Physics. — *"Magnetic resolution of spectral lines and magnetic force."* By Prof. P. ZEEMAN. (Second part). [1])

Asymmetry in strong fields.

2. By means of *the method of the non-uniform field,* described in the first part of this communication, it is possible to survey at one glance a phenomenon dependent upon the intensity of the magnetic field for a series of different intensities, all other circumstances surely being the same.

I there proposed to use this method for a more minute study concerning an asymmetry of the resolution of spectral lines first predicted from theory by VOIGT [2]) and lately considered by LORENTZ [3]) from another point of view.

The theoretical result of VOIGT, applying to the case of resolution into a *triplet,* may be given in his own words: "dass das normal zu den Kraftlinien wahrnehmbare Duplet der parallel zu R [magnetic force] polarisirten Componenten bei kleineren Feldstärken in der Weise unsymmetrisch ist, dass *die nach Rot liegende Componente die grössere Intensität, die nach Violett hin liegende aber den grösseren Abstand von der ursprünglichen Absorptionslinie besitzt.*" VOIGT here mentions an absorption line because he considers the so called inverse effect, by reason however of the parallellism of the phenomena of emission and absorption, the emission lines show analogous phenomena.

The amount of the asymmetry of the distances, i. e. the difference of the distances of the outer components from the middle line, ought to be on VOIGT's theory independent of the strength of the magnetic field. Moreover it is to be inferred that the described asymmetry must be scarcely observable.

On a former occasion [4]) I have given some examples of asymmetrical resolution and measurements since published by other physicists undoubtedly go far towards confirming these results.

A more minute investigation of the course of the magnetic separation, when the scale of field intensities from large to small values is traversed, is I think still of great theoretical interest. The most interesting parts of the scale are of course the very strong and the weak fields.

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[1]) Continued from Proceedings of April 1906.

[2]) VOIGT. Ann. d. Phys 1. p. 376. 1900.

[3]) LORENTZ. These Proceedings November, December 1905.

[4]) ZEEMAN. ibid. December 1899. Archiv. Néerl. (2) T. 5. 237—242. 1900.

The most striking example of asymmetrical resolution that I know of, occurs in the case of one of the yellow mercury lines (5791). The structure of a line like this one cannot be made out by means of MICHELSON's interferometer. Indeed the assumption of symmetry, which is, as has been proved by Lord RAYLEIGH [1]), necessary to deduce the structure from the visibility curve in this case certainly is unjustified.

3. Following the method described in the first part of this paper I have made some experiments concerning the mentioned spectral line in strong fields. For the ROWLAND grating used in my observations I am indebted to the dutch Society of Sciences at Haarlem. Presently I hope to give an account of results obtained in weak fields by means of an interference method.

The grating has 10.000 lines to the inch and a radius of curvature of 6.5 M., the divided part being of 14 cm. width. In the use of my method the grating necessarily should be mounted in such a manner that to every point of the slit corresponds only one point of the spectral image. ROWLAND's concave grating can be mounted in a non-astigmatic manner as has been remarked by RUNGE and PASCHEN [2]) and this arrangement was made use of in former investigations by myself, [3]) HALLO and GEEST. [4])

All observations recorded in the present paper were made with the spectrum of the first order.

4. Whereas the mercury line 5791 is resolved asymmetrically, the neighbouring line 5770 is resolved by the magnetic field into a perfectly symmetrical triplet, or at least very approximately so. I have used this circumstance for applying the optical method of measurement of field (see § 1), the mentioned yellow lines being easily photographed simultaneously.

Fig. 1 represents a ninefold enlargement of one of the negatives. According to measurements of FABRY and PEROT the difference of wavelength of the yellow mercury lines is 5790.66 — 5769.60 = 21.06 A.U., hence 1 m.m. in Fig. 1 corresponds to 0.551 A.U. Inspection of Fig. 1 clearly shows that line 5791 is asymmetrically resolved. Perhaps this is still more evident in the enlargements Figures 2 and 3 of parts of Fig. 1.

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1) RAYLEIGH. Phil. Mag. November 1892.

2) RUNGE and PASCHEN. Wied. Ann. Bd. **61**. p. 641. 1897.

3) ZEEMAN. Archiv. Néerl. (2) T. **5**. 237. 1900, T. 7. 465. 1902. These Proc. May 1902, May 1903, Dec. 1904.

4) HALLO. Archiv. Néerl. (2) T. **10**. p. 148., GEEST. (2) T. **10** p. 291. 1905.

Our object of investigation is the relation between asymmetry and strength of field.

The measurements were made in the following way. The negative was placed on the comparator, in such a manner, that the middle line of one of the triplets was contained between the two parallel wires in the reading microscope. The parallel wires had been placed previously at right angles to the direction of motion of the negative. It appeared that if with one of the triplets the desired coincidence had been obtained, this was also the case with the other. An extra system of cross wires, crossing under an angle of about 50°, was used in the measurements and made it possible to determine the resolution in the selected point of the lines.

The resolution of one line having been measured for a definite value of the magnetic force, the corresponding resolution in the corresponding point of the second line was determined immediately afterwards.

The line 5770 appeared to be divided almost exactly symmetrically, so that the resolution could be taken as a measure of the magnetic force.

On the obtained negatives 34 series of measurements were made. They relate to different points of 10 negatives made at different times.

The vacuum tubes used were intentionally made somewhat dissimilar.

In order to control the results the negatives were taken with different maximum intensity of field.

Finally the negatives obtained can be distributed into two groups, differing by the position of the grating. After taking 7 negatives I resolved to rotate the grating in its own plane through 180° in order to see whether this had some influence on the asymmetry.

This appeared to be not the case, but the apparent distribution of intensities changed in a remarkable manner. Whereas in one position of the grating figures 1—5 were obtained, the middle line being strong and the outer components rather weak, the distribution of light after rotation became reversed. In this position of the grating Fig. 1 of my last paper was taken. (See these Proceedings October 1907). The middle line is very weak and the outer components predominate.

5. The results of the measurements are dealed with in the following way. The amounts of separation of line 5791 towards the red and towards the violet are supposed to be functions of the separation of line 5770, which may be treated as proportional to the magnetic force. The separations of line 5770 may be taken as abscissae, the two other separations as ordinates.

Groups of four or five single proximate results simply were combined by assigning to each mean abscissa the mean ordinate.

The 2 $\times$ 7 principal values thus obtained are given in the first three columns of the following table.

Mean separation 5770	Separation 5791		Asymmetry	Intensity of field in Gauss
	towards red	towards violet		
270	234	259	25	14800
328	283	312	29	18020
362	313	345	32	19860
399	353	388	35	21910
440	394	431	37	24140
453	404	442	38	24880
532	475	523	48	29220

All these differences of wavelength are given in thousandths of an Ångström unit.

The fourth column in like manner gives the amount of the asymmetry.

6. The last column contains the field intensity in Gauss. In calculating it I have assumed proportionality between separation and magnetic force.

Increasing accuracy of the measurements has furnished continually increasing arguments for this proportionality and the investigations of Farber [1]), Weiss and Cotton [2]), Paschen [3]) and Stettenheimer [4]) have given a high degree of certainty to this simple law.

The numbers in the fifth column are deduced from those in the first by means of the separations of line 5770 + 0.414 and — 0.415 given by Runge and Paschen for the field used in their investigation.

The measurements of Runge and Paschen concerning the mercury lines refer, as Prof. Paschen has kindly commmunicated to me, to a field of 22750 Gauss according to measurements made in his laboratory by Frl. Stettenheimer, and of 22780 Gauss according

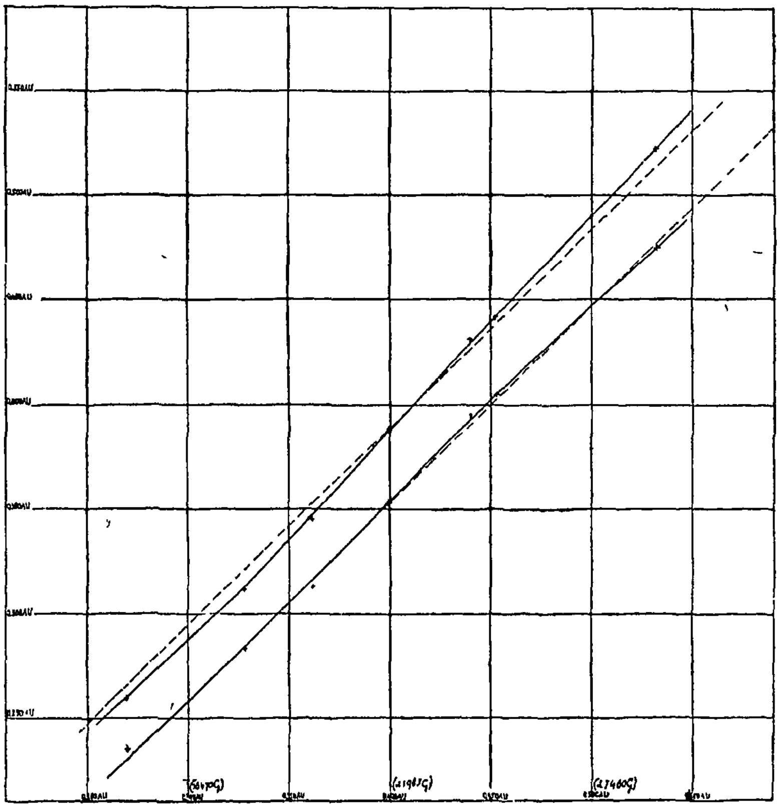
1) Färber. Diss. Tübingen, 1902; Ann. d. Phys. **9**, 886, 1902.
2) Weiss and Cotton. Journal de Physique. Juin 1907.
3) Paschen. Physik. Zeitschr. 8 Jahrgang N°. 16. 522, 1907.
4) Stettenheimer. Diss. Tübingen, 1907; Ann. d. Phys. **24**, 384 1907.

to measurements, not hitherto published, by GMELIN In reducing my observations I have therefore taken 22765 as the value of the magnetic force belonging to the mean of the two numbers mentioned for the separation.

7. The results are graphically represented in the following diagram. The abscissae are the separations of line 5770 in A.U., and the corresponding field intensities in Gauss, the ordinates the corresponding separations of 5791. The small crosses represent the observations of the table in § 5.

The full freehand lines give the best mean result of the observations. (See § 9).

The signification of the upper dotted straight line is the following.



The mean of the 34 single values of the asymmetry is 0.036 A.U. The straight dotted line has been traced parallel to the lower line at a distance of 36 thousandths of an ANGSTRÖM unit, measured along the ordinate.

8. We may infer from our observations that with the fields used, from 15000 to 30000 Gauss, an asymmetry undoubtedly exists which

to say the least bears a very striking resemblance to the one deduced from theory by VOIGT.

In both cases, the theoretical one and that following from our experiments, there is a difference of the distances between the central line and the two outer components, in this sense that the component towards the red is nearer to the middle line than that towards the violet, just as predicted by theory.

There exists also an asymmetry of *the intensities* of the outer components in the sense indicated by theory.

An inspection e. g. of the original negative of which Fig. 1 is a nine-fold reproduction, or of the reproduction Fig. 1, or better of reprints on photographic paper of the 29-fold enlargement given in Fig. 2 or even of that figure reveals the existence of a very small asymmetry of intensity. This is perhaps most clearly seen by looking at the figure from a not too small distance, covering the central line with a small strip of paper. No trace of asymmetry can be seen in the triplet of line 5770, see also the enlargements Fig. 4 and Fig. 5 of the middle and outer parts of the right of Fig. 1.

On the other hand there seems to be a difference between theory and observation in this respect, that the amount of asymmetry appears to be not constant. The table of § 5 and the graphical representation clearly indicate that when the magnetic force decreases from 30000 to 15000 Gauss the asymmetry also is nearly halved. [1]).

An error of an amount sufficient to bring a single point of the upper line on the dotted one is not absolutely excluded (see § 9). [For the right part of the diagram the error ought to be three times the probable error of *one single* of the principal values (see § 5) and would happen therefore on the average in one out of every twenty three cases].

We have however reasonable security against a combination of errors which would move all the points of the full line to the dotted one.

Of course we cannot deduce from the now determined part of the upper line whether or not it will approach asymptotically to a finite distance of the lower one.

9. We may now consider the question as to the best fitting *straight* lines to our two systems of points.

Measuring the divergencies at right angles to the line the best fit will be obtained if we make the sum of the squares of the perpen-

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[1]) An excellent series of measurements made after the writing of this article gives a somewhat lower rate of decrease, the mean value of the asymmetry being the same.

diculars from the system of points to the line a minimum. The line thus determined will be the principal axis of inertia of the system of points [2]).

Performing this calculation we find that the best fitting lower line passes through the point with the coordinates 398, 351, at a slope determined by $\theta_1 = 43°6'$. For the upper line these numbers become 398, 386, whereas $\theta_2 = 45°35'$.

In order to judge of the accuracy obtained in the representation of the observations by these straight lines the following table may serve. The third and fourth column, resp. the sixth and seventh column contain the errors of the abscissae and ordinates of the two point systems to be assumed, if the straight lines are supposed correct.

Mean separation 5770	Separation 5791 towards red	Δa_1	Δo_1	Separation 5791 towards violet	Δa_2	Δo_2
270	234	+ 1 6	— 1 8	259	+ 1.4	— 1 3
328	283	— 1 2	+ 1.4	312	— 1 6	+ 1.6
362	313	— 2 1	+ 2.2	345	— 2 0	+ 2.0
399	353	+ 0.1	— 0.1	388	0	0
440	394	+ 1.3	— 1.5	431	+ 1 0	— 0 9
453	404	+ 0 2	— 0 2	442	0	0
532	475	— 1 0	+ 1.2	523	0	0

It appears from this table that the lines completely represent the observations, if we admit mean uncertainty of 0.0013 A. U. in the observations concerning line 5770 and of 0.0014 A. U. resp. of 0.0011 A. U. in the determination of the components towards the red resp. towards the violet in the case of line 5791; we must admit these as appears from the distribution of deviations.

10. The position of one point of each line may still be checked by the observations of Runge and Paschen. They give for the separation in the case of line 5770 towards the red resp. towards the violet $+ 414 \pm 1.7$ resp. $— 415 \pm 1.7$, whereas for the same magnetic

[2]) Karl Pearson. On Lines and Planes of closest Fit to Systems of Points in space. Phil. Mag. p. 559. Vol. 2. 1901. Here we read: "The best fitting straight line for a system of points in a space of any order goes through the centroid of the system" cf. Keesom. These Proceedings 31 May, 1902.

force these numbers for line 5791 become $+366 \pm 6.7$ resp. 399 ± 6.7, the values preceded by $\pm$ indicating the mean error. According to our observations to the abscissa 415 correspond the ordinates 368 and 403, hence a very good agreement.

11. From the extremely small amount of the asymmetry viz. 0.036 A. U. one might infer after comparison with the width of the spectral lines in our figures that the asymmetry is only a small part of the real width of the line. Such a conclusion would however be too rash.

It is true that from our figures and from their originals follows an apparent width of the outer components of about 0.190 A. U. The negative of Fig. 1 was not taken however with extremely narrow slit, but with a width of slit of 0.08 m.m. Other photographs taken with a width of slit of 0.02 m.m. gave a somewhat smaller apparent width of the spectral line as the first result.

To be sure however of the real width of the line, which is of some importance here, I made an independent determination by means of an echelon spectroscope of high resolving power, the mercury tube being under the same circumstances, as in the experiments under review. The width of the spectral line appeared to be the $^1/_{11}{}^{th}$ part of the distance of two successive orders of the echelon. In the vicinity of the yellow mercury lines this distance is 0.694 A. U. hence the width of these lines is about $\frac{0.694}{11} = 0.063$ A. U.

We may compare this result with a value we may deduce from results obtained by MICHELSON. MICHELSON's analysis [1]) by means of the interferometer shows that in a field of 10000 Gauss the *whole* separation of the yellow mercury lines is 0.36 A. U. From his diagram on pag. 354 l. c. we infer that the width of the spectral line was under the circumstances of the case one fourth part of the separation or 0.09 A. U.

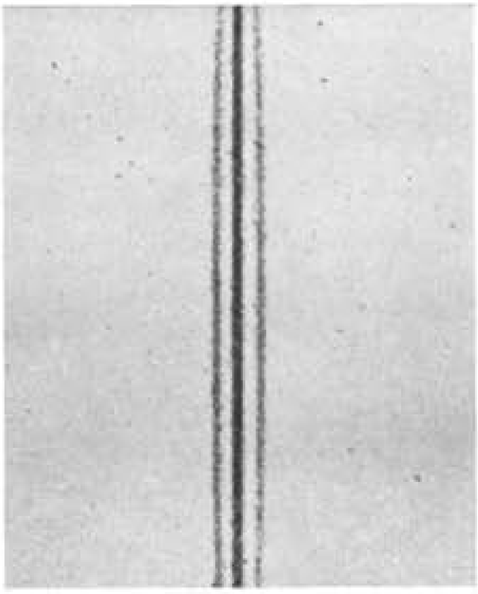
Hence taking a mean value for the width of 0.07 A. U. we conclude that the asymmetry amounts to about one half of the width of the line or at any rate that width and asymmetry are of the same order of magnitude.

[1]) MICHELSON. Phil. mag. Vol. 45, p. 348. 1898.

P. ZEEMAN. "Magnetic resolution of spectral lines and magnetic force." (Second part).

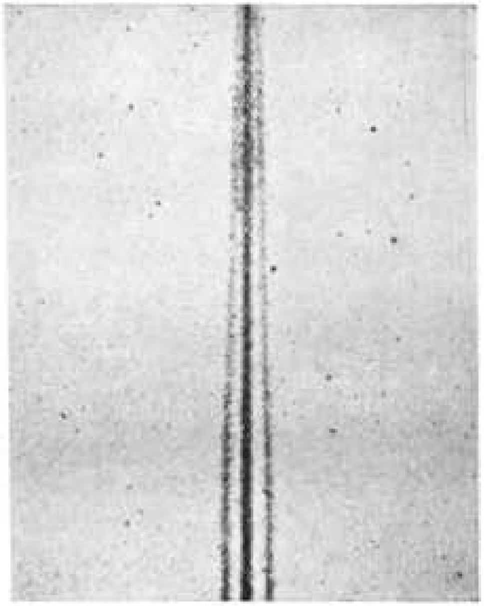
Plate III.

Fig. 2.

r  v

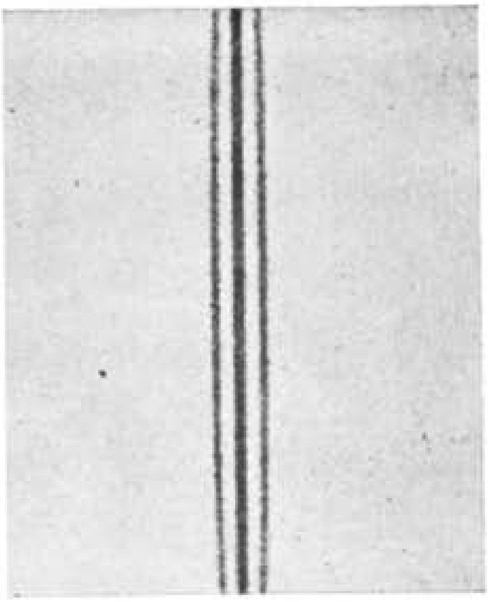
5791
Enlargement of middle part of
5791. Fig. 1.

Fig. 3.

r  v

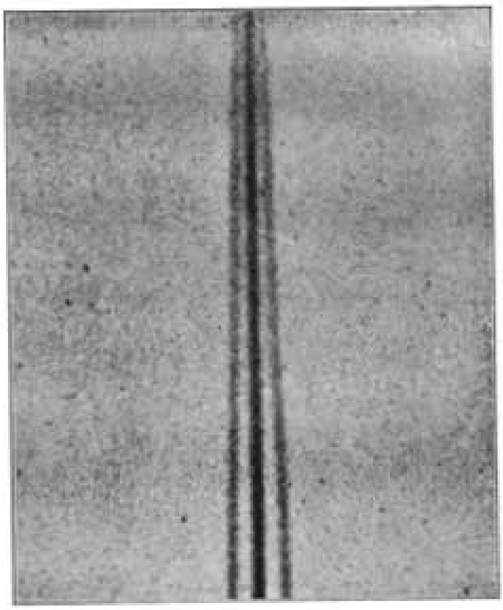
5791
Enlargement of point of
5791. Fig. 1.

Fig. 4.

r  v

5770
Enlargement of middle part of
5770. Fig. 1.

Fig. 5.

r  v

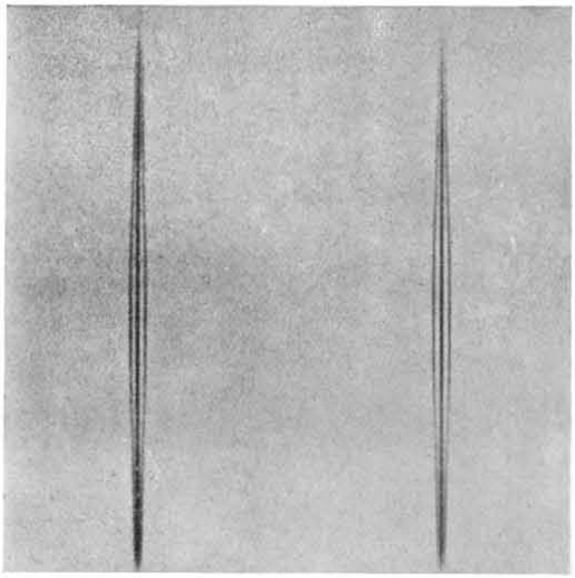
5770
Enlargement of point of
5770. Fig. 1.

P. ZEEMAN. "Magnetic resolution of spectral lines and magnetic force."

(Second part.)

Plate II.

Fig. 1.



r v

5791 5770

5791 asymmetric separation.
5770 symmetric separation.

EXPLANATION OF PLATES II AND III.

Plate II. Fig. 1. The figure is an enlargement (about nine-fold) of the original negative. The yellow mercury lines 5791 and 5770 in a non-uniform field. 1 mm. in figure is 0.551 A. U.

Plate III. Fig. 2—5 enlarged 29 times after the original.

Fig. 2. Middle part of line 5791 in Fig. 1. } asymmetrical separation.
Fig. 3. Point of line 5791 in Fig. 1.

Fig. 4. Middle part of line 5770 in Fig. 1. } symmetrical separation.
Fig 5. Point of line 5770 in Fig. 1.

The letters *r* and *v* indicate the parts towards the red and towards the violet ends of the spectrum.

Botany. — *"On a double reduction of the number of chromosomes during the formation of the sexual cells and on a subsequent double fertilisation in some species of Polytrichum."* By Dr. W. DOCTERS VAN LEEUWEN and Mrs. J. DOCTERS VAN LEEUWEN-REYNVAAN. (Communicated by Prof. F. A. F. C. WENT).

In 1904 there appeared an investigation by IKENO [1]) on spermatogenesis in Marchantia polymorpha. Since then quite a number of researches on this subject have been carried out with liverworts. Here and there an occasional reference to the true Mosses has been made, but, as far as we are aware, nothing has been published on their spermatogenesis since the appearance of IKENO's paper.

The older publications, e. g. those of GUIGNARD [2]) and of STRASBURGER [3]), treat exclusively of the final changes of the spermatids to spermatozoids. For this reason we began the present investigation in 1904 soon after the publication of IKENO's memoir; we obtained results, differing so widely from the ordinary conceptions, that we investigated, not only the spermatogenesis, but also the development and the fertilisation of the ovum.

The material was fixed at a suitable time, mostly in the field, by a sublimate mixture, and was afterwards stained with iron-haematoxylin according to HEIDENHAIN. We used Polytrichum piliferum, P. juniperinum and P. formosum. It is our intention to give a more detailed account of the work and of the methods which we have employed, in the Recueil des Travaux Botaniques Néerlandais.

IKENO made the remarkable discovery that in the antheridial cells, immediately before division, a small round body passed out of the

1) IKENO Beihefte zum Botan. Centralblatt. Bd. 16, 1903.
2) GUIGNARD. Revue gén. de botanique I. 1889.
3) STRASBURGER. Hist. Beitr. Heft IV, 1892.