nerve-endings. It will be difficult to find an object of study as favourable as the organ of EIMER.

Leiden, Anatomical Cabinet.

DESCRIPTION OF THE FIGURES ON THE PLATE

All the figures are drawn from life from preparations made after the method of BIELSCHOWSKY-POLLACK, with a camera lucida of ABBE. Fig. 1 and 5 are enlarged 1200 times, the others 1600 times. Apochromate-oil-immersion. Sections 5 and 6 μ.

Fig 1 Longitudinal section of the upper part of a column of EIMER of the earth-mole. A rand fibre *(rf)* and a part of an axial fibre *(mf)* are seen. The horny layer *(stc)* above the column of EIMER is distinctly thinner than at both sides of it.

Fig. 2. Longitudinal section of a flat cell of the upper part of a column of EIMER, with two tactile discs, growing into the same cell. The netlike structure and the curious drawing in of the connecting fibre, is clearly shown.

Fig. 3. Longitudinal section of the upper part of a column of EIMER, to show the developing of the tactile discs, and the final atrophying of the nerve-fibre.

Fig. 4. From a cross-section through the upper part of a column of EIMER. A nucleus curved in by a tactile disc

Fig. 5. Longitudinal section through the peripheral part of a column of EIMER. Three rand-fibres are shown. The tactile discs lie behind the nerve-fibres. The intracellular position of the tactile discs is clearly to be seen. The upper cell, in which lie four tactile discs, is being transformed into a horny cell. The nerve-fibres degenerate.

Fig. 6. Cross-section through the upper cells of a column of EIMER. In the section of 6 μ four cells were to be seen, lying two and two in the same niveau.

The tactile discs of the rand-fibres all grow centripetally into the cells, the axial fibre runs between the cells.

Astronomy. — *"β Lyrae as a double star."* By J. STEIN, S. J. at Rome. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

1. As far as I know, Professor E. C. PICKERING was the first who, led by his spectroscopic investigations, suggested that β Lyrae might be a close double, the components of which describe circular orbits in a light-period [1]).

This surmise was confirmed by BELOPOLSKY [2]) in 1892. He measured the displacement of the luminous *F*-line on some fourteen spectographs. They were found to show a minimum (in absolute value) at the time of the minima and a maximum at the time of the maxima of

[1]) Spectrum of β Lyrae. By Prof EDWARD C. PICKERING. A. N. 3051 (1891).

[2]) Les changements dans le spectre de β Lyrae. A. BÉLOPOLSKY. Memorie della Società degli Spettroscopisti Italiani. Vol. XXII, 1893.

the star's light, in such a way that they correspond to an approach before the principal minimum and to a recession after that time. From these observations he derived a circular orbit for that component which eclipses the other at the time of this minimum. The investigation of the Potsdam spectographs equally led Prof. VOGEL [1]) to the conclusion that the displacement of the lines can hardly be explained otherwise than as a consequence of the motion of different bodies having unequal spectra. He does not succeed however in determining the position of the lines with sufficient accuracy. He thinks that the photometric data would lead to the assumption of two bodies of unequal luminosity moving either in a fairly circular orbit or in an ellipse having its major axis in the visual line. On the other hand the spectroscopic investigations would lead to the assumption of two bodies, one showing a spectrum with luminous, the other a spectrum with absorption-lines, which would describe very excentric orbits the major axes of which would make a considerable angle with the visual line. It would be impossible, in his opinion, to satisfy the two phenomena at the same time. In 1896 Dr. MYERS [2]) subjected ARGELANDER's lightcurve ("vera" pro 1850) to an elaborate theoretical investigation. His result is that the whole curve of the lightvariation is represented satisfactorily by assuming two elongated revolution ellipsoids the major axes of which are in each other's prolongation, circulating around each other in nearly circular orbits.

The next year BELOPOLSKY found the duplicity confirmed [3]). This time it was the displacements of the dark *Mg*-line ($\lambda = 448.2\ \mu\mu$), which enabled him to derive a slightly excentric orbit for the second component viz. of that component which is eclipsed during the principal minimum. Father W. SIDGREAVES, in his latest spectrographic investigation of β Lyrae [4]) arrives at the same result as Prof. VOGEL: rather considerably excentric orbit, the major axis of which makes a great angle with the visual line.

In conformity with what had already been suggested before by

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1) Ueber das Spectrum von β Lyrae. Von H. C. VOGEL. Sitzungsberichte der K. Preussischen Ak. der Wiss zu Berlin. 8 Februar 1894

2) Untersuchungen über den Lichtwechsel des Sternes β Lyrae. Inauguraldissertation... von G. W MYERS, München 1896. — The system of β Lyrae. id. The Astroph. Journ. Vol. VII No. 1.

3) Recherches nouvelles du spectre de β Lyrae, par A. BÉLOPOLSKY. Memorie della Società degli Spettrosc. It. vol. XXVI, 1897. — New Investigations of the Spectrum of β Lyrae, id Astroph. J. Vol. VI No. 4.

4) A spectrographic Study of β Lyrae. By Rev. WALTER SIDGREAVES S. J. Monthly Notices of R. A. S., Jan. 1904.

Dr. MYERS, Prof. CH. ANDRÉ [1]), basing himself on different numerical data, thinks himself justified in assuming, that the excentricity of the orbit has increased since the time of ARGELANDER, and also that the major axis has been displaced. On this supposition ANDRÉ tries to found an explanation of the terms of a higher order in the formula of ARGELANDER as corrected by Dr. PANNEKOEK [2]). Finally Dr. L. TERKAN has brought forward some short considerations in A. N. n° 4067 [3]). Afterwards a more elaborate investigation has appeared in the Memoirs of the Hungarian Academy of Sciences [4]).

We think that this enumeration covers the principal literature about what has been put forward in *explanation* of the light-variation.

2. The original plan of the author of the present paper was a treatment by the method of MYERS of the light curve derived by Dr. PANNEKOEK, in order to ascertain whether any important change of the elements of the orbits since the time of ARGELANDER, might be established.

The first part of MYERS' thesis in which, as a first approximation, a circular orbit is derived, is generally fairly correct. But the second part in which this orbit is changed to a slightly excentric one, by the aid of differential formulae, appeared to call urgently for a fresh treatment. Erroneous normal equations have been derived from incorrect differential formulae. The former have been wrongly solved and finally the close adjustment of the theoretical curve to that of ARGELANDER, chiefly in the vicinity of the principal minimum, seems to have been obtained by a happy coincidence of numerical errors. It is of no use to enter into further particulars on the subject. As an instance we give in the 2nd column of the following table the light-intensities (I_B), as derived by MYERS from the observed grades (Stufen) of ARGELANDER during the period of from 30 hours before to 30 hours after the principal minimum. In the next column are contained the light-intensities (I_{C_1}) given by MYERS as resulting from the definitive elements of his orbit [5]), the 4th col. shows these same quantities freed from numerical errors. In the three last columns the

[1]) Traité d'Astronomie Stellaire par CH. ANDRÉ, 2me p. NN. 460—1.

[2]) Untersuchungen über den Lichtwechsel von β Lyrae. Dr. A. PANNEKOEK. Verhandelingen der Kon. Ak. van Wetensch. te Amsterdam, Vol. 5, N°. 7. id. A. N. N°. 3456.

[3]) Beitrag zur Berechnung der Bahnelemente van β Lyrae. Dr L. TERKÁN.

[4]) β Lyrae pályaelemeinek kiszámítása spektroskopikai és photometriai adatokból. TERKAN Lajostól. — Mathematikai és Természettudományi Ertesitö, XXIV kötet 3 füzetéböl Budapest 1906.

[5]) Inaugural-dissertation, p. 48; A. J. l.c. p. 16.

same quantities have been given reduced to light-grades (σ). I do not find mentioned what is the value of a light-grade of ARGELANDER according to MYERS. From the light-intensities in the two minima I find 0.130 magnitudes, a value to which I have adhered. The intensity of the maximum has been taken for unit. The light-grades of A which, from 3.35 in the principal minimum, rise to the value 12.35 at a maximum, have been reduced to the interval of 3.00 to 12.00 for the sake of convenience.

t	I_B	I_{C_1}	I_{C_2}	σ_B	σ_{C_1}	σ_{C_2}
-30^h	0 7296	0 7525	0.7586	9.27	9.61	9.67
-24	.5836	.6019	.6674	7.40	7.73	8.60
-18	.4336	4993	.5627	4.83	6.15	7.16
-12	.3661	.4275	.4506	3.55	4.85	5 29
-6	.3484	.3487	.3500	3.10	3.13	3.16
0	.3433	.3433	.3433	3.00	3.00	3 00
$+6$	.3490	3488	3477	3.15	3.12	3.11
$+12$	.3988	.4275	.4462	4.30	4.85	5.21
$+18$	.5306	5591	.5586	6 67	7.11	7.10
$+24$	.6572	.6624	6635	8.46	8 54	8.55
$+30$	.7644	.7528	.7553	9.70	9.61	9.64

In what follows we have tried, first of all, to give correct formulae for the derivation of a slightly excentric orbit from the variation of the light. These have then been used for the curve of ARGELANDER and for that of Dr. PANNEKOEK. Afterwards the spectroscopic data of BELOPOLSKY have also been freshly reduced, because there is some uncertainty about the resulting orbit [1]). This is perhaps to be attributed to the method of LEHMAN—FILHÉS [2]). This method is excellent for a satisfactory determination of the excentricity if it is large; but it is less suitable for a very small excentricity. For the drawing of the graphical velocity-curve remains always slightly arbitrary and this fact exerts too strong an influence in the case that e is small.

3. We thus start from the following hypothesis:

[1]) In the "Recherches nouvelles" (Memorie etc.) BELOPOLSKY gives $e = 0.04$; in his "New investigations" (A. J.) $e = 0.07$, as the result of the same observations.

[2]) A. N. n. 3242.

Two similar, elongated revolution-ellipsoids move about their common centre of gravity in elliptic orbits. We assume that the major axes of the ellipsoids are continually in each other's prolongation while their centres move about their common centre of gravity in obedience to the laws of KEPLER. Required the intensity of the light as it appears to our eye, if we assume that the ellipsoids may be exchanged for their uniformly illuminated projections on the sphere.

As unit of length we take the semi major axis of the larger ellipsoid (E_1); as unit of brightness the maximum of β-Lyrae.

Further let be:

$\varkappa$ the semi major axis of the smaller body:

q the proportion of the major axis to the diameter of the equator;

f the proportion of the major axis of the ellipse, which is the projection of one of the ellipsoids on the sphere, to the major axis of that same ellipsoid;

a the semi major axis of the relative orbit of the smaller body (E_2), e the excentricity, v the true anomaly, r the radius vector in the true relative orbit.

β the angle formed by the radius vector in the true orbit with the projection of the visual line on the plane of the orbit (on the further side of the sphere); this angle increases with the motion in the orbit;

ω the longitude of the periastron, counted in the same way as β;

i the angle between the plane of the orbit and a plane tangent to the sphere;

ϱ the projection of r on the sphere;

M the common part of two circles the radii of which are resp. $= 1$ and $= \varkappa$, having their centres at a distance of $\varrho' = \dfrac{\varrho}{f}$;

λ the proportion of the brightness (per unit of surface) of the larger of the elliptic projections to the smaller one;

J the apparent total light-intensity at the time t, (as seen from the earth).

As long as E_1 and E_2 do not cover each other, we have:

$$J = f.$$

When E_2 is covered by E_1

$$J = f\left(1 - \frac{M}{\pi\,(\lambda + \varkappa^2)}\right).$$

When E_1 is covered by E_2

$$J = f\left(1 - \frac{\lambda M}{\pi\,(\lambda + \varkappa^2)}\right).$$

Let $2\,\varphi'$ and $2\,\varphi$ be the angles formed by the common chord of the circles, which define M, as seen from their respective centres, then

$$M = \tfrac{1}{2}\left\{(2\varphi' - \sin 2\varphi') + \varkappa^2\,(2\varphi - \sin 2\varphi)\right\}$$

$$\sin\varphi' = \varkappa \sin\varphi\,; \qquad \cos\varphi = \frac{\varrho'^2 + \varkappa^2 - 1}{2\varkappa\,\varrho'}.$$

φ' is always $< \frac{\pi}{2}$; φ may become $= \pi$, in the case that the smaller disc is seen projected wholly within the larger one.

Furthermore:

$$\varrho^2 = r^2\,(1 - \cos^2\beta\,\sin^2 i)\,; \qquad \beta = \omega + v.$$

These formulae agree with those of Dr. MYERS.

Computation of f.

The equation of the cylindre, enveloping the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ the axis of which makes the angles φ, χ, ψ with the X-, Y- and Z-axis, is:

$$\left(\frac{\cos^2\varphi}{a^2} + \frac{\cos^2\chi}{b^2} + \frac{\cos^2\psi}{c^2}\right)\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right) =$$
$$= \left(\frac{x\cos\varphi}{a^2} + \frac{y\cos\chi}{b^2} + \frac{z\cos\psi}{c^2}\right)^2.$$

The surface of an orthogonal section of this cylindre is:

$$\Omega = \pi \sqrt{a^2 b^2 \cos^2\psi + b^2 c^2 \cos^2\varphi + c^2 a^2 \cos^2\chi}.$$

or, putting

$$a = b = \frac{1}{q},\ c = 1$$

$$\Omega = \frac{\pi}{q}\sqrt{1 - \varepsilon^2\cos^2\psi}\,, \qquad \varepsilon^2 = \frac{q^2 - 1}{q^2}.$$

The semi minor axis of the section is $\frac{1}{q}$, therefore the semi major axis

$$f = \sqrt{1 - \varepsilon^2\cos^2\psi}\,; \qquad \cos^2\psi = \cos^2\beta\,\sin^2 i\,;$$

because ψ is the angle between the major axis of the ellipsoid and the visual line.

In the computation of f Dr. MYERS, instead of taking the instantaneous projection of the ellipsoid on the sphere, takes the intersection of the ellipsoid with a plane through the centre, at right angles to the visual line. In his opinion this is allowable "wenn die Abplattung nicht ungeheuer gross ist". He therefore puts

$$f = \frac{1}{\sqrt{1 + (q^2 - 1) \cos^2 \psi}}.$$

As the rigorous formula is at least equally simple there is no reason for the substitution. If we put the expression of MYERS $= f'$, we have

$$\frac{f}{f'} = \sqrt{1 + \left(\frac{q^2-1}{2q}\right)^2 \sin^2 2\psi}.$$

Greatest values, for $\psi = \frac{\pi}{4}$

q	1.2	1.3	1.4	1.5
$\frac{f}{f'}$	1.02	1.03	1·06	1.08

There thus is introduced a systematic error, which, already for small elongations, cannot be neglected.

4. As soon as, with the aid of provisional elements, a light-curve has been calculated, we try to vary these elements in such a way that the differences between observation and computation are diminished. We have to investigate, therefore, in what way the light-intensity varies with the elements.

We have already: $J = F(f, M, \lambda, \varkappa^2)$. We will now, first of all, express df and dM in function of $d(\varkappa^2)$, $d\beta$, $d\varrho$.

In the first place we have to consider that λ is fully determined by $\varkappa^2$, in the case, which as we shall presently see must be admitted, that during the minima E_2 is projected wholly on E_1. For, if $f_m = \sqrt{1 - \varepsilon^2 \sin^2 i}$ (= value of f in both the minima), then, on the same supposition:

$$f_m \cdot \frac{\lambda}{\lambda + \varkappa^2} = \text{const.}_1 \quad (= \text{intensity at the principal minimum})$$

$$f_m \left(1 - \frac{\varkappa^2 \lambda}{\lambda + \varkappa^2}\right) = \text{const.}_2 \quad (= \text{ ,, ,, ,, secondary ,, })$$

From these, after division:

$$d\lambda = \frac{\lambda(1-\lambda)}{\varkappa^2} d(\varkappa^2) \text{ and } \frac{df_m}{f_m} = \frac{\lambda}{\lambda + \varkappa^2} d\varkappa^2 = -\frac{d(\varepsilon^2 \sin^2 i)}{2 f_m^2}.$$

With the aid of the latter formula we get without difficulty

$$df = \frac{\varepsilon^2 \sin^2 i}{2f} \sin 2\beta d\beta + \frac{\cos^2 \beta}{f} (1 - \varepsilon^2 \sin^2 i) \frac{\lambda}{\lambda + \varkappa^2} d(\varkappa^2);$$

which is independent of the variation of i.

The computation of dM directly from the formulae is rather lengthy[1]; by considering the geometrical meaning of M, dM is found at once. Evidently M is purely a function of $\varkappa$ and ϱ'. If $\varkappa$ increases by the amount $\Delta\varkappa$, the increment of M is a strip $2\varkappa\varphi\Delta\varkappa$; if ϱ' increases by $\Delta\varrho'$, the increment of M is *negative* and equal to a strip (crescent) $2\sin\varphi' \,.\, \Delta\varrho' = 2\varkappa \sin\varphi \,.\, \Delta\varrho'$. Therefore

$$dM = \varphi d(\varkappa^2) - 2\sin\varphi' \,.\, d\varrho'$$

$$= \varphi d(\varkappa^2) - \frac{2\sin\varphi'}{f} d\varrho + \frac{2\varrho\sin\varphi'}{f^2} \,.\, df$$

If in this expression we substitute the value, given above, of df, we get dM expressed as a function of $d(\varkappa^2)$, $d\beta$, $d\varrho$.

5. *Calculation of* $d\beta$ *and* $d\varrho$ *in function of the variations of the elements of the orbit and of the epoch.*

If $\sin\varphi = e$, then (*vide* BAUSCHINGER, die Bahnbestimmung der Himmelskörper, n°. 197):

$$dv = \left(\frac{a}{r}\right)^2 \cos\varphi\,\{(t-T)\,d\mu - \mu dT\} + \frac{a}{r}\cos\varphi \sin v\left(1 + \frac{r}{p}\right) d\varphi.$$

$$\frac{d\varrho}{\varrho} = \frac{da}{a} - \sin(P-☊)\cos(P-☊)\sin i\, tg\, i \,.\, d\omega - \sin^2(P-☊)\, tg\, i \,.\, di$$

$$+ \left(\frac{a}{r}\right)^2 \{e\sin E - \sin(P-☊)\cos(P-☊)\sin i\, tg\, i\cos\varphi\,\{(t-T)\,d\mu - \mu dT\}$$

$$- \left(\frac{a}{r}\right)^2 \left\{(\cos E - e)\cos\varphi + \sin(P-☊)\cos(P-☊)\sin i\, tg\, i\sin E\left(\frac{r}{a} + \cos^2\varphi\right)\right\} d\varphi.$$

According to the definition of ω adopted above, we have to put:

$$\varrho\sin(P-☊) = r\cos i\cos(\omega + v) = r\cos i\cos\beta$$

$$\varrho\cos(P-☊) = - r\sin(\omega + v) = - r\sin\beta$$

We now pass to the following particular case:

a. the original orbit is circular,

b. if $i = 90° - \iota'$, then ι' is so small that 3rd and higher powers may be neglected; the same is true for $\sin\varphi$.

Furthermore let $d\mu = 0$; $d\omega = 0$; $2e\cos\omega = x$, $2e\sin\omega = y$; $n = \frac{2\pi}{U}$ (U = period = 12.91 days), t_1 = time counted from "superior conjunction"; $dM_0 = -\mu dT$.

Then with sufficient approximation

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[1]) See: Untersuchungen über den Lichtwechsel des Sternes β Persei, von J. HARTING. (München 1889) p. 41.

$$d\beta = dM_0 + x\sin nt_1 - y\cos nt_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (a)$$

$$\frac{d\varrho}{\varrho} = \frac{da}{a} + \frac{a^2}{2\varrho^2}\cos^2 nt_1\, i'^2 + \frac{a^2}{2\varrho^2}\sin 2nt_1 \,.\, dM_0 -$$

$$- \tfrac{1}{2}x\left(\cos nt_1 - \frac{a^2}{\varrho^2}\sin 2nt_1 \sin nt_1\right) - \tfrac{1}{2}y\left(\sin nt_1 + \frac{a^2}{\varrho^2}\sin 2\,nt_1 \sin nt_1\right) \quad . \qquad (b)$$

As these differential expressions have led several astronomers [1]) into error, we will derive them in still another way.

From:

$$\beta = v + \omega$$

we get:

$$d\beta = dv + d\omega.$$

In the circular orbit $v = M$; in the elliptic orbit this becomes:

$$v = M + 2e\sin M + \ldots + dM_0.$$

If we substitute $M = nt_1 - \omega$, and put $d\omega = 0$, we get, neglecting higher powers of e:

$$d\beta = dM_0 + x\sin nt_1 - y\cos nt_1$$

If in:

$$\varrho^2 = r^2\sin^2\beta + r^2\cos^2 i\cos^2\beta,$$

$$i = 90^\circ - i',$$

then, neglecting higher powers of i':

$$\frac{d\varrho}{\varrho} = \frac{dr}{r} + \frac{r^2}{2\varrho^2}\sin 2\beta\, d\beta + \frac{r^2}{2\varrho^2}\cos^2\beta .\, i'^2 \quad . \quad . \quad . \quad . \qquad (c)$$

In the elliptic orbit we have:

$$r = \frac{a\,(1-e^2)}{1+e\cos v} = a_0 + da - ae\cos(\beta-\omega) + \ldots =$$

$$= a_0 + da - \frac{1}{2}a\,x\cos nt_1 - \frac{1}{2}ay\sin nt_1 \ldots$$

Therefore:

$$dr = da - \frac{1}{2}ax\cos nt_1 - \frac{1}{2}ay\sin nt_1$$

and, substituting this in (c), we get the expression already given of $\dfrac{d\varrho}{\varrho}$.

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1) Dr. Myers puts $d\beta = 0$ for $t_1 = 0$ and at the same time $dM_0 = 0$; this is incompatible with (a). Prof. Hartwig, in his paper: "Der veränderliche Stern vom Algoltypus Z Herculis" (Bamberg 1900) p. 39, puts $\sin(P\text{-}☊)\cos(P\text{-}☊)\sin i\, tg\, i = 0$ for $i = 90^\circ$, whereas, according to our formulas, it becomes $-\dfrac{r^2}{2\varrho^2}\sin 2\beta$. (See also A.N. 3644).

Dr. Pannekoek quotes another instance in his Thesis on Algol (p. 22—3).

6. We thus have consecutively expressed dM and df in function of $d(\varkappa^2)$, $d\beta$ and $d\varrho$; and afterwards $d\beta$ and $d\varrho$ in function of da, i'^2, dM_0, x and y. If now we differentiate the expression

$$J_1 = f\left(1 - \frac{M}{\pi(\lambda + \varkappa^2)}\right),$$

valid in the vicinity of the first minimum, we find, by consecutive substitution, the following expression for dJ_1:

$$\pi(\lambda + \varkappa^2)\, dJ_1 = K_1\, d(\varkappa^2) + A_1\, da + I_1\, i'^2 + X_1 x + Y_1 y + \Delta_1 (dM_0 - y),$$

in which:

$$K_1 = \frac{\pi\lambda}{q^2 f^2} J_1 \cos^2 nt_1 + \pi\left(1 + \frac{\lambda - \lambda^2}{\varkappa^2}\right)(f - J_1) - f\varphi - \frac{2\lambda}{q(\lambda+\varkappa^2)} \cdot \frac{\varrho}{f^2} \cdot \cos^2 nt_1 \sin\varphi' ;$$

$$A_1 = \frac{2\varrho \sin\varphi'}{r} \ ; \ I_1 = \frac{r^2 \cos^2 nt_1}{\varrho} \sin\varphi' ;$$

$$X_1 = \Delta_1 \sin nt_1 - \varrho \sin\varphi' \cos nt_1 \ ; \ Y_1 = \Delta_1 (1 - \cos nt_1) - \varrho \sin\varphi' \sin nt_1 \ ;$$

$$\Delta_1 = \frac{r^2 \sin 2nt_1 \sin\varphi'}{\varrho} + \frac{\varepsilon^2}{2f^2} \sin 2nt_1 \{\pi(\lambda + \varkappa^2) J_1 - 2\varrho \sin\varphi'\} .$$

If we treat in the same way the expression:

$$J_2 = f\left(1 - \frac{\lambda M}{\pi(\lambda + \varkappa^2)}\right),$$

valid in the vicinity of the second minimum, we find, putting $t_2 = t_1 - \frac{U}{2}$:

$$\pi \frac{\lambda + \varkappa^2}{\lambda}\, dJ_2 = K_2\, d(\varkappa^2) + A_2\, da + I_2 i'^2 + X_2 x + Y_2 y + \Delta_2 (dM_0 + y)$$

in which:

$$K_2 = \frac{\pi}{q^2 f^2} J_2 \cos^2 nt_2 + \pi(f - J_2) - f\varphi - \frac{2\lambda}{q(\lambda + \varkappa^2)} \cdot \frac{\varrho}{f^2} \cdot \cos^2 nt_2 \sin\varphi' \ ;$$

$$A_2 = \frac{2\varrho \sin\varphi'}{r} \ ; \ I_2 = \frac{r^2 \cos^2 nt_2}{\varrho} \sin\varphi' ;$$

$$X_2 = -\Delta_2 \sin nt_2 + \varrho \sin\varphi' \cos nt_2 ; \ Y_2 = -\Delta_2 (1 - \cos nt_2) + \varrho \sin\varphi' \sin nt_2$$

$$\Delta_2 = \frac{r^2 \sin 2nt_2 \sin\varphi'}{\varrho} + \frac{\varepsilon^2}{2f^2} \sin 2nt_2 \left(\pi \frac{\lambda + \varkappa^2}{\lambda} J_2 - 2\varrho \sin\varphi'\right).$$

7. If the observations do not give the light-intensity, but the brightness expressed in magnitudes or in grades, then we have still to express the variation of the number indicating the magnitude or the grade, in the variation of the light-intensity.

Let J_0 represent the intensity at the maximum. G_0 the corresponding magnitude, J and G the same quantities at the time t, then, by the formula of POGSON:

$$G - G_0 = 2.512\ (log\ J_0 - log\ J).$$

Consequently:

$$dG = -\ 2.512\ m. \frac{dJ}{J},\ (m = modulus\ of\ Brigg's\ log.)$$

$$dG = -\ 1.092\ \frac{dJ}{J}.$$

Now, if $-\frac{1}{v}$ is the equivalent in magnitudes of a grade, then, σ_0 and σ being the number of grades:

$$\sigma_0 - \sigma = v(G - G_0) = 2.512v\ (log\ J_0 - log\ J)$$

Therefore:

$$d\sigma = 1.092v \frac{dJ}{J}$$

Putting the value of ARGELANDER's grade for the light-curve of β Lyrae at 0.130 magnitudes, then:

$$d\sigma = 8.413\ \frac{dJ}{J}.$$

8. In the hypothesis which we adopted, the main phases (min.$_{1}$, max.$_{1}$, min.$_{2}$, max.$_{2}$) take place for the values $\beta = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ of β_1. Let v_1, v_2, v_3, v_4 represent the true anomalies for these values; M_1, M_2, M_3, M_4 the corresponding mean anomalies. If, as is the case with β Lyrae, the intervals are nearly equal, e must be small and we may put approximately:

$$v_1 = M_1 - y;\ v_2 = M_2 + x;\ v_3 = M_3 + y;\ v_4 = M_4 - x.$$

$$(x = 2e\ cos\ \omega;\ y = 2e\ sin\ \omega)$$

or:

$$v_2 - v_1 = \frac{\pi}{2} = (M_2 - M_1) + x + y$$

$$v_3 - v_2 = \frac{\pi}{2} = M_3 - M_2) - x + y$$

$$v_4 - v_3 = \frac{\pi}{2} = (M_4 - M_3) - x - y$$

If the differences $M_2 - M_1 \ldots$ [1]) are known with equal and

[1]) The time-equation for the reduction to the common centre of gravity, computed from the spectroscopic orbit, is found to reach a value of somewhat over $\pm$ 100 seconds and may consequently be neglected.

sufficient precision, we find from these formulae the most probable values of x and y as follows:

$$\left.\begin{aligned} 4x &= -\pi + (M_4 - M_3) + 2(M_3 - M_2) - (M_2 - M_1) \\ 4y &= \pi + (M_4 - M_3) - 2(M_3 - M_2) - (M_2 - M_1) \end{aligned}\right\} \quad . \quad . \quad I$$

If we combine only similar phases, we get

$$\left.\begin{aligned} 2x &= -\pi + (M_4 - M_2) \\ 2y &= \pi - (M_3 - M_1) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad II$$

The two solutions are identical, if

$$(M_2 - M_1) + (M_4 - M_3) = \pi.$$

In A. N. n°. 3456 Dr. PANNEKOEK summarises the intervals, counted from the principal minimum, for different observers between 1842 to 1895.

Dividing this period in two, he finds on an average: ($U = 12^d.91$)

	max_1—min_1	min_2—min_1	max_2—min_1
1842—1870	$3^d.12$	$6^d.40$	$9^d.54$
1870—1895	$3^d.32$	$6^d.48$	$9^d.73$.

From these values we find, for the first period:

according to form. (I):

$$\begin{cases} e \sin\omega = -0.0052; & e = 0.009 \\ e \cos\omega = +0.0076; & \omega = 326^\circ \end{cases}$$

according to form. (II):

$$\begin{cases} e \sin\omega = +0.0067; & e = 0.008 \\ e \cos\omega = -0.0043; & \omega = 123^\circ. \end{cases}$$

Similarly for the second period:

$$\begin{cases} e \sin\omega = +0.0040; & e = 0.013 \\ e \cos\omega = -0.0125; & \omega = 162^\circ \end{cases}$$

$$\begin{cases} e \sin\omega = -0.0030; & e = 0.006 \\ e \cos\omega = -0.0055; & \omega = 209^\circ \end{cases}$$

The only conclusion to be derived from these results is that e was very minute in both periods, and hardly exceeding 0.01.

9. A single glance at the numbers communicated by Dr. PANNEKOEK shows that a trial to derive something more definite from the results of the *separate* observers would be quite hopeless. In particular we may allege the considerable difference between the results obtained by LINDEMANN and PANNEKOEK, in their reduction of the observations of PLASSMANN. It thus seems to be out of place, from these observations alone, to draw the conclusion that the excentricity has increased.

Dr. L. TERKAN has proposed the following method of deriving the inclination of the orbit. [1])

[1]) A. N. nr 4067.

The minimum or maximum of light takes place when ϱ takes "extreme" values, consequently when

$$\frac{d\varrho}{dv} = \cos v \sin v \sin^2 i + e \sin v = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In formula (2) $\sin v = 0$ for the principal minimum, $\cos v = -\frac{e}{\sin^2 i}$ for the "secondary maximum". This is in the assumption that the time of the principal minimum coincides with the time of periastron. Therefore if, at the moment of the maximum, we know e and v, then we know also i. TERKAN adopts the value 0.07, derived for e by BELOPOLSKY from his spectroscopic observations [1]). He determines the mean anomaly at the maximum from the interval found by PLASSMANN [2]):

II min. — II max. = 3,05 days,

and then expands this anomaly in a series [3]) by the aid of $\cos v = -\frac{e}{\sin^2 i}$. This series has an argument α, which contains $\sin^2 i$. He thus finds

$$i = 51^\circ.3.$$

Afterwards, in his Hungarian paper [4]), he takes $e = 0.06$. From his own observations he derives: I min. — I max. = 3,48 days and then finds, using the usual equations of KEPLER, by the aid of $\cos v = -\frac{e}{\sin^2 i}$:

$$i = 30^\circ.$$

Even if we disregard the very doubtful value of the numerical data, the hypothesis seems unfounded that the maximum of the light occurs at the moment that ϱ is a maximum. If, moreover, we assume with Dr. TERKAN, both the celestial bodies to be spherical, then the light must be constant as long as the two spheres do not cover each other as seen by the observer. This is not confirmed by observation. Besides there can be no question of a clearly defined epoch of maximum in such a case. The way in which Dr. TERKAN meets this objection by saying: "that our eye or the telescope is unable to separate the system and that the rays of light which in space come

[1]) See p. 462. 1st footnote.

[2]) A. N. nr 3242.

[3]) In this series e has been erroneously substituted for $\sin\varphi \cos\varphi$ and $\frac{\sqrt{4-e^2}-e}{\sqrt{4-e^2}+e}$ (instead of $\sqrt{\left(\frac{1-e}{1+e}\right)}$ for $tg\,(45^\circ - \frac{1}{2}\varphi)$.

[4]) β Lyrae palyaelemeinek etc. p. 412.

from the same distance, but from a larger field, are united to a larger disc" [1]), seems little satisfactory.

10. *Determination of the elements of the orbit etc. by means of the light-curve of* ARGELANDER [2]).

As a first approximation we put $i = 90°$, $e = 0$.

An approximate value of q is furnished by the general course of the curve in the vicinity of the maxima. As long as it is symmetrical in regard to the ordinate of the maximum, we may assume that the eclipse has not yet begun, so that

$$J = \sqrt{(1-\varepsilon^2 \sin^2 nt_M)}\,;$$

t_M being the time counted from the maximum.

From the light-curve we take the decrease of $(\sigma_0-\sigma)$ in grades, for equal intervals of time before and after the two maxima.

t_M	$(\sigma_0-\sigma)_I$	$(\sigma_0-\sigma)_{II}$
-30^h	0 76	0.50
-24	0 47	0.31
-18	0.25	0.18
-12	0 10	0.07
-6	0 02	0 02
$+6$	0 03	0.03
$+12$	0.09	0 11
$+18$	0.21	0.24
$+24$	0.38	0 43
$+30$	0.59	0.67

t_M	$(\sigma_0-\sigma)_{mean}$
$\pm 6^h$	0.025
± 12	0.093
± 18	0 220

t_M	$(O-C)_I$	$(O-C)_{II}$
-18^h	-0.03	$+0.04$
-12	0 00	$+0.03$
-6	0.00	0.00
$+6$	-0.01	-0.01
$+12$	$+0.01$	-0.01
$+18$	$+0.01$	-0.01

An increasing dissymmetry begins to show itself for both the maxima at about 24 hours distance from these epochs.

With the mean values $(\sigma_0-\sigma)_{mean}$ the light-intensities were now computed by the formula:

$$\sigma_0-\sigma = -\frac{2.512}{0.13}\log J$$

and the relation

$$\varepsilon^2 \sin^2 nt_M = 1-J^2$$

[1]) β Lyrae palyaelemeinek etc. p. 417.

[2]) De Stella β Lyrae variabili commentatio altera. Scripsit FREDERICUS ARGELANDER Bonnae a. 1859. — Curva "vera" pro 1850.

furnished the data:

$$0.015\,\varepsilon^2 = 0.006$$
$$0.058\,\varepsilon^2 = 0.022$$
$$0.127\,\varepsilon^2 = 0.051$$

leading to the most probable values $\varepsilon^2 = 0.397$; $q = 1.288$.

The deviations Obs.-Comp. have been given in the two last columns.

Having found q, we get $\varkappa$ and λ from the light-intensities at the two minima. The values of these being 0.3433 and 0.6365, we obtain, for $i = 90°$, the two relations

$$\frac{1}{q} \cdot \frac{\lambda}{\lambda + \varkappa^2} = 0.3433\,; \quad \frac{1}{q}\left(1 - \frac{\varkappa^2 \lambda}{\lambda + \varkappa^2}\right) = 0.6365,$$

whence:

$$\varkappa = 0.6387\,; \quad \lambda = 0.3233.$$

Finally, at the moment at which the eclipse begins:

$$\frac{\varrho}{f} = \varrho' = 1 + \varkappa.$$

The consideration of the asymmetry, shows that this must be the case shortly after 18^h ($nt_M = \pm\, 20°55'$). We therefore put:

$$\frac{\varrho}{f} = \frac{a \cos 21°}{\sqrt{1 - \varepsilon^2 \sin^2 21°}} = 1.6387 = 1 + \varkappa$$

from which:

$$a = 1.710.$$

We thus have, as a first approximation, the following elements: $\varkappa = 0.6387$; $\lambda = 0.3233$; $q = 1.288$; $a = 1.710$; $e = 0$; $i = 90°$ and, as for the "epoch", we assume, that the central eclipse of E_2 by E_1, coincides with the principal minimum of ARGELANDER's curve.

11. In the following table the 2nd column, headed O_1, shows the light-grades of ARGELANDER's curve for equal intervals before and after the principal minimum; the 7th column, headed O_2, similarly shows the same element before and after the half period $= 6^d.455$ (*not* therefore before and after the secondary minimum, which ARGELANDER places at $6^d.375$ from the principal minimum). The columns C_{s_1} and C_{s_2} contain the light-grades, computed by the aid of the elements given just now.

12. As will be remarked, the deviations $O - C_s$ are in the main negative before, positive after the two minima. We conclude that, by shifting the theoretical light-curve in a negative direction with regard to the time, we may obtain improved agreement. The excen-

t	O_1	C_{S_1}	C_{M_1}	O_1-C_{S_1}	O_1-C_{M_1}	O_2	C_{S_2}	C_{M_2}	O_2-C_{S_2}	O_2-C_{M_2}
—72	11.95	11.98	11.98	—0.03	—0.03	11.87	11 98	11 98	—0 11	—0.11
—66	11 84	11.91	11.93	—0.07	—0 09	11 79	11.91	11.93	—0.12	—0.14
—60	11.69	11.80	11.84	—0 11	—0.15	11.66	11.80	11.84	—0.14	—0.16
—54	11.48	11.58	11.72	—0 10	—0.24	11.48	11.62	11 72	—0.14	—0.24
—48	11.20	11.34	11.45	—0 14	—0.25	11 25	11 37	11.52	—0 12	—0.27
—42	10 82	10.75	11.04	+0 07	—0.22	10.99	11.06	11.26	—0.07	—0.27
—36	10.29	10.07	10 44	+0.22	—0 15	10.68	10.68	10.92	0.00	—0.24
—30	9.27	9.14	9.64	+0.13	—0 37	10.30	10.22	10.50	+0.08	—0.20
—24	7.40	7.91	8.55	—0.51	—1.15	9 78	9.70	10 00	+0.08	—0.22
—18	4.83	6 28	7.13	—1.45	—2.30	9.04	9.13	9.44	—0.09	—0.40
—12	3.55	4.22	5 26	—0 67	—1.71	8.42	8.56	8.82	—0.14	—0.40
— 6	3.10	3 05	3.15	+0 05	—0 05	8.21	8 23	8 25	—0.02	—0.04
0	3.00	3 00	3.00	0.00	0.00	8.20	8.19	8.19	+0.01	0.00
+ 6	3 15	3.05	3.15	+0.10	0.00	8 38	8 23	8 25	+0 15	+0.13
+12	4 30	4 22	5.26	+0.08	—0 96	8.91	8.56	8.82	+0.35	+0.09
+18	6.67	6.28	7.13	+0.39	—0.46	9.64	9.13	9.44	+0.51	+0 20
+24	8.46	7.91	8.55	+0 55	—0 09	10.25	9.70	10.00	+0.55	+0.25
+30	9.70	9 14	9 64	+0 56	+0.06	10.75	10.22	10 50	+0.53	+0.25
+36	10.50	10.07	10 44	+0.43	+0 06	11.14	10.68	10.92	+0.46	+0.22
+42	10.97	10.75	11.04	+0.20	—0.07	11.43	11.06	11.26	+0.37	+0.17
+48	11.31	11.34	11.45	—0 03	—0.14	11.64	11 37	11.52	+0.27	+0.12
+54	11.57	11.58	11.72	—0.01	—0.15	11.79	11.62	11.72	+0 17	+0.07
+60	11 75	11.80	11 84	—0.05	—0.09	11 91	11.80	11.84	+0.11	+0.07
+66	11.88	11.91	11 93	—0.03	—0 05	11.98	11.91	11.93	+0.07	+0.05
+72	11.91	11.98	11.98	—0.07	—0 07	12.01	11.98	11.98	+0.03	+0.03

tricity, besides displacing the maxima and minima, also causes a slight dissymmetry in regard to the minima. In order to separate the influence of the excentricity on the asymmetry from that of the epoch, we may divide the equations of condition into two groups. For the coefficients K, A, I and X are even functions; Y and Δ uneven functions of nt_1, resp. nt_2. If, therefore, we take the sum and the difference of two equations of condition, corresponding to the times $+t_1$ and $-t_1$, the former of the resulting equations will only contain the quantities $d(\varkappa^2)$, δa, ι'^2 and x, the latter only y and dM.

In this way the following equations have been derived for successive intervals of six hours counted from the 1[th], resp. 2[nd] minimum.

They do not rest however on the above elements but on those

PRINCIPAL MINIMUM.

t_1	$d(\varkappa^2)$	δa	$(10\iota'^2)$	x	=	y	(dM_0-y)	=
$\pm 72^h$	+ 0.03	+ 0.00	+ 0.00	+ 0.70	− 0.05	+ 0.62	+ 0.70	− 0.02
66	+ .13	+ .00	+ .00	+ 1 41	− .07	+ 1.12	+ 1.45	+ .02
60	+ .30	+ .00	+ .00	+ 2.02	− .12	+ 1.41	+ 2.15	+ .03
54	+ .04	+ 60	+ .02	+ 2.59	− .19	+ 1.55	+ 3.20	+ 04
48	− .40	+ 1 46	+ .06	+ 3.16	− 19	+ 0.94	+ 4.76	+ .05
42	− .50	+ 1.92	+ .14	+ 3.55	− .14	+ .76	+ 6.37	+ 07
36	− .40	+ 2 34	+ .28	+ 3 80	− .04	+ 60	+ 8.18	+ .10
30	− .06	+ 2.58	+ .50	+ 3.89	− .15	+ .44	+ 10.32	+ .21
24	+ .60	+ 2.70	+ .91	+ 3.78	− 62	+ .31	+ 12.92	+ 53
18	+ 1.75	+ 2.60	+ 1 70	+ 3.43	− 1.38	+ .18	+ 16.08	+ .92
12	+ 3.42	+ 2.20	+ 3.38	+ 2.69	− 1.33	+ .07	+ 19 57	+ .37
6	+ 2.97	+ 0 79	+ 4 98	+ 1 02	− 0.02	+ .01	+ 14.11	+ .02

SECONDARY MINIMUM.

t_2	$d(\varkappa^2)$	δa	$(10\iota'^2)$	x	=	y	(dM_0+y)	=
$\pm 72^h$	+ 0.03	+ 0.00	+ 0.00	− 0 05	− 0.04	− 0.04	+ 0.05	+ .02
66	+ .12	+ .00	+ .00	− .10	− .04	− .07	+ .10	+ .09
60	+ .30	+ .00	+ .00	− .14	− .05	− .09	+ .15	+ .12
54	+ .33	+ .24	+ .00	− .26	− .08	− 01	+ .40	+ .15
48	+ .30	+ .58	+ .02	− .44	− 07	+ .07	+ .90	+ .19
42	+ 38	+ 78	+ .05	− .59	− .05	+ .06	+ 1.42	+ .22
36	+ .52	+ .89	+ .10	− .71	− .01	+ .05	+ 2 01	+ .23
30	+ .70	+ .94	+ .18	− .79	+ .02	+ .04	+ 2.65	+ .22
24	+ .90	+ .91	+ 30	− 80	+ .01	+ .03	+ 3.35	+ 23
18	+ 1.11	+ .80	+ .52	− .73	− .10	− .01	+ 4.02	+ .30
12	+ 1.22	+ .58	+ .89	− .55	− .15	− .00	+ 4.48	+ .24
6	+ 0.62	+ .17	+ 1.09	− .17	+ .04	+ .00	+ 2.71	+ .08

which have been derived by repeated approximations from ARGELANDER's curve by Dr. MYERS. In his opinion these are the best possible circular elements:

$$a = 1\,8955\,;\; \varkappa = 0.7580\,;\; q = 1.1993\,;\; \lambda = 0.4023\,;\; i = 0$$

By their aid I computed the light-grades C_{M_1} and C_{M_2} of the preceding table. As will be remarked, the deviations $O - C_{M_1}$ are rather considerable in the vicinity of the principal minimum.

In deriving the following normal equations, the equations of condition for $t_1 = \pm 6^h$ and $t_2 = \pm 6^h$ have been neglected.

Normal-equations:

$$\begin{array}{l}
18.54\, d(\varkappa^2) + 15.17\, da + 16.98\,(10\iota'^2) + 15.31\, x = -\;\; 7.670 \\
15.17 \;\;,, \;\; + 41.68 \;,, + 18.14 \;\;,, \;\; + 53.17 \;,, = -\;\; 9.673 \\
16.98 \;\;,, \;\; + 18.14 \;,, + 20.29 \;\;,, \;\; + 20.75 \;,, = -\;\; 7.720 \\
15.31 \;\;,, \;\; + 53.17 \;,, + 20.75 \;\;,, \;\; + 101.96 \;,, = - 13.227
\end{array}$$

$$255.69\, y + \;\; 160.37\,(dM_0 - y) = 10.242$$
$$160.37\, y + 1125.52\,(dM_0 - y) = 37.759$$

Solution of the first four equations:

$$x = -0.026\,;\;\; 10\iota'^2 = -0.044\,;\;\; da = -0.0741\,;\;\; d(\varkappa^2) = -0.2915.$$

As ι' becomes imaginary, we put $\iota' = 0$ in the equations of condition, and then find:

$$x = -0.026\,;\; da = -0.0799\,;\; d(\varkappa^2) = -0.3268.$$
$$y = +0{,}021\,;\; dM_0 - y = +0.031,$$

which lead to the improved elements:

$$a = 1.8156\,;\, \varkappa = 0.4978\,;\, \lambda = 0.2249\,;\, q = 1\,3859\,;\, e = 0.017;\, \omega = 141^\circ.3.$$

The correction for $\varkappa^2$ is particularly large, more than half its original value (0.5746). As in such a case dJ cannot any longer be considered to be proportinal to $d(\varkappa^2)$, we should have to compute a new light-curve by the aid of the new elements; we should then have to calculate the differential coefficients in order — if necessary — to find a new approximation.

In the following table the columns C_1 and C_2 show the light-grades calculated by means of the improved elements.

In fig. I has been given a graphical representation of these numbers. The agreement in the vicinity of the principal minimum is considerably improved. It is true that there remains a deviation exceeding a light-grade, at 18 hours before the minimum. It might perhaps be further diminished by a repetition of the whole process. If, however, we take into account the uncertainty mostly existing when we draw the curve for the vicinity of the minimum, then it seems hardly worth while to repeat the elaborate calculation. At all

t	C_1	O_1	O_1-C_1	O_2	C_2	O_2-C_2
-72^h	11.95	11.95	0.00	11.87	11.97	−0.10
−66	11.84	11 84	.00	11 79	11.88	− .09
−60	11.69	11.68	+ .01	11.66	11.73	− .07
−54	11.48	11.47	+ .01	11.48	11.52	− .04
−48	11.20	11.22	− .02	11.25	11.26	− .01
−42	10.82	10.93	− .11	10.99	10.96	+ .03
−36	10.29	10.42	− .13	10.68	10.55	+ .13
−30	9.27	9.47	− 20	10.30	10 05	+ .25
−24	7.40	8.06	− .66	9.78	9 47	+ .31
−18	4 83	6 01	−1.18	9.04	8.84	+ .20
−12	3.55	3.46	+ .09	8.42	8.35	+ .07
− 6	3.10	3.05	+ .05	8.21	8.22	− .01
0	3.00	3.00	.00	8.20	8 21	− .01
+ 6	3.15	3 06	+ .09	8 38	8.30	+ .05
+12	4 30	3.88	+ .42	8.91	8.73	+ .18
+18	6.67	6.39	+ .28	9.64	9.35	+ .29
+24	8.46	8.32	+ .14	10.25	9.94	+ .31
+30	9.70	9.66	+ .04	10.75	10 45	+ .30
+36	10.50	10.54	− .04	11.14	10.87	+ .27
+42	10.97	11.01	− .04	11.43	11.18	+ .25
+48	11.31	11 30	+ .01	11.64	11.44	+ .20
+54	11.57	11.55	+ 02	11.79	11 66	+ .13
+60	11.75	11.75	.00	11.91	11.83	+ .08
+66	11.88	11.89	− .01	11.98	11.94	+ .04
+72	11.91	11.98	− .07	12.01	12 00	+ .01

events the small coefficients of y in the equations of condition, show clearly that it is impossible to explain any appreciable asymmetry by an excentricity of a few hundredths. The improved agreement near the principal minimum is obtained in the main by shifting the theoretical principal minimum by $\frac{dM_0 - y}{2\pi} \times U = 0^d.063$ in the direction of the negative time-axis, while at the same time the secondary minimum occurs 0.069 days before the minimum of ARGELANDER's curve.

13. A second set of elements was derived by making the plausible assumption, that the first minimum in ARGELANDER's curve occurs 0.08 days earlier. We thus get rid of the greater part of the dissymmetry. The second theoretical minimum is assumed to coincide with the observed minimum. The interval between the two minima thus becomes just half the period ($6^d.375 + 0^d.08 = 6^d.455$). As a consequence the orbit must be either circular or elliptic with

t	O_1	C_1	C'_1	$O_1 - C_1$	$O_1 - C'_1$	O_2	C_2	C'_2	$O_2 - C_2$	$O_2 - C'_2$
— 72^h	11.97	11.97	11.97	0.00	0.00	11.89	11.97	11.97	—.08	—.08
— 66	11.88	11.89	11.83	—.01	+.05	11.83	11.89	11.95	—.06	—.12
— 60	11.74	11.75	11.66	—.01	+.08	11.71	11.75	11.84	—.04	—.13
— 54	11.54	11.56	11.44	—.02	+.10	11.54	11.56	11.67	—.02	—.13
— 48	11.29	11.31	11.18	—.02	+.11	11.33	11.31	11.44	+.02	—.11
— 42	10.95	10.98	10.86	—.03	+.09	11.08	11.01	11.15	+.07	—.07
— 36	10.48	10.35	10.16	+.13	+.32	10.78	10.62	10.71	+.16	+.07
— 30	9.65	9.42	9.20	+.23	+.45	10.42	10.26	10.40	+.16	+.02
— 24	8.06	8.09	7.85	—.03	+.21	9.95	9.63	9.75	+.32	+.20
— 18	5.48	6.29	6.05	—.81	—.57	9.28	9.06	9.15	+.22	+.13
— 12	3.80	4.07	3.90	—.27	—.10	8.56	8.52	8.57	+.04	—.01
— 6	3.18	3.05	3.05	+.13	+.13	8.25	8.25	8.26	.00	—.01
0	3.00	3.00	3.00	.00	.00	8.19	8.20	8.20	—.01	—.01
+ 6	3.07	3.05	3.05	+.02	+.02	8.30	8.25	8.26	+.05	+.04
+ 12	3.76	4.07	3.90	—.31	—.14	8.71	8.52	8.57	+.19	+.14
+ 18	6.01	6.29	6.05	—.28	—.04	9.40	9.06	9.15	+.34	+.25
+ 24	7.98	8.09	7.85	—.11	+.13	10.08	9.63	9.75	+.45	+.33
+ 30	9.37	9.42	9.20	—.05	+.17	10.61	10.26	10.40	+.35	+.21
+ 36	10.31	10.35	10.16	—.04	+.15	11.04	10.62	10.71	+.42	+.33
+ 42	10.84	10.98	10.86	—.14	—.02	11.35	11.01	11.15	+.34	+.20
+ 48	11.22	11.31	11.18	—.09	+.04	11.58	11.31	11.44	+.27	+.14
+ 54	11.50	11.56	11.44	—.06	+.06	11.75	11.56	11.67	+.19	+.08
+ 60	11.71	11.75	11.66	—.04	+.05	11.87	11.75	11.84	+.12	+.03
+ 66	11.84	11.89	11.83	—.05	+.01	11.96	11.89	11.95	+.07	+.01
+ 72	11.91	11.97	11.97	—.06	—.06	12.00	11.97	11.97	+.03	+.03

the major axis at right angles to the line of the nodes. In this way we find:

a=1.7209; $\varkappa$=0.5015; λ=0.2276; q=1.3944; i'=7°,25; e=0.04; ω=180°.

In the next tables the columns C_1 and C_2 show the light-grades computed by means of the first five elements, neglecting the excentricity. The columns C_1 and C_3 contain the same quantities taking into account the excentricity.

The mean deviation of the values in the columns O_1-C_1' and O_2-C_2' is $\pm$ 0.17 light-grades, whereas ARGELANDER assigns the value $\pm$ 0.16 to the mean error of the ordinates of his light-curve (prob. error 0.1095). It would be quite illusory therefore to endeavour to obtain an improved agreement. Against the elliptic orbit there is however the grave objection that it gives the first maximum 0.18 days after —, the second maximum 0.10 days before the corresponding maxima of ARGELANDER's light-curve. In the circular orbit the first maximum lies only 0.02 days, the second 0.06 days later, whereas the agreement is still very satisfactory.

14. Finally we communicate a set of circular elements obtained by a repeated approximation from the light-curve of Dr. PANNEKOEK:

$$a = 1.5378\ ;\ \varkappa = 0.5378\ ;\ \lambda = 0.2900\ ;\ q = 1\ 4609.$$

In deriving them we assumed that a cannot fall short of $1+\varkappa$. We further assumed the theoretical principal minimum to coincide with the observed minimum.

In the following table t is the number of hours before and after the *theoretical* principal and secondary minimum; O_1 and O_2 are the light-grades at the same moments, as read off from Dr. P.'s light-curve; C_1 and C_2 the light-grades of the theoretical curve.

The results have been graphically represented in Fig. II. The remaining deviations are mainly positive before the first minimum; after that they are negative. At the secondary minimum the signs are reversed. The deviations might be rather considerably diminished if, with a small excentricity ($e\ sin\ \omega = 0.016$), we place the principal minimum in Dr. PANNEKOEK's light-curve 0.063 days later, the secondary minimum 0.069 earlier. In this way, however, the interval in time min. I — min. II is diminished more considerably than seems admissible.

For the rest it need not be said, that in the present case, where two gaseous bodies seem to be in contact, the Keplerian equations of motion must give only a rough approximation, while the action of the tides must contribute its part to mask the influence of the

t	O_1	C_1	$O_1—C_1$	O_2	C_2	$O_2—C_2$
—72h	11.96	11.97	—0 01	11.96	11 97	—0.01
—66	11.88	11.87	+ .01	11 87	11.88	— .01
—60	11.72	11 67	+ .05	11.62	11.71	— 09
—54	11.53	11.37	+ 16	11.33	11.46	— .13
—48	11.26	10.95	+ 31	10.89	11.15	— 26
—42	10.87	10.37	+ .50	10 42	10.75	— .33
—36	10.23	9.61	+ .62	9 92	10 27	— .35
—30	9.06	8.60	+ .46	9 38	9 72	— 34
—24	7 17	7.27	— 10	8 86	9.10	— 24
—18	5.15	5.53	— .38	8.34	8.43	— .09
—12	3.80	3.53	+ .27	7 92	7.82	+ .10
— 6	3 20	3 09	+ .11	7 61	7.57	+ .04
0	3.00	3 00	.00	7.50	7.50	.00
+ 6	3.16	3.09	+ .07	7.75	7.57	+ .18
+12	3.77	3 53	+ .24	8.11	7.82	+ .29
+18	5.07	5.53	— .46	8.68	8.43	+ .25
+24	6.75	7.27	— 52	9.26	9.10	+ .16
+30	8 36	8.60	— .24	9.80	9.72	+ .08
+36	9 40	9.61	— .21	10 30	10.27	+ .03
+42	10.07	10.37	— .30	10.75	10.75	.00
+48	10.62	10.95	— .33	11.08	11 15	— .07
+54	11.13	11.37	— .24	11.42	11.46	— .04
+60	11.52	11 67	— .15	11.61	11.71	— .10
+66	11.78	11.87	— .09	11.87	11.88	— .01
+72	11.93	11.97	— .04	11.94	11.97	— .03

excentricity on the course of the light-curve. We conclude that, from the light-curves we can only infer that the orbit is nearly circular.

At all events there is no reason to assume an increase or decrease of the, certainly very small, excentricity. A comparison of the elements a and q might lead us to conjecture that the distance of the two celestial bodies has diminished since the time of ARGELANDER. The increase in q is in agreement with such a supposition, but the continual lengthening of the period seems to clash with it.

15. *Computation of the orbit from the spectrographs of* BELOPOLSKY.

In the computation of the orbit from the velocities in the line of sight as derived by B. from the displacement of the bright *F*-line in the spectrographs of 1892, the method of WILSING [1]) has been adopted. For very small excentricities it is to be preferred to that of LEHMAN-FILHÉS.

The first column contains the mean time of observation at PULKOWA; the 2nd gives the phase in the light-period of 12.91 days. We have assumed, in accordance with the formula of ARGELANDER, as corrected by PANNEKOEK, that the principal minimum occurred on 1892 Sept 25. 781 M. T. Greenwich (= 25d.865 M. T. PULKOWA).

The 3rd column contains the velocities, expressed in geographical miles, reduced to the sun. They have been taken, with slight modifications, from the Memorie della Soc. d. Spettr. It., vol. XXII. For BELOPOLSKY has applied a constant correction — 2.1 G.M. for the velocity of the earth, whereas in reality this velocity varies between — 1.6 and — 2.3 G.M.

T	Phase	Veloc. in G.M.	V_r	O-C
	d			
Sept. 23 3	10.34	—11.2	—11.25	+0 05
24.4	11.44	—11 6	—10.09	—1.51
25.4	12.44	— 4.4	— 2.58	+1.82
27.3	1.44	+ 4.8	+ 5.28	—0 48
30.3	4.44	+10.7	+10.50	+0.20
Oct. 2.3	6.44	+ 1 7	+ 2.09	—0.39
3.3	7.44	— 3.6	— 2 71	—0.89
7.3	11 44	— 9.5	—10.09	+0.59
11.3	2.53	+10.1	+10.29	—0.19
19.3	10 53	—12.4	—11.29	—1.11
20.3	11.53	—10.3	— 9.84	—0.46
26.3	4.62	+10.6	+ 9 98	+0.62
Nov. 25.2	8.70	— 6.7	— 7.85	+1.15
26.2	9.70	—10.2	—10 62	+0 42

[1]) Dr. J. WILSING. Ueber die Bestimmung von Bahnelementen enger Doppelsterne aus spectroskopischen Messungen der Geschwindigkeitscomponenten. A. N. no. 3198.

With the notations of WILSING these observations lead to the following normal equations:

$$
\begin{aligned}
&+\ 14\,g_0 - 3.17\ a_1 + 1.56\ b_1 - 3.41\,a_2 - 1.28\,b_2 = -42.00\\
&-3.17\ g_0 + 7.64\ a_1 - 1.68\ b_1 + 1.81\,a_2 + 1.02\,b_2 = +90.78\\
&+1.56\ g_0 - 1.68\ a_1 + 6.36\ b_1 - 2.71\,a_2 - 0.27\,b_2 = -37.81\\
&-3.41\ g_0 + 1.81\ a_1 - 2.17\ b_1 + 8.52\,a_2 + 0.32\,b_2 = +30.74\\
&-1.28\ g_0 + 1.02\ a_1 - 0.27\ b_1 + 0.32\,a_2 + 5.48\,b_2 = +\ 8.57
\end{aligned}
$$

Solution :

$g_0 = -\ 0.097$ G.M. = constant velocity towards the sun.

$a_1 = -\,an\,sin\,i\,sin\,(\omega'+M_0) = +11.196$; $b_1 = an\,sin\,i\,cos\,(\omega'+M_0) = -2.953$

$a_2 = -\,ean\,sin\,i\,sin\,(\omega'+2M_0) = +0.498$; $b_2 = ean\,sin\,i\,cos(\omega'+2M_0) = -0.708$

ω' is the longitude of the periastron, counted from ☊ ; M_0 the mean anomaly at the beginning of the light-period, consequently:

$$an\,sin\,i = 11.579\,;\ \omega' = 115°20'\,;\ M_0 = 139°54'\,;\ e = 0.075.$$

As $\omega' + M_0 = 180° + 75°12'$, the elements belong to the body which, during the principal minimum, eclipses the other. Conjunction takes place, when $\omega' + v = 270°$, i.e. $0^d.39$ days after the time of the principal minimum, as computed from the empirical formula.

In the following table the 4th column shows the computed velocities, the 6th the outstanding deviations.

16. *Spectrographs of* 1897.

The velocities (in G. M), derived by *B.* from the displacements of the dark *Mg*-line $\lambda = 448.16\ \mu\mu$, have been taken unchanged from the Memorie della Soc. degli Spettr. It. vol XXVI. The empirical formula leads to the epoch 1897 June 22 16th. 24 M. T. PULKOWA for the principal minimum.

In a first approximation I determined a circular orbit and found:

$$g_0 = -\ 2.094\,;\ an\,sin\,i = 24.210\,;\ \omega' + M_0 = 89°.30'.1.$$

Afterwards corrections were derived by the aid of the formula :

$$d\frac{dz}{dt} = dg_0 + KndT\,sin\,(\omega' + v) + dK\,.\,cos\,(\omega' + v) +$$
$$+ Ke\,cos\,\omega'\,cos\,2\,(\omega' + v) + Ke\,sin\,\omega'\,sin\,2\,(\omega' + v).$$

in which $K = an\,sin\,i$, $T =$ time of periastron-passage.

This formula is obtained from the general differential-formula [1]), by putting $d\mu = 0$, $d\omega = 0$ and by further neglecting 2nd and higher powers of e. We thus find the following normal equations :

[1]) Vide: BAUSCHINGER, Die Bahnbestimmung der Himmelskörper, N⁰. 199.

$$
\begin{array}{llllll}
+\ 26dg_0 - & 1.35K\mu dt - & 2.01dK - & 2.22Ke\cos\omega' - & 2.03Ke\sin\omega' & = -0\ 05 \\
-1.35\ ,, & +13.18\ ,, & -\ 1.02\ ,, & +\ 1.31\ ,, & +\ 0.16\ ,, & = +6.17 \\
-2.01\ ,, & -\ 1.02\ ,, & +12.82\ ,, & -\ 0.28\ ,, & +\ 0.26\ ,, & =\ \ 0\ 00 \\
-2.22\ ,, & +\ 1.31\ ,, & -\ 0.28\ ,, & +13.88\ ,, & -\ 0.31\ ,, & = +4.75 \\
-2.03\ ,, & +\ 0.16\ ,, & +\ 0.26\ ,, & -\ 0.31\ ,, & +12.12\ ,, & = -5.98
\end{array}
$$

	T	Phase	Veloc. in G.M.	V_r	$O-C$
June 20	$11^h.5$	$10^d\ 17^h.0$	+18.27	+19.29	—1.02
22	12.0	12 17.6	— 2.60	+ 0.28	—2.88
23	12.4	0 20.2	—10.62	—10 98	+0.36
24	12.1	1 19.9	—20.40*)	—19.92	—0.48
28	11.6	5 19.4	—11.14	—10.77	—0.37
30	11.1	7 18.9	+14.16	+12.41	+1.75
July 2	11.9	9 19.7	+21.38	+22.42	—1.04
8	12.3	2 22.3	—24.97*)	—25.56	+0.59
9	11.4	3 21.4	—25.68	—25.32	—0 36
10	11.1	4 21.1	—21.27	—19.87	—1.40
11	11.0	5 21.0	— 8.83	—10.00	+1.17
12	11.5	6 21.5	+ 3.24	+ 2.34	+0.90
13	11.4	7 21.4	+13.15	+13.43	—0.28
15	11.4	9 21.4	+24 15	+22.34	+1.81
17	11 2	11 21.2	+10.34	+ 9.35	+0.99
21	11.2	2 23.4	—27.52	—25.65	—1.87
22	11.2	3 23.4	—23.48	—25.07	+1.59
24	10.3	5 22.3	— 9.28	— 9.37	+0.09
25	10.2	6 22.2	+ 0.53	+ 2.70	—2.17
26	10.0	7 22.0	+12.77	+13.68	—0.91
27	10.2	8 22.0	+21.03	+20.86	+0.17
30	10.1	11 22.1	+10.11	+ 9.16	+0.95
31	10.2	0 0.6	— 1.03	— 2.01	+0.98
Aug. 2	9.7	2 0.1	—20.36	—21.64	+1.28

*) Mean of two observations.

Solution:

$$dg_0 = + 0.0124 \quad ; \quad K\mu dT = + 0.450 \quad ; \quad dK = + 0.054 ;$$
$$Ke \cos \omega' = + 0.292 \quad ; \quad Ke \sin \omega' = - 0.489.$$

from which we get the elements:

$$g_0 = - 2.082 \; GM. \quad ; \quad an \sin i = 24.264 \quad ; \quad e = 0.0235 ;$$
$$\omega' = 300°51' \quad ; \quad \omega' + M_0 = 88°26' ;$$

whereas the conjunction coincides perfectly with the principal minimum, *the difference in time amounting to less than* 0.01 *days.* Evidently this is the principal minimum. This is in accordance with the fact that the difference of the longitudes found for the two periastra deviates but slightly from 180° (300°51′ — 115°20′). This may be partly due to a fortunate coincidence.

Meanwhile the excentricity of the 2nd orbit is more than three times *smaller* than that of the 1st, while the velocity in the direction towards the sun found for the whole system is 2 Geogr. miles greater in the 2nd case.

If the latter difference is real, this acceleration would have caused a *shortening* of the period between 1892 and 1897. As, however the measures of 1892, according to the judgment of Prof. H. C. VOGEL "nicht als ganz einwurfsfrei angesehen werden können" [1]), we suspend our judgment to the time that Prof. BELOPOLSKY will again have taken up his beautiful investigations on the spectrum of β Lyrae, particularly about the F- line. Already in 1897 he communicated his intention to do so.

If we put $i = 90°$, the semiaxes major are:

$$a_1 = 2056000 \; G.M. \quad ; \quad a_2 = 4307000 \; G.M.$$

From KEPLER's third law we derive therefrom, roughly

$$m_1 = 17.1 \text{ sun's masses} \quad ; \quad m_2 = 8.1 \text{ sun's masses.}$$

17. In our opinion the preceding considerations justify the conclusion that the data about β Lyrae do not furnish a sufficient basis for a decision about any change in the elements, in particular in the excentricity. For the rest, owing to our ignorance on the circumstances in such a close system, the adopted explanation of the light variation can only claim to give a rough approximation — a rude imitation of a very complicated mechanism.

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[1]) Ueber das Spectrum von β Lyrae. Sitzungsb. Ak. Berlin. 1894 VI.

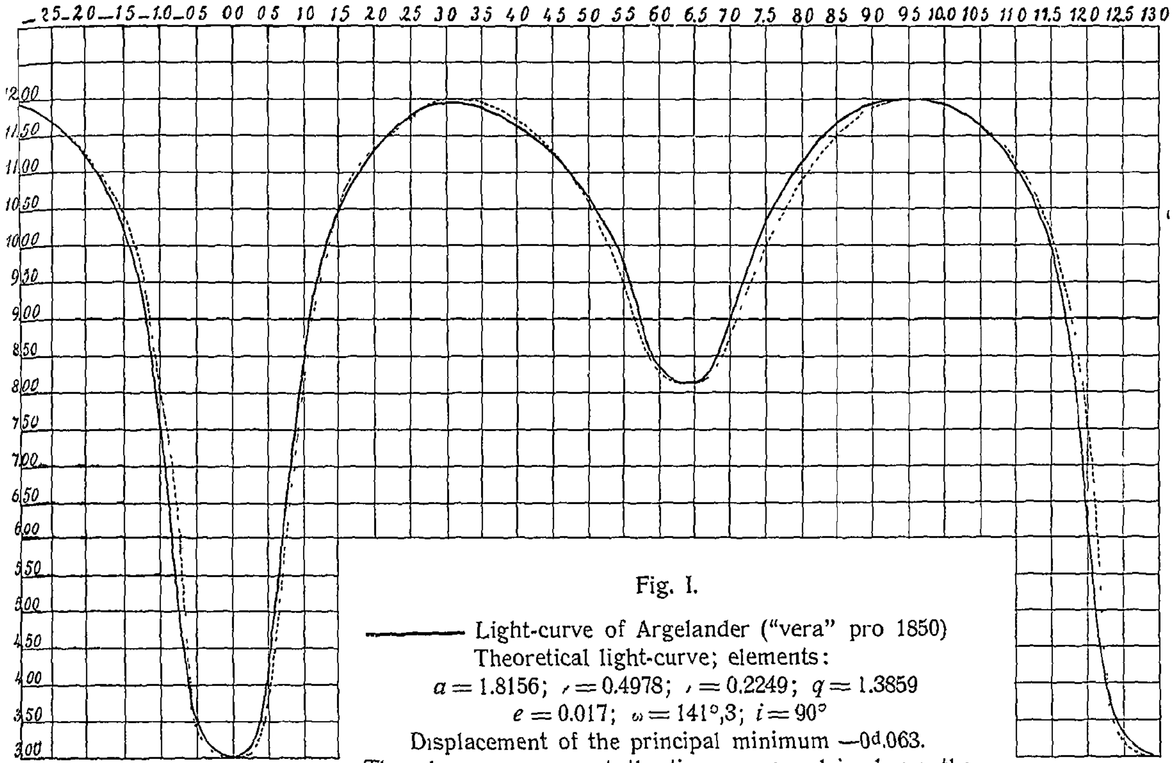

Fig. I.

——— Light-curve of Argelander ("vera" pro 1850)
Theoretical light-curve; elements:
$a = 1.8156$; ' $= 0.4978$; ' $= 0.2249$; $q = 1.3859$
$e = 0.017$; $\omega = 141°,3$; $i = 90°$
Displacement of the principal minimum $-0^d.063$.
The abcissae represent the time expressed in days; the ordinates the light-intensity in grades.

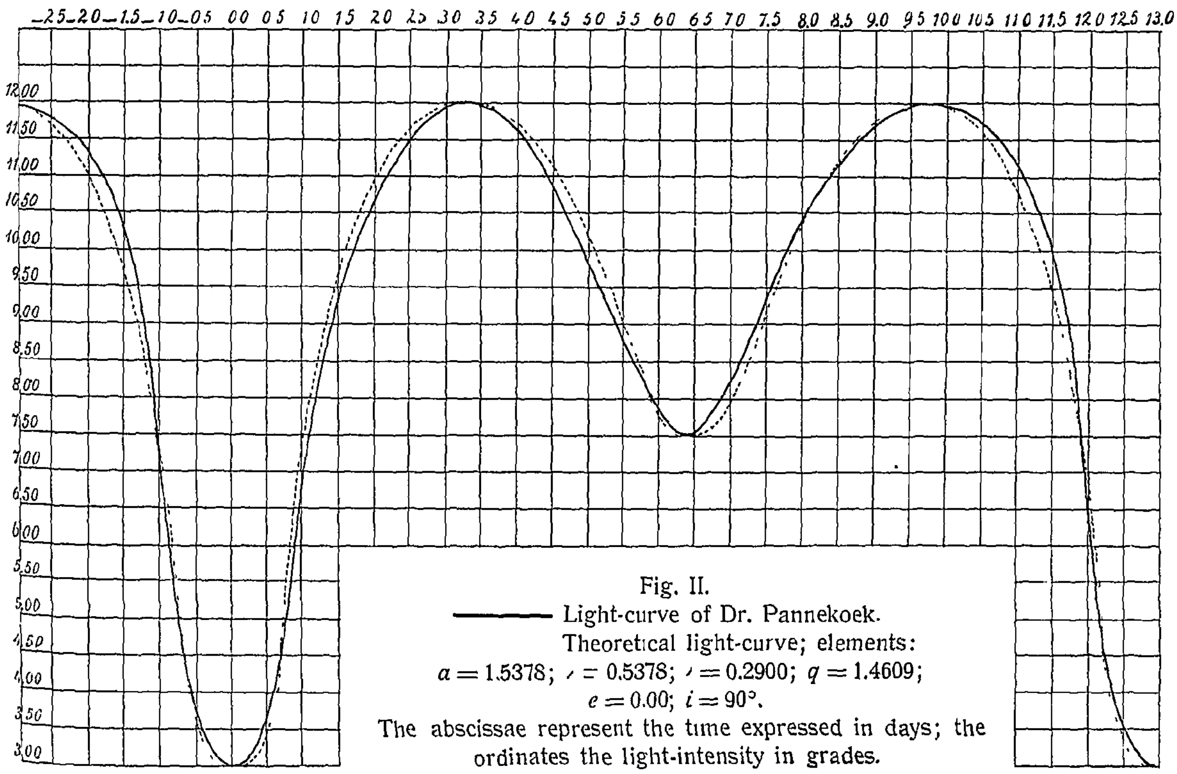

Fig. II.

——— Light-curve of Dr. Pannekoek.
Theoretical light-curve; elements:
$a = 1.5378$; ' $= 0.5378$; ' $= 0.2900$; $q = 1.4609$;
$e = 0.00$; $i = 90°$.
The abscissae represent the time expressed in days; the ordinates the light-intensity in grades.