup-shaped and has short teeth, whereas in *Polyporandra Junghuhnii* it is 5-partite, with pointed segments 1 millim. long.

Superficially the flowers of *Polyporandra Junghuhnii* somewhat resemble these of *Natsiatum herpeticum* BUCHAN, but our *Polyporandra* is sharply differentiated from their species by the characteristic structure of the anthers, described above.

That our species must be included in the above-mentioned genus *Polyporandra* BECC., seems to me to be highly probable. Since however, all the material, which has as yet been found, consists of a single dried branch with young, not completely developed male flowers, floral bud and three leaves, I consider it possible that afterwards, when the as yet unknown female flower, fruits and seeds of *Polyporandra Junghuhnii* shall have been found, this species will prove to be the type of a new sub-genus of *Polyporandra* or of a new

genus, directly intermediate between *Polyporandra* BECC. and *Natsiatum* BUCHAN. Because the material is so incomplete, I have, however, thought it advisable to refrain even from proposing a new sub-genus and to assign to this species a place in the genus *Polyporandra*.

In the Herbarium of the Royal Botanic Gardens at Kew I last year compared authentic specimens of the only hitherto described species of *Polyporandra* (*P. scandens* BECCARI and *P. Hansemanni* ENGLER) with JUNGHUHN's unicum of the Leiden Herbarium. In so doing I have become convinced that *Polyporandra scandens* and *Hansemanni* are very closely related but that, as was indicated above, our species (*Polyporandra Junghuhnii*) is sharply marked off from these specifically.

In conclusion I wish to tender my best thanks to the Director of the Royal Botanic Gardens in Kew, for the facilities given me for the comparison of the above-mentioned authentic specimens of BECCARI and of ENGLER.

Leiden, February 26th, 1909.

Physics. — "*On the solid state*". By Mr. J. J. VAN LAAR. (Communicated by Prof. H. A. LORENTZ.)

(Communicated in the meeting of February 27, 1909).

1. In a recently published paper [1]) I treated the complete theory of association, not only for gases and vapours, but also for liquids.

If we assume that only two simple molecules combine to a double molecule, the formula:

$$\frac{\beta^2}{1-\beta^2} = \frac{cT^{\gamma+1}\, e^{-\frac{q_0+(p+a/v^2)\Delta b}{RT}}}{p+a/v^2} \quad , \quad . \quad . \quad . \quad . \quad (\alpha)$$

holds universally, in which c is a constant to be determined, and further:

$$\gamma = \frac{-k_1+2k_2}{R} \quad ; \quad \Delta b = -b_1+2b_2 \, .$$

So the quantity γR is the change of the specific heat for infinitely great constant volume, when 1 Gr. mol. of double molecules passes into 2 Gr. mol. of single molecules, while Δb is the change of the volume of the molecules in this transition. The quantity $q_0 = -(e_1)_0 + 2(e_2)_0$ repre-

[1]) In the *Arch. Teyler* (2) T. 11, Troisième partie, p. 235—331 (1909).