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Crystallography. — *"About the oblique extinction of rhombic crystals."* By J. SCHMUTZER. (Communicated by Prof. A. WICHMANN).

(Communicated in the meeting of November 30, 1907).

As late as the year 1901 ALFRED HARKER in a brief communication [1]) pointed out the fact that with rhombic crystals the oblique extinction on planes making a small angle with the *c*-axis, is to be neglected only when the angle of the optic axes has no great value. That this was not superfluous is perhaps partly owing to the fact that, in the application of the theoretically deduced results concerning the extinction of crystal-sections mineralogists have confined themselves to monoclinic and triclinic minerals, preferably to the feldspars. [2]) It seems, therefore, that the circumstance that rhombic minerals as a rule show an oblique extinction and only exceptionally a straight one, is not laid sufficient stress on, though the fact is of course well-known. [3]) That is why, even in the younger petrographical literature, it is often alleged, in verification of the rhombic nature of a mineral, that all its sections show a straight extinction, whilst at partly straight, partly oblique extinction of the crystal-sections the monoclinic nature of the mineral is considered to have been proved. [4]) A separation of rhombic and monoclinic pyroxenes, olivine and diopside, zoisite and klinozoisite however on the ground of the character of the extinction is not to be insisted upon; only in case of small axis-angles this characteristic has some value as a criterion. What gave rise to the calculation of the angles of extinction for olivine was that considerable extinctions were found with respect to a particularly well developed pinacoidal cleavage of this mineral, whilst, to compare them with the results obtained from this, I have also made the same calculations for talc.

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1) Mineralogical Magazine, XIII, 1903, p. 66—68.

2) MICHEL LÉVY, Ann. d. Mines, (7), XII, 1877, p. 392—471, Abstract Zeitschr. f. Kryst. III, 1879, 217—231; Minéraux des Roches, 1888. p. 9 seq.; FOUQUÉ et MICHEL LÉVY, Minéralogie Micrographique, Paris 1879; A. HARKER, Min. Mag. X, 1893, p. 239—240; G. CÉSARO, Mém. cour. Acad. Roy. Belg. LIV, 1895; DALY, Proc. Americ. Acad. Arts a. Sc. XXXIV, 1899, p. 311—328; A. A. FERRO, Riv. di Min.; Padua XX, 1898; Atti Soc. Lig. di Sc. nat. Genova, IX, 1898, Abstract Zeitschr. f. Kryst. XXXII, 1900, 532; VICENTE DE SOUSA BRANDÃO, Communicações da direcção d serviç. geol. de Portug. IV, 1901, 13—126.

3) Cf. FOUQUÉ et MICHEL LÉVY, Minéralogie Micrographique, p. 55—57.

4) Cf. LACROIX, about Fouqueit in Contributions à l'étude des gneiss à pyroxène et des roches à wernérite, Bull. Soc. franç. de Minéralogie XII, Paris 1889, p. 328.

Olivine and Talc.

If O be the intersection of the acute bisectrix with the globe of projection $\rho = 1$, A and B the projections of the optic axes, ZO the axis of a zone, from which ZQb represents an arbitrary plane with its pole N, then, according to FRESNEL, the extinction on the plane ZQb, with respect to the zone-axis, is represented by the curve Zc, when the plane cN divides the angle BNA into two equal parts. Suppose we call $\frown OQ$, the inclination of the plane (N) with regard to the acute bisectrix, x, and the angle of extinction with respect to the zone-axis, $\frown Zc = y$, then, according to MICHEL LÉVY [1]) the value of y can be calculated from the equation:

$$cot\, 2y = cot\, (aZ + bZ).$$

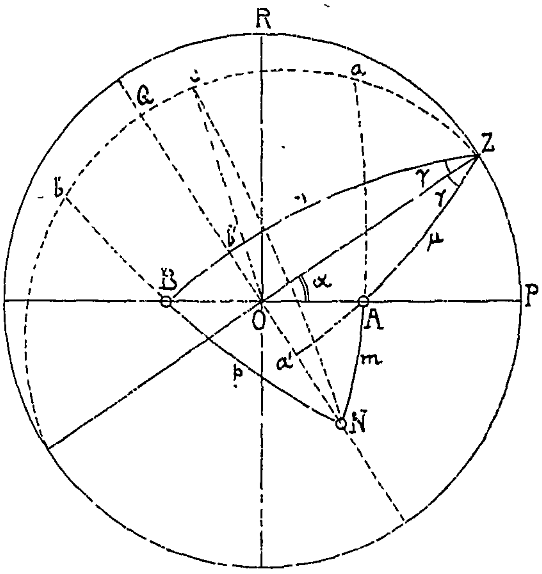

Fig. 1.

$$\frown aZ = \frac{\pi}{2} - \angle ANa'.$$

Now in the right-angled $\Delta ANa'$

$$tg\, ANa' = \frac{tg\, Aa'}{sin\, Na'} = \frac{cot\, \mu}{cos\, (x + \gamma)}$$

so that:

$$tg\, aZ = cot\, ANa' = tg\, \mu\, cos\, (x + \gamma).$$

In the same way we find:

[1]) Les Minéraux des Roches, p. 9.

$$tg\, bZ = tg\, \nu \cos(x - \gamma).$$

Now

$$\cot 2y = \cot(aZ + bZ) = \frac{1 - tg\, aZ\, tg\, bZ}{tg\, aZ + tg\, bZ} = \frac{1 - tg\, \mu\, tg\, \nu \cos(x+\gamma)\cos(x-\gamma)}{tg\, \mu \cos(x+\gamma) + tg\, \nu \cos(x-\gamma)} \,. \quad (1)$$

As is indicated in the figure, here the particular case is considered that the zone-axis lies in the plane forming right angles with the acute bisectrix, so that $\mu + \nu = \pi$. If we let the zone-axis successively make different angles α with OP, varying from 0 to π, and if, at the same time, we let this plane perform a revolution about this axis, then N passes through the whole surface of the globe and consequently the extinction with regard to OZ can be calculated as a function of α and x for each arbitrary section through the crystal.

As $\mu + \nu = \pi$, the formula (1) can be simplified as follows:

$$\cot 2y = \frac{1 + tg^2 \mu \cos(x + \gamma)\cos(x - \gamma)}{tg\, \mu\, [\cos(x + \gamma) - \cos(x - \gamma)]}$$

from which we derive:

$$\cot 2y = \frac{-(\cos^2 \mu + \sin^2 \mu \cos^2 \gamma) + \sin^2 \mu \sin^2 x}{\sin 2\mu \sin \gamma \sin x} =$$

$$= -\frac{\cos^2 \mu + \sin^2 \mu \cos^2 \gamma}{\sin 2\mu \sin \gamma} \cdot \frac{1}{\sin x} + \frac{\sin^2 \mu}{\sin 2\mu \sin \gamma}, \sin x \quad . \quad . \quad (2)$$

$$= \frac{A}{\sin x} + B \sin x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now in $\Delta\, ZOA$ $\cos \mu = \sin OA \cos \alpha = \sin V \cos \alpha$

and in $\Delta\, ZPA$ $\cos \angle AZP = \dfrac{tg\, PZ}{tg\, AZ}$

or

$$\cos\left(\frac{\pi}{2} - \gamma\right) = \sin \gamma = \frac{tg\, \alpha}{tg\, \mu}.$$

If in (2) we substitute the values in α and V for μ and γ, we get:

$$\cot 2y = -\frac{1 - \sin^2 \mu \sin^2 \gamma}{\sin 2\mu \sin \gamma} \cdot \frac{1}{\sin x} + \frac{\sin^2 \mu}{\sin 2\mu \sin \gamma} \cdot \sin x =$$

$$= -\frac{1 - tg^2 \alpha \cos^2 \mu}{2 \cos^2 \mu\, tg\, \alpha} \cdot \frac{1}{\sin x} + \frac{1 - \cos^2 \mu}{2 \cos^2 \mu\, tg\, \alpha} \cdot \sin x =$$

$$= -\frac{1 - \sin^2 V \sin^2 \alpha}{\sin 2\alpha \,.\, \sin^2 V} \cdot \frac{1}{\sin x} + \frac{1 - \sin^2 V \cos^2 \alpha}{\sin 2\alpha \sin^2 V} \cdot \sin x \quad . \quad . \quad (4)$$

From this form we can deduce what follows. For $x = 0$, y

becomes $=\frac{\pi}{2}$ for all values of a, and the same thing takes place with $a=0$ for all values of x. Consequently on all planes parallel to the acute or to the obtuse bisectrix the extinction with respect to these bisectrices is straight. If the direction of the mean index of refraction (OR) becomes zone-axis, with $a=\frac{\pi}{2}$, a certain particularity shows itself. For this value of a (4) assumes the following form:

$$\cot 2y = \frac{1}{\underset{(2a=\pi)}{\sin 2a}}\left(-\frac{1-\sin^2 V}{\sin^2 V}\cdot\frac{1}{\sin x}+\frac{\sin x}{\sin^2 V}\right)=$$

$$=\frac{1}{\underset{(2a=\pi)}{\sin 2a}}\left(-\frac{\cos^2 V}{\sin^2 V.\sin x}+\frac{\sin x}{\sin^2 V}\right) \quad . \quad . \quad . \quad . \quad (5)$$

For $x=0$ becomes $y=\frac{\pi}{2}$.

For $x=\frac{\pi}{2}-V$ (5) changes into:

$$\cot 2y = \frac{1}{\underset{(2a=\pi)}{\sin 2a}}\left(-\frac{\cos V}{\sin^2 V}+\frac{\cos V}{\sin^2 V}\right)=\frac{0}{0}.$$

y becomes indefinite; the pole N of the plane at this moment coincides with an optical axis.

Finally for $x=\frac{\pi}{2}$ y becomes $=0°$. So the extinction is $\frac{\pi}{2}$ for a value of x between $0°$ and $\frac{\pi}{2}-V$, next becomes indefinite and remains $0°$ for $x=\frac{\pi}{2}-V$ to $\frac{\pi}{2}$, as the sign for $\cot 2y$ shows.

As to the values of y in general, the following may be observed. In (4), if $\frac{\pi}{2}>V>0$ is assumed, is always

$$1 \geqq 1-\sin^2 V \sin^2 a > 0$$
$$1 \geqq 1-\sin^2 V \cos^2 a > 0$$

For a given value of a $\cot 2y$ keeps the same sign, if x varies between 0 and π; it gets, however, negative values for x between 0 and $-\pi$. If we confine ourselves to a variation of x between the limits 0 and $\frac{\pi}{2}$, then the sign of $\cot 2y$ becomes negative for the values of α, lying between 0 and $\frac{\pi}{2}$; it becomes positive, however for $\frac{\pi}{2} < \alpha < \pi$, whilst the absolute values of y are equal for two poles, lying symmetrically with regard to the plane RO. The same thing holds good for the extinction on planes, lying symmetrically with regard to the plane OP, so that the isogyres drawn upon the globe will lie symmetrically with respect to the planes RO, OP and also RP. Just as the symmetry with regard to RO and OP is accompanied by a change of sign, so also for the plane RP.

The extinction with regard to the variable zone-axis OZ is easy to reduce to that with respect to the acute bisectrix, as the latter is yielded by $\angle ONc = \frown Qc = \frac{\pi}{2} - y = y'$.

$$y = \frac{\pi}{2} - y'$$

$$\cot 2y = \cot(\pi - 2y') = -\cot 2y'.$$

from which follows according to (3)

$$\cot 2y' = -\frac{A}{\sin x} - B \sin x \qquad \ldots\ldots (6)$$

in which:

$$A = -\frac{\cos^2 \mu + \sin^2 \mu \cos^2 \gamma}{\sin 2\mu \sin \gamma}$$

$$B = \frac{\sin^2 \mu}{\sin 2\mu \sin \gamma}.$$

For the determination of the greatest extinction with regard to the acute bisectrix with $x =$ constant and a variable angle α, we may set about as follows[1]). If we call $\angle ANO = \psi$, $\angle BNO = \psi'$, we find from the triangles ANO and BNO

$$tg\,\psi = \frac{\sin \angle AON}{\sin ON \cot V - \cos ON \cos \angle AON} = $$

$$= \frac{\sin V \cos \alpha}{\cos x \cos V - \sin x \sin V \sin \alpha}$$

and

$$tg\,\psi' = \frac{\sin V \cos \alpha}{\cos V \cos x + \sin V \sin x \sin \alpha}.$$

Now $2y' = \psi - \psi'$

$$tg\,2y' = tg(\psi - \psi') = \frac{-2 \sin^2 V \sin \alpha \cos \alpha \sin x}{\sin^2 V \cos^2 \alpha + \cos^2 V \cos^2 x - \sin^2 V \sin^2 \alpha \sin^2 x},$$

which gives:

1) A. Harker, Min. Mag. XIII, 1903, p. 66—67.

$$\cot^2 \alpha (\sin^2 V + \cos^2 V \cos^2 x) - 2 \cot \alpha \sin^2 V \sin x \cot 2y' + (\cos^2 x - \sin^2 V) = 0$$

$$\cot \alpha = \frac{\sin^2 V \sin x \cot 2y'}{\sin^2 V + \cos^2 V \cos^2 x} \pm$$

$$\pm \sqrt{\left(\frac{\sin^2 V \sin x \cot 2y'}{\sin^2 V + \cos^2 V \cos^2 x}\right)^2 - \left(\frac{\cos^2 x - \sin^2 V}{\sin^2 V + \cos^2 V \cos^2 x}\right)} .$$

As long as the second term remains smaller than the first, the condition for which being $\cos x > \sin V$, or $x < \frac{\pi}{2} - V$, this equation will yield two positive roots, and accordingly two values of α between 0 and $\frac{\pi}{2}$ will satisfy it at a given value of $2y$ smaller than the maximum. The extinction will have reached the maximum, when the two roots are equal, so if

$$(\sin^2 V \sin x \cot 2y')^2 = (\sin^2 V + \cos^2 V \cos^2 x)(\cos^2 x - \sin^2 V)$$

or:

$$\sin 2y'_{max} = \frac{\sin V \operatorname{tg} V}{\cos x \cot x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

whilst the corresponding value of α [1]) is found from

$$\cot \alpha_{max} = \frac{\sin^2 V \sin x \cot 2y'_{max}}{\sin^2 V + \cos^2 V \cos^2 x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The plane ON, in which lies the corresponding pole, then makes with the plane OP an angle $\left(\frac{\pi}{2} - \alpha\right)$.

If we take for olivine the value $2V = 87°$ [2]), this gives according to the above formulas the following figures:

TABLE I.

x	μ	γ	A	B
15°	48°19' 35"	13°48' 20"	—4.0852	2.3539
30	53 24 25	25 23 10	—2 1480	1.5708
45	60 52 23	33 51 47	—1.6103	1.6103
60	69 52 7	39 25 0	—1.5708	2.1482
75	79 44 14	42 30 39	—2.3548	4.0868

1) $\alpha_{(y'=max)}$ is denoted by α_{max}, wherever it could not give rise to ambiguity

2) Min. d. Roches, p. 248.

from which the following extinctions with respect to the acute bisectrix are calculated:

TABLE II.

x	Values of y' at $\nu =$				
	$\nu = 15°$	$\nu = 30°$	$\nu = 45°$	$\nu = 60°$	$\alpha = 75°$
15°	1°53′ 6″	3°36′ 38″	4°53′ 14″	5° 8′ 26″	3°32′ 41″
30	4 4 28	7 56 59	11 14 42	12 54 21	10 16 52
45	6 49 59	13 42 48	20 38 46	27 27 29	33 6 56
60	10 14 10	20 52 53	32 32 7	46 20 3	64 40 45
75	13 32 29	27 22 46	41 47 36	57 4 59	73 14 21
90	15 (+18″)	30 (+11″)	45	60 (—4″)	75 (—1″)

The values for y' found by calculation with $x = 90°$, which accordingly represent the limit of extinction with regard to the acute bisectrix on the plane making right angles with the latter, give a measure for the exactitude of the values found. The errors successively amount to $+18''$, $+11''$, $0''$, $-4''$, $-1''$.

The greatest extinction for different values of x with the corresponding angle $\left(\frac{\pi}{2} - \alpha\right)$ are now to be calculated from the formulas (7) and (8); we come to the following result:

x	y'_{max}	$\frac{\pi}{2} - \alpha$
15°	5°13′ 12″	34°36′ 4″
30	12 54 29	29 25 6
45	33 44 35	10 40 21
$\frac{\pi}{2} - V =$ 46°30′	45°	0°

To calculate from

$$\cot \alpha_{max} = \frac{\sin^2 V \sin x \cot 2y'_{max}}{\sin^2 V + \cos^2 V \cos^2 x}$$

the value of α_{max} when $\sin x = 0$, we eliminate y'.

As

$$\sin 2y'_{max} = \frac{\sin V \, tg \, V}{\cos x \ \cot x}$$

we get

$$\cot a_{max} = \frac{\dfrac{\sin^2 V \sin x \sqrt{1 - \left(\dfrac{\sin V \, tg \, V}{\cos x \ \cot x}\right)^2}}{\dfrac{\sin V \, tg \, V}{\cos x \ \cot x}}}{\sin^2 V + \cos^2 V \cos^2 x} =$$

$$= \frac{\sin^2 V \sin x \sqrt{\cos^2 x \cot^2 x - \sin^2 V \, tg^2 \, V}}{\sin V \, tg \, V (\sin^2 V + \cos^2 V \cos^2 x)} =$$

$$= \frac{\sin x \cos V \sqrt{\dfrac{\cos^4 x}{\sin^2 x} - \dfrac{\sin^4 V}{\cos^2 V}}}{\sin^2 V + \cos^2 V \cos^2 x} =$$

$$= \frac{\sqrt{\cos^4 x \cos^2 V - \sin^2 x \sin^4 V}}{\sin^2 V + \cos^2 V \cos^2 x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When $x = 0$, is

$$\cot a_{max} = \pm \cos V.$$

From which for olivine follows the value:

$$\left(\frac{\pi}{2} - a\right) = bg \, tg \, (\pm) \cos V =$$

$$= bg \, tg \, (\pm) \cos 43°30'$$

$$= (\pm) \, 35°57'22''.$$

In the following figure these results are graphically represented. The black lines connect the poles of planes with equal positive, the lines in black and white those of planes with equal negative extinction. Herein the angles have been considered positive from the acute bisectrix in the direction of the hands of the clock; negative in the opposite direction.

The curves MM' and NN', going through the optic axes, connect the poles of the planes with the greatest (positive and negative) extinction and with the same inclination with regard to the acute bisectrix. The point in which the curves mentioned intersect an isogyre, has on that isogyre the greatest angulardistance from O. For the rest very little need be added to what is to be read from the figure. It shows clearly that an extinction with regard to the acute bisectrix, which deviates little from 0°, is confined to the immediate neighbourhood of the principal planes of symmetry.

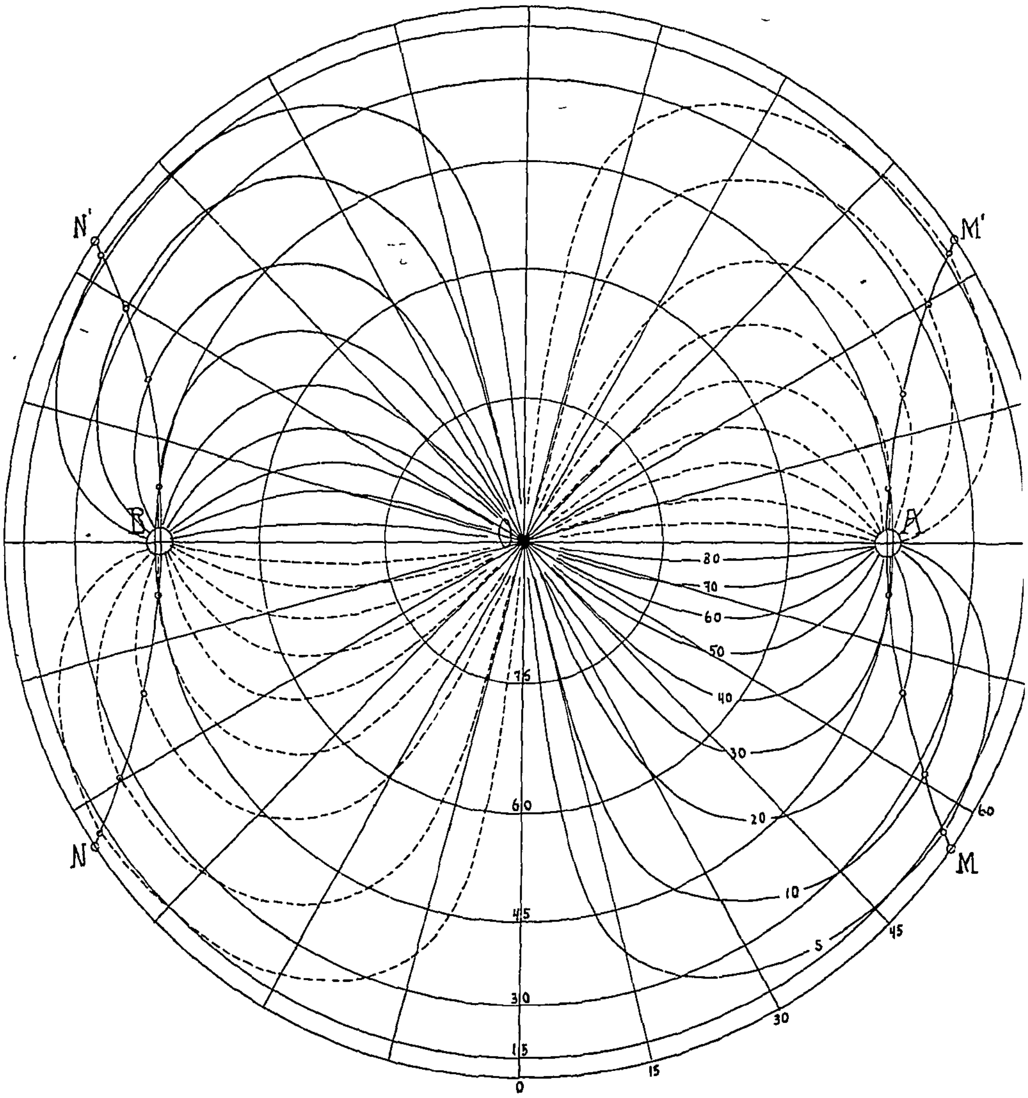

Fig. 2

For talc ($2V = 13°$)[1] the following values are calculated in the same way:

[1]) HINTZE, Handbuch der Mineralogie, II, 1897, p. 815, cf. BAUER, Pogg. Ann. 1869, CXXXVIII, 368.

	μ	ι	*A*	*B*
15°	83°43′ 21″	1°42′ (± 1′)	— 156.05	154.32
30	84 22 26	3 15	— 89 846	89 268
35	84 40 45	3 44	— 82 815	82 45
40	85 1 25	4 11	— 78 895	78 718
45	85 24 31	4 36	— 77 628	77.628
50	85 49 38	4 59	— 78 717	78.891
55	86 16 38	5 20	— 82 31	82 672
60	86 45 19	5 38	— 89 268	89 846
65	87 15 5	5 53 30″	— 100.62	101 46
70	87 46 48	6 6	— 120.16	121 38
75	88 19 16	6 16 30	— 154 32	156.05
80	88 52 25	6 24	— 225.49	228.14
85	89 26 4	6 28 30	— 443.61	449 28

to which correspond the extinctions:

x	$\alpha = 15$	$\alpha = 30$	$\alpha = 35$	$\alpha = 40$	$\alpha = 45$	$\alpha = 50$	$\alpha = 55$
15	0° 3′ 4″	0° 4′ 54″	—	—	0° 6′ 9″	—	—
30	0 7 9	0 12 44	—	—	0 14 46	—	—
45	0 8 12	0 26 53	—	—	0 31 19	—	—
60	0 36 55	1 4 59	—	—	1 16 39	—	—
70	1 21 38	2 26 12	2°40′ 54″	2°51′ 34″	2 57 13	2°57′ 48″	2°52′ 14″
75	2 17 13	4 11 16	4 39 53	5 1 35	5 15 26	5 21 1	5 17 6
80	4 23 11	8 24 35	9 31 30	10 33 35	11 23 43	12 1 43	12 24 4
85	9 27 9	19 10 28	22 33 26	26 2 27	29 36 9	33 23 25	37 31 53
90	15°(+53″)	30°(—50″)	—	—	45°	—	—

x	$\varphi = 60$	$\varphi = 65$	$\varphi = 70$	$\varphi = 75$	$\varphi = 80$	$\varphi = 85$	$\varphi = 90$
15	0° 5′ 21″	—	—	0° 3′ 14″	—	—	0°
30	0 12 52	—	—	0 7 28	—	—	0
45	0 27 21	—	—	0 15 56	—	—	0
60	1 7 57	—	—	0 39 55	—	—	0
70	2 41 57	2°26′ 6″	2° 4′ 20″	1 7 40	1° 7′ 27″	0°34′ 26″	0
75	5 1 13	4 35 53	3 58 51	3 9 35	2 12 10	1 7 58	0
80	12 19 51	11 53 14	10 58 49	9 9 38	6 42 42	3 33 47	0
85	41 34 43	46 43 1	53 20 59	59 24 19	67 38 33	78 3 57	90
90	60°(+51″)	—	—	75°(— 8″)	—	—	90°

For the greatest extinction and corresponding angle we find:

x	$y'_{max.}$	$\frac{\pi}{2} - \varphi$
0°	0°	44°48′ 56″
15	0° 6′ 9″	44 48 31
30	0 14 47	44 41 24
45	0 31 21	44 27 2
60	1 16 50	43 38 28
75	5 21 33	39 38 9
$\frac{\pi}{2} - V =$ 83°30′	45°	0°

Figure 3 affords a general view of the results. Suppose the part of the globe-surface, falling outside the parallel-circle of 60° but within the isogyre of 1°, to be equal to the part falling within the same circle outside the isogyre, then it appears that at about $^7/_8$ of the sphere an extinction of less than 1° is observed, so practically a straight extinction. Now the sections, yielding greater extinctions, lie so much in the neighbourhood of the planes, making right angles with the optic axes, that they are for the greater part impracticable for the determination of the direction of extinction. A comparison of figures 2 and 3 shows the result that with rhombic

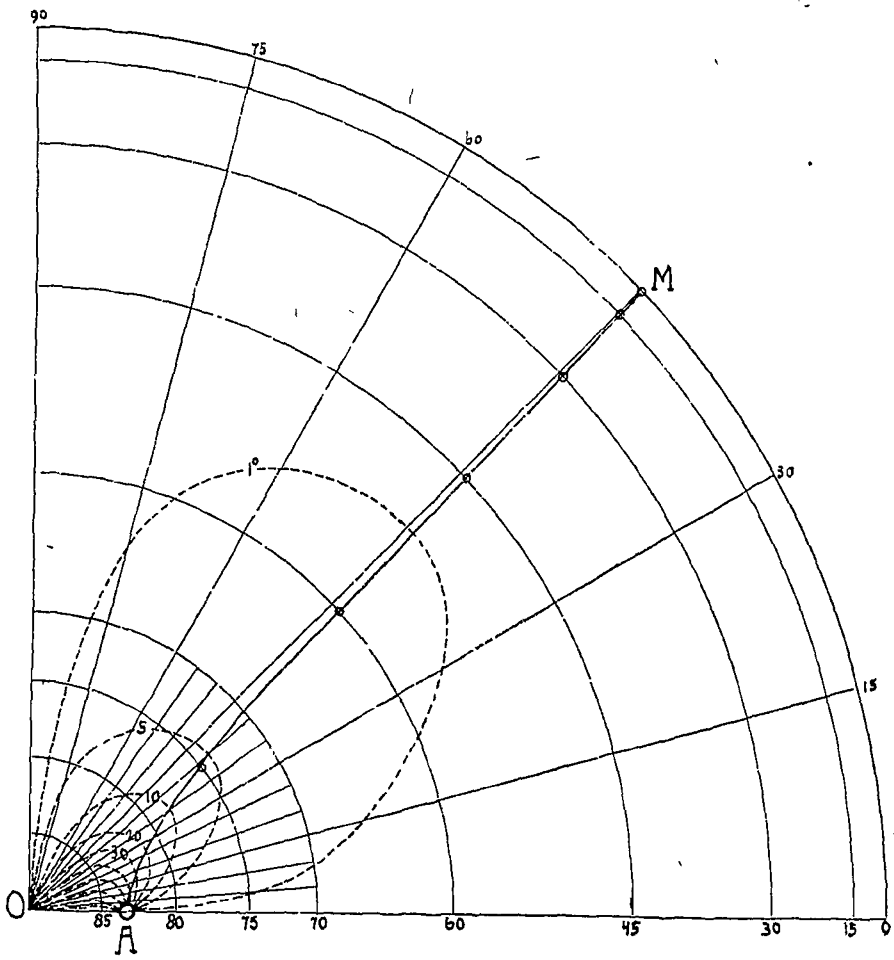

Fig. 3.

crystals with a great axis-angle the oblique, with those with a small axis-angle the straight extinction will predominate, when we have to do with arbitrary sections, as in a rock slice. However in the two cases we as rarely find an absolutely straight extinction. With hexagonal and tetragonal crystals, however, exclusively straight extinction with regard to the optic axis occurs, as for $V=0$ the equation (4)

$$\cot 2y = \frac{1}{\sin^2 V}\left(-\frac{1-\sin^2 V \sin^2 \alpha}{\sin 2\alpha} \cdot \frac{1}{\sin x} + \frac{1-\sin^2 V \cdot \cos^2 \alpha}{\sin 2\alpha} \sin x\right)$$

always becomes ∞.

In fig. 4 the maximum extinction as a function of x is represented for one globeoctant; MA_1 refers to talc, MA_2 to olivine, MA_3 to a mineral with an axis-angle $2V=160°$. The values $O'A_1$, $O'A_2$

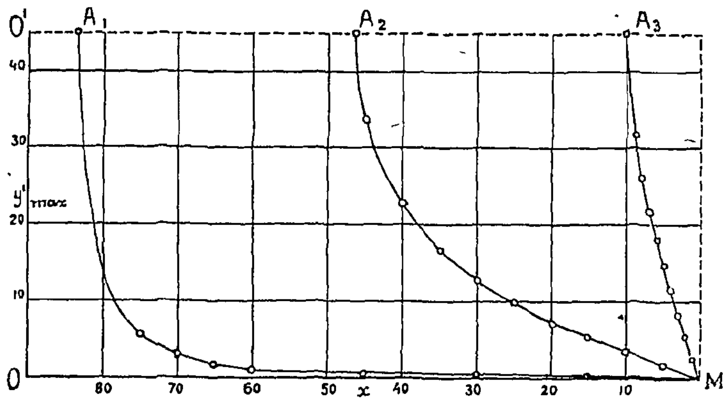

Fig. 4.

and $O'A_3$ give the size of V. The general èquation of the curves MA is:

$$\sin 2y'_{max} = \frac{\sin V \, tg\, V}{\cos x \cot x},$$

$$y'_{max} = \tfrac{1}{2}\, bg \sin \left(\frac{\sin V \, tg\, V}{\cos x \cot x}\right)$$

from which

$$\frac{dy'_{max}}{dx} = \frac{\sin V \, tg\, V \cdot}{2\sqrt{1-\left(\frac{\sin V \, tg\, V}{\cos x \cot x}\right)^2}} \cdot \left(\frac{1+2tg^2 x}{\cos x}\right) =$$

$$= \frac{\sin V \, tg\, V\, (1+2tg^2 x) \cot x}{2\sqrt{\cos^2 x \cot^2 x - \sin^2 V \, tg^2 V}} =$$

$$= \frac{\sin^2 V}{2} \cdot \frac{1+\sin^2 x}{\cos x \sqrt{\cos^4 x \cos^2 V - \sin^2 x \sin^4 V}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

So for $x = 0$ the direction of the tangent is given by:

$$\frac{dy'_{max}}{dx} = \frac{\overset{+}{(-)}\sin^2 V}{2\cos V}.$$

For $x = \frac{\pi}{2} - V$ by

$$\frac{dy'_{max}}{dx} = \infty$$

as the term under the root-mark becomes $= 0$. So the tangents in A on the curves form right angles with the direction MO.

It further follows from the formula (10) that the rise of the curve for the same value of x grows smaller as the value of V diminishes, as is also shown by fig. 4. If V becomes $= 0$, as in the hexagonal and tetragonal system, then we also have

$$\frac{dy'_{max}}{dx} = 0\,,$$

so that the curve MA coincides with the abscissa-axis MO. Finally with regard to the form of the curve which represents the angle $\left(\frac{\pi}{2} - \alpha_{max}\right)$ as a function of x, it appears already from a comparison of figures 2 and 3, that this curve MA, the axis-angle becoming smaller, gradually approaches the straight line that divides into two equal parts the angle between OA and the normal to it in O.

Indeed (9) yields

$$cot\ \alpha_{max} = \frac{\sqrt{cos^4 x\, cos^2 V - sin^2 x\, sin^4 V}}{sin^2 V + cos^2 V\, cos^2 x}\,,$$

$$= tg\left(\frac{\pi}{2} - \alpha_{max}\right) = \frac{\sqrt{cos^4 x - (cos^4 x + sin^2 x\, sin^2 V)\, sin^2 V}}{cos^2 x + sin^2 x\, sin^2 V}\,.$$

If V becomes smaller, $tg\left(\frac{\pi}{2} - \alpha_{max}\right)$ increases, and with $V = 0$ reaches the greatest value $\overset{+}{(-)}$ 1, so that then α_{max} becomes $= 45°$. The curves $M'AM$ and NBN' then pass into two straight lines which intersect in O, thus forming right angles, whilst they have shifted 45° with regard to direction AB.

Of great practical importance is the solution of the problem, how

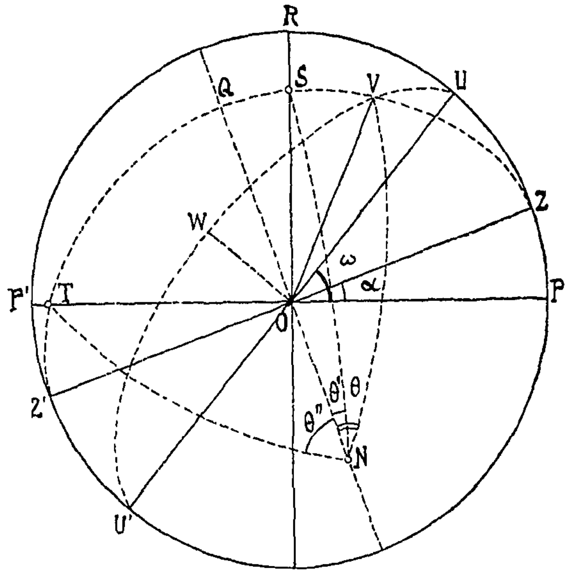

Fig 5.

great the extinction is with regard to the trace of a cleavage plane. For this, if the angle of extinction with regard to the acute bisectrix be known, is only necessary the value of the apparent angle between the trace mentioned and the same bisectrix. One value need only be subtracted from the other.

If ZZ' be the axis of a zone, in which ZQZ_1 represents an arbitrary plane, N the corresponding pole, and be the plane determined again by a and $OQ = x$; if UWU' be an arbitrary cleavage plane, determined by ω and $WO = y$, then VO is the line of intersection of both planes, $VQ = \angle QNV = \theta$, the apparent angle between the acute bisectrix (O) and VO.

Now $VQ = \frac{\pi}{2} - VZ$, and in ΔVUZ is

$$\angle Z = \frac{\pi}{2} - x$$

$$\angle U = \frac{\pi}{2} + y$$

$$UZ = \omega - a.$$

So

$$cot\ VZ = tg\ \theta = \frac{sin\left(\frac{\pi}{2} - x\right) cot\left(\frac{\pi}{2} + y\right) + cos\left(\frac{\pi}{2} - x\right) cos\,(\omega - a)}{sin\,(\omega - a)}$$

$$= \frac{-\,cos\ x\ tg\ y + sin\ x\ cos\,(\omega - a)}{sin\,(\omega - a)} \quad . \ . \ . \ . \ . \ . \ . \quad (11)$$

If we apply this formula to the cleavage planes h^1 (100) and g^1 (010) of olivine, then with

$$h^1\,(100) \ . \ . \ . \quad \omega = 0\,,\ y = 0$$

$$g^1\,(010) \ . \ . \ . \ . \ \omega = \frac{\pi}{2}\,,\ y = 0$$

and (11) passes into:

$$tg\ \theta' = sin\ x\ tg\ a$$

$$tg\ \theta'' = -\ sin\ x\ cot\ a$$

in which θ' and θ'' are successively the apparent angles between the traces of g' (010) and h' (100) on the plane (N).

Now if we think both x and a to vary between 0 and $\frac{\pi}{2}$, we find the following values for θ' and θ'':

i′	$x=0$	$x=15$	$x=30$	$x=45$	$x=60$	$x=75$	$x=90$	ϑ at θ' (+)
ɔ	indef.	90°	90°	90°	90°	90°	90°	90°
	0°	86° 8′ 30″	88° 0′ 2″	88°35′ 50″	88°50′ 43″	88°57′ 53″	89	89
	0	82 18 57	86 0 17	87 10 22	87 41 27	87 55 46	88	88
	0	78 33 11	84 0 59	85 45 40	86 32 13	86 53 39	87	87
	0	74 52 5	82 2 19	84 21 9	85 23 2	85 51 34	86	86
	0	71 19 25	80 4 30	82 56 43	84 13 53	84 49 28	85	85
	0	55 44 4	70 34 28	75 59 53	78 29 29	79 39 17	80	80
	0	44 0 26	61 48 38	69 14 48	72 48 29	74 29 45	75	75
	0	24 8 46	40 33 34	50 46 7	56 18 35	59 7 56	60	60
	0	14 30 39	26 33 55	35 15 53	40 53 36	44 0 24	45	45
	0	8 29 56	16 6 8	22 12 28	26 33 54	29 8 50	30	20
	0	3 58 2	7 37 51	10 43 43	13 3 52	14 30 38	15	15
	0	2 36 57	5 2 18	7 6 25	8 40 56	9 39 57	10	10
	0	1 17 49	2 30 17	3 32 24	4 19 58	4 49 49	5	5
	0	1 2 13	2 0 9	2 49 50	3 27 56	3 51 50	4	4
	0	0 46 38	1 30 4	2 7 20	2 35 55	2 53 52	3	3
	0	0 31 4	1 0 1	1 24 52	1 43 56	1 55 55	2	2
	0	0 15 32	0 30 0	0 42 26	0 51 53	0 57 57	1	1
	0	0°	0°	0°	0°	0°	0	0

s is also shown in the table

$$\theta' = bg\ tg\ \sin x\ tg\ \alpha$$

mes:

$x=0,\ \alpha < \frac{\pi}{2}$ equal to 0,

$x=0,\ \alpha = \frac{\pi}{2}$ indefinite, and

$\alpha = 0$ also every time 0, whilst

$$-\theta'' = bg\ tg\ \sin x\ \cot \alpha$$

$x=0,\ \alpha > 0$ equal to 0,

$x=0,\ \alpha = 0$ indefinite, and lastly

$\alpha = \frac{\pi}{2}$ always becomes 0.

order to know the extinction of the plane (N) with respect to trace of g' (010) or h' (100), we combine this table with table II.

For $\frac{\pi}{2} \geqq \alpha \geqq 0$, $\frac{\pi}{2} \geqq x \geqq 0$ the extinctions successively become $(-\theta' + y')$ and $(-\theta'' + y')$; we find the following values:

$$\varphi' = y' - \theta^1.$$

α	$x=0$	$x=15$	$x=30$	$x=45$	$x=60$	$x=75$	$x=9$[illegible]
0°	0°	0°	0°	0°	0°	0°	0°
15	0	— 2° 4′ 56″	— 3°33′ 23″	— 3°33′ 44″	— 2°49′ 42″	— 0°58′ 9″	0
30	0	— 4 53 18	— 8 9 9	— 8 29 40	— 5 41 1	— 1 46 4	0
45	0	— 9 37 25	—15 19 13	—14 37 7	— 8 21 29	— 2 12 48	0
60	0	—19 0 20	—27 59 13	—23 18 38	— 9 58 32	— 2 2 57	0
75	0	—40 27 45	—51 31 46	—36 7 52	— 8 7 44	— 1 15 24	0
90	indif.	—90°	—90°	—90°	0°	0°	0

When $\alpha = 90°$ and $x = \left(\frac{\pi}{2} - V\right) = 46°30'$ the extinction becomes indefinite; it is here that the transition from 90° to 0° takes place, In the same way we find for

$$\varphi'' = y' - \theta''.$$

α	$x=0$	$x=15$	$x=30$	$x=45$	$x=60$	$x=75$	$x=90$
0°	indefinite	90°	90°	90°	90°	90°	90°
15	0°	45°53′ 32″	65°53′ 6″	76° 4′ 47′	83° 2′ 39″	88° 2′ 14″	90
30	0	27 45 24	48 50 33	64 28 55	77 11 28	86 30 42	90
45	0	19 23 53	37 48 37	55 54 59	73 25 43	85 48 0	90
60	0	13 38 22	29 0 29	49 39 57	72 53 57	86 13 49	90
75	0	7 30 43	17 54 43	43 50 39	77 44 37	87 44 59	90
90	0	0°	0°	0°	90°	90°	90

Also here the extinction for $\alpha = 90°$ and $x = \left(\frac{\pi}{2} - V\right) = 46°30'$ becomes indefinite.

The shape of the φ-isogyres is represented in fig. 6. The black lines refer to φ'', those in black and white to φ', φ'' gives the value of the positive extinction with regard to the trace of h' (100), φ'

that of the negative extinction with reference to the trace of g' (010) on the plane (N).

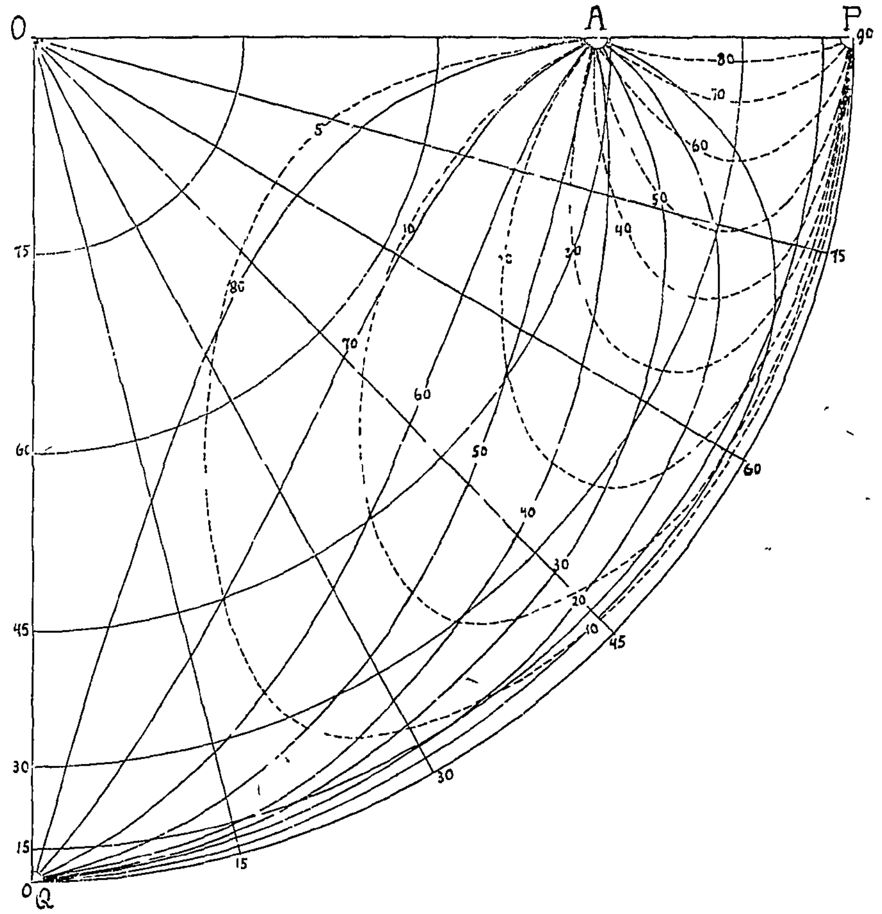

Fig. 6.

The figure is with regard to the axes OP and OQ quite symmetrical again, in the same way as fig. 2 shows for the isogyres with respect to the acute bisectrix, because the points here correspond to the value:

$$\varphi' = \tfrac{1}{2}\, lg\, cot \left(\frac{1 - sin^2 V\, sin^2 \alpha}{sin\, 2\alpha \,.\, sin^2 V} \cdot \frac{1}{sin\, x} + \frac{1 - sin^2 V\, cos^2 \alpha}{sin\, 2\alpha\, sin^2 V} \,.\, sin\, x \right) -$$

$$- lg\, tg\, (sin\, x\, tg\, \alpha) = y' - \theta'.$$

$$\varphi'' = \tfrac{1}{2}\, lg\, cot \left(\frac{1 - sin^2 V\, sin^2 \alpha}{sin\, 2\alpha \,.\, sin^2 V} \cdot \frac{1}{sin\, x} + \frac{1 - sin^2 V\, cos^2 \alpha}{sin\, 2\alpha\, sin^2 V} \,.\, sin\, x \right) +$$

$$+ lg\, tg\, (sin\, x\, cot\, \alpha) = y' - \theta''.$$

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Now the signs of y', θ' and θ'' remain unchanged for $0 < x < \pi$ and for a varying between 0 and $\frac{\pi}{2}$, so that also here the sign of the extinction in adjoining globe octants will be alternately positive and negative. The points in which equivalent isogyres in the systems φ' and φ'' intersect, indicate the places of the poles of the sections, in which symmetrical extinction with respect to the cleavage planes h' (100) and g' (010) visible in the slice is observed. So there

$$\varphi' + \varphi'' = 2y' - (\theta' + \theta'') = 0.$$

$$2y' = \theta' + \theta''$$

The curve $OA = V$. If the axis-angle diminishes, A gradually approaches O; the isogyres φ' and φ'' approach a symmetrical direction with regard to the axes OP and OQ, so that the curve, connecting their points of intersection, draws nearer and nearer to the straight line, and at last, when V has become $= 0$, and A coincides with O, passes into the straight line which divides the angle POQ into two equal parts. For $V = 0$ y' becomes $= 0$, so:

$$\theta' + \theta'' = 0$$

$$\sin x \; tg\, a - \sin x \cot a = 0$$

$$tg\, a = \cot a$$

so that the geometrical place of the points of intersection of the isogyres φ' and φ'', i.e. that of the points of symmetrical extinction, is represented by the line

$$a = 45°.$$

Anatomy. — *"On ascending degeneration after partial section of the spinal cord."* By Dr. S. J. de Lange. (Communicated by Prof. C. Winkler).

(Communicated in the meeting of November 30, 1907).

The following researches have been made with the purpose of investigating whether there exist any connections between the spinal cord and the ascending fasciculus longitudinalis dorsalis, and on the other hand to ascertain once more the course of the ascending anterolateral fascicle of Gowers and its relation to the dorso-lateral fasciculi of the cerebellum.

From the extensive literature on this subject I will but rarely quote something, whenever the results obtained by others do not accord with my observations. The drawings are taken after four