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If we introduce this value of ψ in equation (8) we get

$$2 : \frac{3}{2} : 1 = z_1 : z_2 : z_3 ;$$

which is entirely consistent with the double proportion.

Consequently $n=6$ is the only value ≥ 5 that is consistent with the principle of the rational indices. If we extend the requirement that $\cos \psi$ must be rational to the case $n < 5$ then the values $n=2, 3, 4$, are also permissible for axes of the first kind, and the values $n=2, 4$ for axes of the second kind.

Göttingen, November 1907.

Physics. — *“Isotherms of diatomic gases and their binary mixtures.*

VI. *Isotherms of hydrogen between -104°C. and -217°C. ”*

(Continued). By Prof. H. KAMERLINGH ONNES and C. BRAAK.
Communication N°. 100^a from the Physical Laboratory at Leiden.

(Communicated in the meeting of November 30, 1907).

§ 17. *Survey of the determinations. Remark on the apparatus.*

The measurements mentioned in this Communication comprise in the first place the supplementary determinations to which we already alluded in § 14 of Comm. N°. 99^a (Sept. 1907). These are three determinations at -217° at a density of about 170 times the normal one.

The obvious thing to do further was to repeat the other determinations of series II with the same piezometer arranged for the determinations mentioned above, this piezometer being one of about the same dimensions as that of series II of Comm. N°. 97^a (March 1907). As a matter of fact a comparison of the values of p_{vA} obtained in this series with those yielded by the series III and IV teaches that the former lie somewhat, though only slightly, lower than the latter. This may be due to a systematical error as the filling in the later series was accomplished with more precautions (compare § 5 of Comm. N°. 97^a). In the series now given, just as in series IV, distilled hydrogen was used.

Both the steel tubes on the stem of the piezometer and those on the stem of the piezometer reservoir were soldered to the glass (cf. § 15 of Comm. N°. 99^a). This ensures a gas-proof connection with the steel capillary. With sealing wax it is difficult to make the connection gas-proof, because sometimes the nut begins to slide off when the flange is tightly screwed on.

§ 18. *Values of pv_A of series V.*

In table XX the results of the determinations have been represented in the same way as in table XII of Comm. N°. 97^a. The temperatures at which the measurements were made were: $-182^{\circ}.74$, $-195^{\circ}.16$, $-204^{\circ}.62$, $-212^{\circ}.91$ and $-215^{\circ}.94$. In table XX the reduction to the standard temperatures of table XII has been carried out. It was effected by interpolation by means of virial coefficients, which were derived in § 12 of Comm. N°. 97^a, which enabled us to abandon the elaborate method of § 8. The computation of the temperatures took place in the same way as in Comm. N°. 95^c (Nov. 1906). They may be reduced to the normal hydrogen scale by means of table XVIII of Comm. N°. 97^b (March 1907).

TABLE XX, H ₂ . Series V. Values of pv_A .				
Nº.	t_s	p	pv_A	d_A
1	$-182^{\circ}.81$	48.431	0.32746	147.90
2		55.499	0.32857	168.91
3		62.889	0.33028	190.41
4	$-195^{\circ}.27$	42.304	0.27362	154.61
5		47.782	0.27351	174.70
6		52.808	0.27360	193.01
7	$-204^{\circ}.70$	36.999	0.23165	159.72
8		41.258	0.23061	178.91
9		44.631	0.23001	194.04
10	$-212^{\circ}.82$	32.035	0.12414	165.01
11		34.611	0.19270	179.61
12		37.275	0.19149	194.66
13	$-217^{\circ}.41$	28.955	0.17318	167.63
14		31.191	0.17152	181.85
15		33.180	0.17005	195.12

§ 19. *Values of pv_A of series IV.*

If in the same way as in the preceding § the results of table XIX of Comm. N°. 99^a are reduced to the standard temperatures the values of the adjoined table are obtained. For $-139^{\circ}.88$ this

TABLE XXI, H₂. Series IV. Values of pv_A .

N ^o .	t_s	p	pv_A	d_A
1	— 103°.57	28.447	0.63261	44.967
2		38.186	0.63702	59.944
3		48.724	0.64198	75.897
4		58.368	0.64694	90.222
5	— 139°.88	25.406	0.49459	51.368
6		33.774	0.49697	67.960
7		41.273	0.49967	82.600
8		48.558	0.50232	96.667
9	— 164° 14 ¹⁾	22.818	0.40065	56.972
10		28.688	0.40164	71.427
11		34.387	0.40253	85.427
12		39.947	0.40376	98.936
13	— 182°.81	20.496	0.32704	62.670
14		24.818	0.32699	75.898
15		28.506	0.32672	87.248
16		32.568	0.32675	99.673
17	— 195°.27	18.527	0.27827	66.581
18		23.303	0.27724	84.055
19		27.837	0.27580	100.933
20		16.749	0.24036	69.684
21	— 204°.70	20.453	0.23876	85.658
22		24.015	0.23691	101.367
23		15.416	0.20644	74.679
24		18.038	0.20430	88.296
25	— 212°.82	20.643	0.20228	102.051
26		14.635	0.18738	78.103
27		16.784	0.18491	90.766
28		18.853	0.18289	103.080

¹⁾ This temperature has been derived from the comparison of the resistance thermometer with the hydrogen thermometer of July 3, '07 (see table I, Comm. N^o. 101a).

reduction has not been carried out, as it is better to take this temperature as standard temperature instead of $-135^{\circ}.71$. Here the reduction would have to be made for a comparatively large difference of temperature, and would become inaccurate. It is, therefore, preferable to leave the values of table XIX for this temperature intact, and necessarily apply the reduction to those of series I, which are much less reliable. The temperature $-164^{\circ}.14$ has been adopted as new standard temperature.

The determinations 5 and 9 as well as 14 and 18 of table XII have been united to a mean.

§ 20. *Comparison of the series I and II with the control-determinations.*

For reasons mentioned in § 17 the points of series I and II have been doubly determined in a mutually perfectly independent way. They can be easily compared with the control determinations of IV and V by reducing them to the same density and temperature by means of virial coefficients. If in this way for $-103^{\circ}.57$ Nos. 2, 3 and 4 of series I (see table XII) are compared with 1, 2 and 3 of series IV (see table XXI), we find for the differences of pv_A for IV—I respectively:

$$+0.00001, +0.00007, -0.00019$$

and for $-139^{\circ}.88$ for IV (5,6) — I (2,3):

$$-0.00085, -0.00036.$$

Dealing in the same way with the series II and V (see table XII and XX), we find respectively for the temperatures $-182^{\circ}.81$, $-195^{\circ}.27$, $-204^{\circ}.70$, and $-212^{\circ}.82$,

$$V(1,2) - II(2,3) = +0.00007, +0.00010$$

$$V(4,5,6) - II(2,3,4) = +0.00012, +0.00026, +0.00017$$

$$V(7,8,9) - II(2,3,4) = +0.00020, +0.00019, +0.00034$$

$$V(10,11,12) - II(2,3,4) = +0.00013, +0.00008, +0.00021$$

The differences are to be ascribed chiefly to condensation or impurities, as they diminish with increase of the temperature. This was considered as sufficient ground to reject the results of the series I and II for the further calculations as less reliable. This was also done for -104° , though the series I and IV harmonize very well for this temperature. When we disregard the influence of the probable condensation the very regular course in the situation of the points is an indication about the accuracy of the measurements themselves also for the other isotherms.

So there remain the determinations of the series III, IV and V, which, reduced to the standard temperatures, occur in the tables XII, XX and XXI. With these data the further calculations have been carried out. Plate I gives a survey of the situation of the points in the diagram of isotherms; on this plate $\frac{pv_A}{T}$ has been given as function of the density. (T absolute temperature). By I and II the isotherms of $100^{\circ}.20$, and 0° , which will be treated in the following communication, are indicated, by the other Roman figures ascending with decrease of temperature, those to $-217^{\circ}.41$.

§. 21. *Individual virial coefficients.*

In the same way as has been explained in § 12 of Comm. N^o. 97^a the first three virial coefficients of the development into series considered there were calculated for every isotherm, by means of the earlier and the new data. They have been put together in the subjoined table.

TABLE XXII. H ₂ Individual virial coefficients.					
t_s	A_A	$10^3 B_A$	$10^6 C_A$	$10^{12} D_A$	$10^{18} E_A$
$-103^{\circ}.57$	0.62748	+ 0.24409	+ 0.5300	+ 0.9113	- 0.648
$-139^{\circ}.88$	0.48765	+ 0.11175	+ 0.4034	+ 0.6753	- 0.378
$-164^{\circ}.14$	0.39891	+ 0.00732	+ 0.4148	+ 0.4970	- 0.208
$-182^{\circ}.81$	0.33063	- 0.07947	+ 0.3908	+ 0.3809	- 0.088
$-195^{\circ}.27$	0.28508	- 0.12309	+ 0.3165	+ 0.2892	- 0.016
$-204^{\circ}.70$	0.25074	- 0.17328	+ 0.3398	+ 0.2166	+ 0.031
$-212^{\circ}.82$	0.22103	- 0.22271	+ 0.3599	+ 0.1514	+ 0.066
$-217^{\circ}.41$	0.20424	- 0.24539	+ 0.3558	+ 0.1122	+ 0.082

In the same way as in Comm. N^o. 97^a the differences between the observed values of pv_A and those calculated by means of the found virial coefficients are put together in a table, which we subjoin.

The second column contains the differences for the points of the hydrogen thermometer (see table XII of Comm. N^o. 97^a), the following columns refer to the series IV, V and III in the order given here, the values being arranged according to the ascending densities for each series.

TABLE XXIII H₂. Deviations from the formula.

t_s	$10^5 (O_i - C_i)$									
-103° 57	-1	+8	-1	-11	+6					
-139° 88	-2	+13	-15	0	+4					
-164° 14	0	-3	+7	-6	+2					
-182° 81	-1	-15	+13	+3	+12	-15	-9	+12		
-195° 27	-4	-2	+25	-11	-16	0	+8			
-204° 70	-18	+4	+36	+22	-22	-24	-21	+23		
-212° 82	-14	+3	+12	+22	-5	-10	-4	-22	+18	
-217° 41	-14	+13	0	+15	-2	+1	-3	-22	-6	+18

It appears that on the whole series IV gives higher values than series V. The calculated curves may serve for the adjustment of the series mutually. Undoubtedly their points will be more reliable than those of the separate determinations. In future we shall, therefore, start from the virial coefficients of XXII.

§ 22. *Minima of pv_A .*

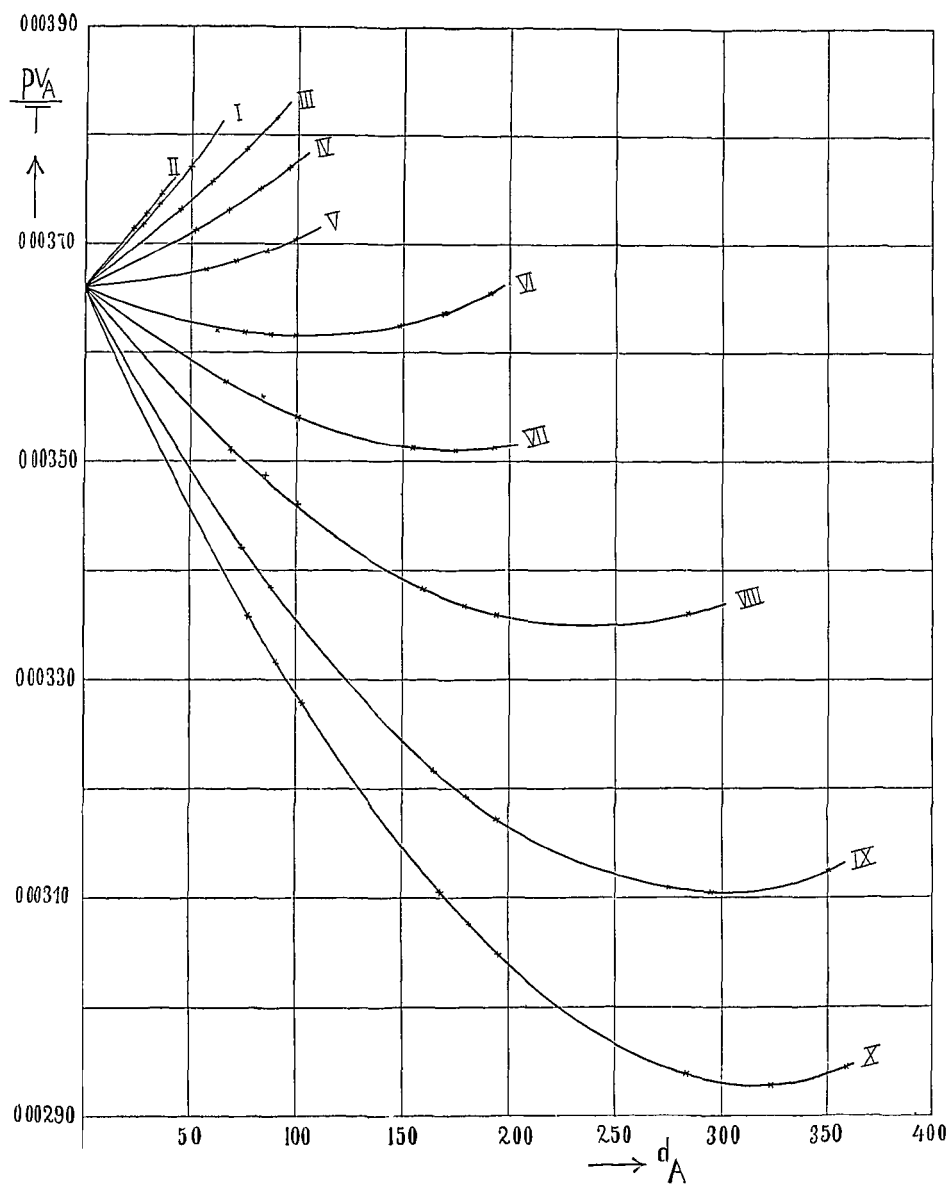
With the now available data the minima of the pv_A curves were now again determined for the lowest five temperatures, and as before the coefficients P_0 , P_1 and P_2 of a parabola calculated. The columns of table XXIV have the same meaning as those of table XV of Comm. N^o. 97^a.

TABLE XXIV. H₂. Minima of pv_A .

t_s	pv_A	d_A	p	$O-C$
-182° 81	0 32663	99 70	32 57	-0 29
-195° 27	0 57348	183 10	50 07	+1 36
-204° 70	0 22945	238 27	54 67	-0 21
-212° 82	0 18782	287 99	54 09	-0 84
-217° 41	0 16342	321 51	42 54	+0 19
	0 39292	0	0	-0 23

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Pl. I.



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For the calculation of the coefficients of the parabola a sixth point was used which has been inserted at the bottom of the table, and was obtained by means of the isotherm of $-164^{\circ}.14$. For this temperature the value of B_A is very slight, and by means of interpolation the BOYLE-point can be easily determined. For this is found, measured on the absolute scale:

$$\theta = -165^{\circ}.72$$

to which corresponds a value of $pv_A = 0.39292$.

For the coefficients of the parabola we find:

$$P_0 = -14.8370$$

$$P_1 = +676.563$$

$$P_2 = -1624.31$$

The differences of the last column are slight, except for $-195^{\circ}.27$. For this temperature C_A appears also to be too small (see table XXII).

Both deviations must be owing to the not quite accurate position of one or more of the points of the isotherms. From the diagram of Plate I it is already to be seen that the middle point of series IV probably lies too high.

The parabola cuts the ordinate $p = 0$ in two points where pv_A is respectively $= 0.39330$ and 0.02323 , with which agree the absolute temperatures:

$$T_1 = 6.3 \quad T_2 = 107^{\circ}.5.$$

For the top of the parabola $pv_A = 0.20826$, with which corresponds a pressure of 55.61 atmospheres. From this follows for the absolute temperature of the isotherm which passes through the top:

$$T = 64^{\circ}.2.$$

Physics. — *“Isotherms of diatomic gases and their binary mixtures.*

VII. *Isotherms of hydrogen between 0° C. and 100° C.”* By Prof. H. KAMERLINGH ONNES and C. BRAAK. Communication N^o. 100^b from the Physical Laboratory at Leiden.

(Communicated in the meeting of November 30, 1907).

§ 1. *Survey of the determinations.*

The reservoir of 5 cm.³ of the piezometer of series IV (Comm. N^o. 99^a Sept. 1907) was replaced by one of 10 cm.³ With this apparatus two isotherms were determined, in ice and in vapour of boiling water. To obtain constant temperatures the same instruments were used as in Comm. N^o. 60 (Sept. 1900). The water manometer (c.f. § 8 of Comm. N^o. 27 (June 1896)) was read, but the difference