in the usual manner. On microscopic examination the blood-vessel were found to be filled now *with a perfectly homogeneous black mass*

This method has an advantage over that of GROSSER in the fac that we need not fear the injection-fluid becoming solidified befor the injection; besides the preparation of the suspension requires *muc less time.*

In another direction too we have simplified the method, viz. by substituting blood-serum for egg white. A mixture of 3 parts o blood-serum with 2 parts of the above named Indian ink gave excellent results.

The blood-serum need not be derived from the same species of animal. For injections of caviae or rabbits we got good results by using horse-serum or cow-serum, fluids that are easily obtained.

Here too fixation was brought about by means of sublimate-formol.

As yet kidneys and liver were microscopically examined. But the injection fluid also penetrated skin, muscles and brain.

An attempt to prepare suspensions of carmine grains in serum suggested itself now, but these experiments failed as the carmine particles conglomerated. Perhaps, however, mixtures of dissolved carmine or of colloidal fluids may be prepared with serum, giving good results.

The above mentioned experiments were made in cooperation with Mr. A. F. DE BOER and Mr. G. A. KALVERKAMP, medical students.

Groningen, March 1908.

Mathematics. — "*The sections of the net of measure-polytopes M_n of space Sp_n with a space Sp_{n-1} normal to a diagonal.*" By Prof. P. H. SCHOUTE.

1. In the first part of a communication on fourdimensional nets and their sections by spaces (*Proceedings*, Febr. 1908) we have i.a. transformed the net (C_8) into a net (C_{16}) and a net (C_{24}); so here the regular simplex, the fivecell C_5, was not considered. Whereas the regular simplex of Sp_2, the equilateral triangle, furnishes a plane-filling all by itself as well as in connection with some other regular polygons, and the regular simplex of Sp_3, the tetrahedron, can fill the space in combination with the octahedron, it is impossible, as was shown in the quoted paper, to find for the regular simplex C_5 of Sp_4 other regular cells, which can together fill the space of Sp_4.

This leads us gradually to the question, whether it is not possible to point out one or more polytopes — if not quite regular ones — which with C_5 fill the fourdimensional space. We have here in view to give to this question an answer, emanating from the connection of a few results formerly arrived at.

2. We consider the net (M_5) of the measure-polytopes M_5 of space Sp_5 and cut this by a space Sp_4 normal to a diagonal. This work breaks immediately up into two parts. First the section of space Sp_4 with a definite measure-polytope M_5 must be found, e. g. with the one, the centre of which has been taken for origin of a rectangular system of coordinates with axes parallel to the edges; we must next investigate how we can prove from this section in which way the intersecting space Sp_4 affects the other measure-polytopes of the net.

The answer to the first part of this question can be found by means of one of the two diagrams 1 and 2, which we shall therefore discuss successively. Of these diagram 1 is what we arrive at when we project

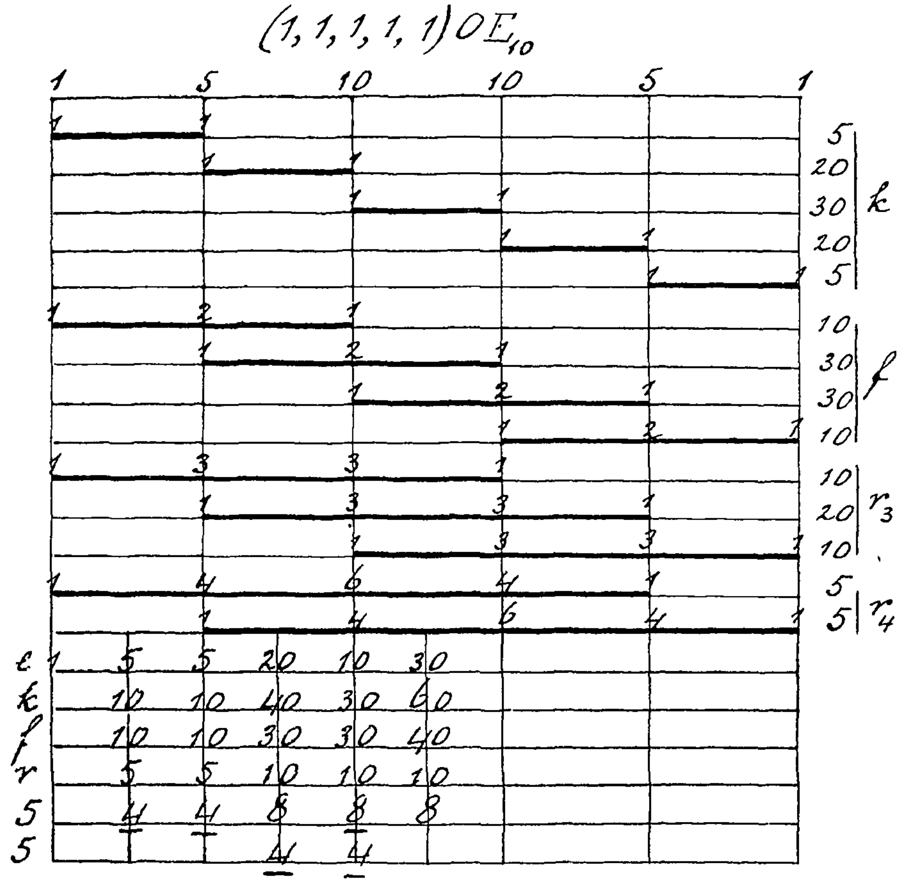

Fig. 1.

the bounding elements of M_5 on the diagonal; it is an extension of the second diagram $n=5$ of the plate, added to the communication on the section of the measure-polytope M_n of the space S_n with a central space Sp_{n-1} normal to a-diagonal (*Proceedings*, Jan. 1908). Here, too, we restrict ourselves to a few sections, viz. to the transition forms and to those intermediary forms which bisect the distance of two adjacent transition forms; according to the notation introduced there, we distinguish the transition forms by the symbols $\frac{1}{5}M_5$, $\frac{2}{5}M_5, \frac{3}{5}M_5, \frac{4}{5}M_5$, the intermediary forms by the symbols $\frac{1}{10}M_5, \frac{3}{10}M_5$, $\frac{5}{10}M_5, \frac{7}{10}M_5, \frac{9}{10}M_5$. As these sections have been incidentally already found in the last quoted paper, we can suffice here by a mere enumeration; to be able to indicate relations in measure we again assume that we have taken half the edge of M_5 as measure-unit.

Transition forms. As two sections pM_5 and qM_5 of which the fractional symbols p and q complete each other to unity, form two oppositely orientated positions of the same polytope, we have here to deal with but two transition forms, viz. $\frac{1}{5}M_5 = -\frac{4}{5}M_5$ and $\frac{2}{5}M_5 = -\frac{3}{5}M_5$. Of these $\frac{1}{5}M_5$ is a regular fivecell $C_5^{(2\sqrt{2})}$, whilst $\frac{2}{5}M_5$ is formed (see *Proceedings*, page 488 under $n=6$) by truncating a fivecell $C_5^{(4\sqrt{2})}$ at the vertices as far as halfway the edges and hence transforming it into a polytope (10, 30, 30, 10) with edges $2\sqrt{2}$; for the last form *Proceedings*, page 503, can be compared.

Intermediary forms. Of the three intermediary forms $\frac{1}{10}M_5 = -\frac{9}{10}M_5$, $\frac{3}{10}M_5 = -\frac{7}{10}M_5$, $\frac{5}{10}M_5$ the first is a $C_5^{(\sqrt{2})}$, the second (*Proceedings*, page 488 under $n=7$) a fivecell $C_5^{(3\sqrt{2})}$ truncated as far as a third of the edges, passing by this proceeding into a polytope (20, 40, 30, 10) with edges $\sqrt{2}$, the third (*Proceedings*, page 487 under $n=5$) a $C_5^{(5\sqrt{2})}$ truncated as far as three fifths of the edges, which has on account of this passed into a polytope (30, 60, 40, 10) with edges $\sqrt{2}$.

We shall now pass to diagram 2 where the plane through two opposite edges PQ, $P'Q'$ intersecting the diagonal $P'Q$ has been taken as plane of projection. The projection of $M_5^{(2)}$ on this plane is the

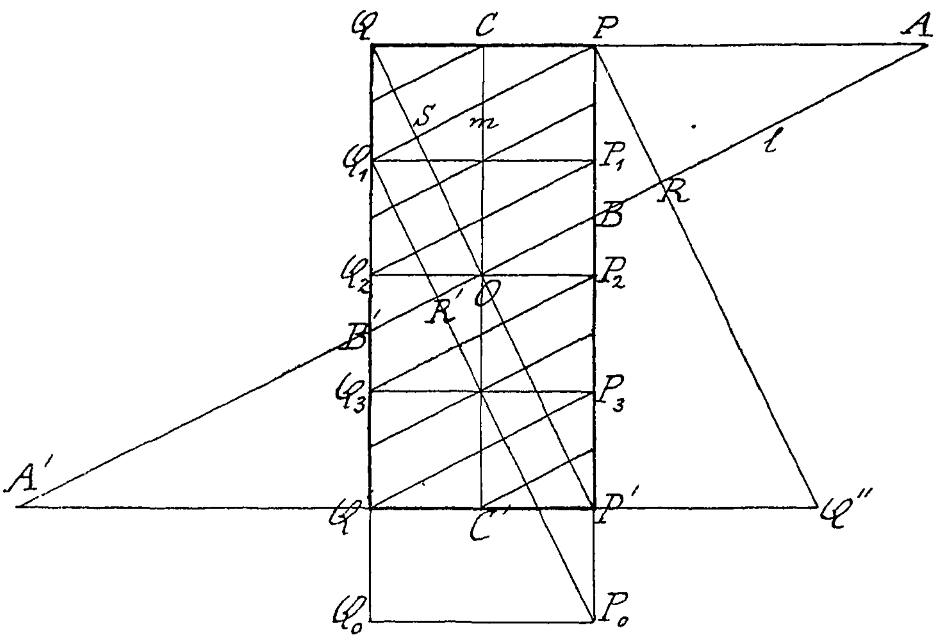

Fig. 2.

rectangle $PQQ'P'$ with edges $PQ = 2$, $PP' = 4$, which is divided by three lines parallel to PQ into four equal rectangles. The intersecting space Sp_4 passing through the centre O stands according to the perpendicular l, erected in O on the diagonal $P'Q$, normal to the plane of projection. If we suppose (*Proceedings*, page 491) a few measure-polytopes $M_5^{(2)}$, which are laid against each other in the direction of the edge PQ on either side, to be united to a prism of which the basis is an $M_4^{(2)}$ and the edges normal to OA have the direction PQ, then the section of the space Sp_4 through O with this prism is a rhombotope Rh_4 of which AA' — with a length of $4\sqrt{5}$ — represents the axis with the period 4. Comparison of this rhombotope with the measure polytope $M_4^{(2)}$ of $M_5^{(2)}$ lying in the space Sp_4 perpendicular according to m on the plane of projection shows us that the rhombotope can be obtained by stretching this polytope $M_4^{(2)}$ in the direction of the diagonal CC' to an amount of $OA : OC = \sqrt{5}$. This rhombotope is truncated perpendicularly by the spaces Sp_3 projecting themselves in the points of intersection B, B' of the axis AA' with the sides PP', QQ' of the rectangle. If again we make

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use of the annotation $a\,(p, q)$ formerly introduced (*Verhandelingen*, vol. IX, N°. 7, page 17) then the central section is a polytope $4\sqrt{5}\left(\frac{3}{8}, \frac{5}{8}\right)$ and we find, omitting the length of axis $4\sqrt{5}$ alike for all sections, for the transition forms and the intermediary forms described above the following rhombotope symbols:

$$\frac{1}{10} M_5 = \left(0, \frac{1}{8}\right), \quad \frac{3}{10} M_5 = \left(\frac{1}{8}, \frac{3}{8}\right), \quad \frac{5}{10} M_5 = \left(\frac{3}{8}, \frac{5}{8}\right), \quad \frac{7}{10} M_5 = \left(\frac{5}{8}, \frac{7}{8}\right), \quad \frac{9}{10} M_5 = \left(\frac{7}{8}, \frac{8}{8}\right),$$

$$\frac{1}{5} M_5 = \left(0, \frac{1}{4}\right), \quad \frac{2}{5} M_5 = \left(\frac{1}{4}, \frac{2}{4}\right), \quad \frac{3}{5} M_5 = \left(\frac{2}{4}, \frac{3}{4}\right), \quad \frac{4}{5} M_5 = \left(\frac{3}{4}, \frac{4}{4}\right).$$

3. The second part of the question, viz. how the intersecting space Sp_4 affects the other measure-polytopes can now be answered by means of analytical geometry as well as by descriptive geometry.

With reference to the system of coordinates assumed above the centres and vertices of all cells $M_5^{(2)}$ of the net have all nothing but integers as coordinates, the centres only even integers, the vertices only odd ones. From this follows in general that the distances from the centres to the central space $\sum_1^5 x_i = 0$ are multiples of fifth parts of the diagonal, those of the vertices to the same space odd multiples of tenth parts of the diagonal. In this way a space of intersection $\sum_1^5 x_i = p$ in general furnishes five different sections of which the fractions placed before M_5 differ respectively $\frac{1}{5}$. If the space of intersection passes through a vertex we find the transition sections; if it passes though a centre we find the intermediary forms.

We arrive at the same result by diagram 2. If we allow the same space Sp_4 bisecting perpendicularly the diagonal $P'Q$ of the central cell to intersect the right adjacent cell with the diagonal P_1Q'', then the segment QO cut from the diagonal of the central cell passes into PR, which means a decrease of $QS = \frac{1}{5} QP'$, and this is repeated every time a cell is taken further to the right. If we exchange the central cell by an other one of which the projection $P_0 P_1 Q_1 Q_0$ covers for three fourths that of the central one, then QO passes into $Q_1 R'$, again a decrease of $\frac{1}{5}$, and this too is repeated every time the projection moves onward in the direction PP' to an amount of PP_1. So here too we find five different symbols pM_5, of which the fractions gradually increase with $\frac{1}{5}$. With the aid of the above table this result of the notation pM_5 can be transformed into that of the rhombotope symbols.

We have now answered the question put at the commencement. If we wish to fill Sp_4 with C_5 and a single other groundform, then the form (10, 30, 30, 10) with the same length of edges can do service; both forms appear then in two oppositely orientated positions. If by the side of C_5 we allow two other groundforms to fill Sp_4, we can make use of the forms (20, 40, 30, 10) and (30, 60, 40, 10) of the same length of edges, if we take into consideration difference in orientation, then this space-filling demands five forms. And if one does not object to connecting more than two really different groundforms we can take the five forms

$$\frac{1}{20} M_5,\ \frac{5}{20} M_5,\ \frac{9}{20} M_5,\ \frac{13}{20} M_5,\ \frac{17}{20} M_5,$$

i. e.

$$\left(0, \frac{1}{16}\right),\ \left(\frac{1}{16}, \frac{5}{16}\right),\ \left(\frac{5}{16}, \frac{9}{16}\right),\ \left(\frac{9}{16}, \frac{13}{16}\right),\ \left(\frac{13}{16}, 1\right),$$

of which the first is a $C_5^{(\frac{1}{4}\sqrt{2})}$; these appear in only one position.

4. Before passing on to the general case of Sp_n we indicate the shortest way, by which one can calculate the number of component parts when filling a fourdimensional block of one of the found forms but of k-times larger linear dimension. To prepare the general case of an arbitrary n we introduce a simpler notation. We distinguish the transition forms and the intermediary forms by the letters T and I and then indicate by exponent — this, to avoid rootsigns, in $\sqrt{2}$ as new unit — the size, by a footindex the place of the section. We then represent the polytope, formed by truncating regularly a regular fivecell with a length of edges $p\sqrt{2}$ at the five corners to the fraction q of the edge by the symbol $qS^{(p)}$. Thus each of the five different forms is represented by four different signs as follows:

$$\frac{1}{10}M_5^{(2)}=\left(0,\frac{1}{8}\right)=I_1^{(1)}=\quad S_,^{(1)}$$

$$\frac{3}{10}M_5^{(2)}=\left(\frac{1}{8},\frac{3}{8}\right)=I_2^{(1)}=\frac{1}{3}S_,^{(3)}$$

$$\frac{5}{10}M_5^{(2)}=\left(\frac{3}{8},\frac{5}{8}\right)=I_3^{(1)}=\frac{3}{5}S_,^{(5)}$$

$$\frac{1}{5}M_5^{(2)}=\left(0,\frac{1}{4}\right)=T_1^{(2)}=\quad S_,^{(2)}$$

$$\frac{2}{5}M_5^{(2)}=\left(\frac{1}{4},\frac{2}{4}\right)=T_2^{(2)}=\frac{1}{2}S_,^{(4)}$$

whilst the forms appearing past the middle $\frac{7}{10}M_5^{(2)}, \frac{9}{10}M_5^{(2)}$ and $\frac{3}{5}M_5^{(2)}, \frac{4}{5}M_5^{(2)}$ of opposite orientation are indicated by $I_{-2}^{(1)}$, $I_{-1}^{(1)}$ and $T_{-2}^{(2)}$, $T_{-1}^{(2)}$.

By considering the truncated fivecells $qS_{(p)}$ we find immeditately:

$$\left.\begin{array}{l} T_1^{(2k)}=I_1^{(2k)} \\ I_2^{(k)}=I_1^{(3k)}-5I_1^{(k)} \\ T_2^{(2k)}=I_1^{(4k)}-5I_1^{(2k)} \\ I_3^{(k)}=I_1^{(5k)}-5I_1^{(3k)}+10I_1^{(k)} \end{array}\right\} \quad \ldots\ldots \quad (1)$$

Of these relations e. g. the last one is deduced in the following way: The form $I_3^{(k)}=\frac{3}{5}S^{(5k)}$ appears by truncating the fivecell $S^{(5k)}=I_1^{(5k)}$ to $\frac{3}{5}$ of the edges. As each two of the five polytopes $S^{(3k)}=I_1^{(3k)}$, which are taken off by the truncation, have an $S^{(k)}=I_1^{(k)}$ in common, we subtract when diminishing $I_1^{(5k)}$ by $5\,I_1^{(3k)}$ ten times $I_1^{(k)}$ too much.

Together the equations (1) lead to the relations of volume:

$$\frac{I_1^{(k)}}{1}=\frac{T_1^{(2k)}}{16}=\frac{I_2^{(k)}}{76}=\frac{T_2^{(2k)}}{176}=\frac{I_3^{(k)}}{230}=\frac{R^{(2k)}}{384},$$

where $R^{(2k)}$ is the rhombotope formed by the required stretching of an $M_4^{(2k)}$ in the direction of a diagonal. If the number 384 is deduced from the remark that $T_1^{(2k)}=\frac{1}{4!}R^{(2k)}$, then the two relations

$$2\,(16+176)=384 \quad , \quad 2\,(1+76)+230=384,$$

which express that $R^{(2k)}$ can be built up either out of the four forms $T_i^{(2k)}$ or out of the five forms $I_i^{(k)}$ can serve to control.

We shall now indicate at full length how the obtained relations will serve to get us over the entire difficulty of the determination of the demanded numbers. To this end we notice that the vertices of the k^5 measure polytopes $M_5^{(2)}$ forming together a block $M_5^{(2k)}$ project themselves on a diagonal of that block except in the ends in the $5k-1$ points dividing this diagonal into $5k$ equal parts. If we indicate (diagram 3) the $5k+1$ points obtained in this way on the diagonal by $A_0, A_1, A_2, \ldots, A_{5k}$, then the segment A_0A_5 bears the projection of a single $M_5^{(2)}$, the segment A_1A_6 that of a group of five, the segment A_2A_7 that of a group of fifteen measure-polytopes,

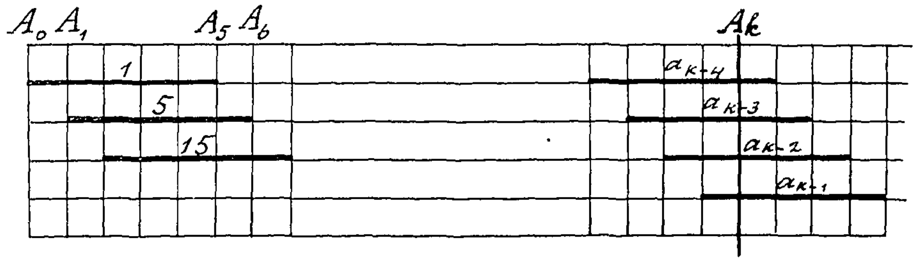

Fig. 3.

etc., where the numbers 1, 5, 15, etc. of the measure polytopes with the same projection are the coefficients a_p of the terms x^p in $(1+x+x^2+\ldots+x^{k-1})^5$ for $p=0,1,2$, etc. When determining the section $\frac{1}{5}M_5^{(2k)}$ we find that the intersecting space Sp_4 hits the diagonal of projection in the point of division A_k, from which ensues that the groups of polytopes $M_5^{(2)}$ corresponding to the coefficients $a_0, a_1, \ldots a_{k-5}$ are not yet cut, the groups corresponding to the coefficients $a_k, a_{k+1}, \ldots a_{5k-5}$ are no more cut, so that we have but to deal with the four groups shown to the right of the diagram:

$$a_{k-4}T_{-1}^{(2)}, \quad a_{k-3}T_{-2}^{(2)}, \quad a_{k-2}T_2^{(2)}, \quad a_{k-1}\,T_1^{(2)}.$$

Now for the coefficients a_p the particularity appears that for $p \leqq k$ they can be represented as binominal coefficients viz. by the equation

$$a_p = (p+4)_4,$$

whilst for greater values of p they are "gnawed" binominal coefficients. So we find here immediately

$$T_1^{(2k)} = I_1^{(2k)} = (k+3)_4\,T_1^{(2)} + (k+2)_4\,T_2^{(2)} + (k+1)_4\,T_{-2}^{(2)} + (k)_4\,T_{-1}^{(2)}, \ldots (2)$$

and in quite the same way

$$I_1^{(2k+1)} = (k+4)_4\, I_1^{(1)} + (k+3)_4\, I_2^{(1)} + (k+2)_4\, I_3^{(1)} + (k+1)_4\, I_{-2}^{(1)} + (k)_4\, I_{-1}^{(1)} \ldots (3)$$

which two relations in connection with the ratios of volume lead back to the identities

$$\left.\begin{array}{l} k^4 = (k+3)_4 + 11\,(k+2)_4 + 11\,(k+1)_4 + (k)_4 \\ (2k+1)^4 = (k+4)_4 + 76\,(k+3)_4 + 230\,(k+2)_4 + 76\,(k+1)_4 + (k)_4 \end{array}\right\}.$$

From (1), (2), (3) we can now easily deduce all results. To prove this we mention for the two cases, in which the block consists of an even or of an odd number of measure-polytopes, the composition of the central section in the form

$$I_3^{(2k)} = \frac{5}{12} k^2 \left\{ (23k^2 - 11)\, T_1^{(2)} + (23k^2 + 1)\, T_2^{(2)} + \right.$$
$$\left. + (23k^2 - 1)\, T_{-2}^{(2)} + (23k^2 + 11)\, T_{-1}^{(2)} \right\},$$

$$I_3^{(2k+1)} = \frac{5}{12} k(k+1) \left\{ (23k^2 + 23k - 10) \left(I_1^{(1)} + I_{-1}^{(1)} \right) + \right.$$
$$\left. + (23k^2 + 23k + 8) \left(I_2^{(1)} + I_{-2}^{(1)} \right) \right\}$$
$$+ \frac{1}{12} (115k^4 + 230k^3 + 185k^2 + 70k + 12)\, I_3^{(1)}.$$

5. We shall now consider in the space Sp_n the net of measure-polytopes $M_n^{(2)}$ and shall discuss the transition sections and the intermediary forms situated in the middle between two adjacent transition sections furnished by spaces Sp_{n-1} perpendicular to a diagonal. We then find

$$\frac{1}{2n} M_n^{(2)} = \left(0, \frac{1}{2n-2}\right) = I_1^{(1)} = S^{(1)},$$

$$\frac{3}{2n} M_n^{(2)} = \left(\frac{1}{2n-2}, \frac{3}{2n-2}\right) = I_2^{(1)} = \frac{1}{3} S^{(3)},$$

$$\frac{5}{2n} M_n^{(2)} = \left(\frac{3}{2n-2}, \frac{5}{2n-2}\right) = I_3^{(1)} = \frac{3}{5} S^{(5)},$$

$$\vdots$$

for n even

$$\frac{n-1}{2n} M_n^{(2)} = \left(\frac{n-3}{2n-2}, \frac{n-1}{2n-2}\right) = I_{\frac{1}{2}n}^{(1)} = \frac{n-3}{n-1} S^{(n-1)},$$

for n odd

$$\frac{1}{2} M_n^{(2)} = \left(\frac{n-2}{2n-2}, \frac{n}{2n-2}\right) = I_{\frac{1}{2}(n+1)}^{(1)} = \frac{n-2}{n} S^{(n)},$$

$$\frac{1}{n} M_n^{(2)} = \left(0, \frac{1}{n-1}\right) = T_1^{(2)} = S^{(2)},$$

$$\frac{2}{n} M_n^{(1)} = \left(\frac{1}{n-1}, \frac{2}{n-1}\right) = T_2^{(2)} = \frac{1}{2} S_2^{(4)},$$

$$\frac{3}{n} M_n^{(2)} = \left(\frac{2}{n-1}, \frac{3}{n-1}\right) = T_3^{(2)} = \frac{2}{3} S^{(6)},$$

$$\vdots$$

for n even

$$\frac{1}{2} M_n^{(2)} = \left(\frac{n-2}{2n-2}, \frac{n}{2n-2}\right) = T_{\frac{1}{2}n}^{(2)} = \frac{n-2}{n} S^{(n)},$$

for n odd

$$\frac{n-1}{2n} M_n^{(2)} = \left(\frac{n-3}{2n-2}, \frac{n-1}{2n-2}\right) = T_{\frac{1}{2}(n-1)}^{(2)} = \frac{n-3}{n-1} S^{(n-1)}.$$

If we restrict ourselves to these forms and if again we do not take the transition form consisting of a single vertex into consideration, we have in both cases to deal with n different forms, namely for n even with $\frac{1}{2}n$ transition and $\frac{1}{2}n$ intermediary forms, for n odd with $\frac{1}{2}(n-1)$ transition and $\frac{1}{2}(n+1)$ intermediary forms. Thus we get again in Sp_{n-1} two more or less regular space-fillings in which the regular simplex of that space shares.

In connection with the symbols ${}_qS^{(p)}$ the relations hold here

$$\overset{(1)}{I_2} = \overset{(3)}{I_1} - (n)_1\, \overset{(1)}{I_1},$$
$$\overset{(2)}{T_2} = \overset{(4)}{I_1} - (n)_1\, \overset{(2)}{I_1},$$
$$\overset{(1)}{I_3} = \overset{(5)}{I_1} - (n)_1\, \overset{(3)}{I_1} + (n)_2\, \overset{(1)}{I_1},$$
$$\overset{(2)}{T_3} = \overset{(6)}{I_1} - (n)_1\, \overset{(4)}{I_1} + (n)_2\, \overset{(2)}{I_1},$$
$$\vdots$$

which leads
for n even to

$$\overset{(2)}{T_{\frac{1}{2}n}} = \overset{(n)}{I_1} - (n)_1 \overset{(n-2)}{I_1} + (n)_2 \overset{(n-4)}{I_1} - \ldots + (-1)^{(\frac{1}{2}n-1)}\, (n)_{\frac{1}{2}n-1}\, I_1,$$

for n odd to

$$\overset{(1)}{I_{\frac{1}{2}(n+1)}} = \overset{(n)}{I_1} - (n)_1 \overset{(n-2)}{I_1} + (n)_2 \overset{n-4}{I_1} - \ldots + (-1)^{\frac{1}{2}(n-1)} (n)_{\frac{1}{2}(n-1)}\, I_1,$$

whilst the ratios of volume are determined by

$$\frac{\overset{(1)}{I_1}}{1} = \frac{\overset{(2)}{T_1}}{2^{n-1}} = \frac{\overset{(1)}{I_2}}{3^{n-1} - (n)_1} = \frac{\overset{(2)}{T_2}}{4^{n-1} - (n)_1 2^{n-1}} = \frac{\overset{(1)}{I_3}}{5^{n-1} - (n)_1 3^{n-1} + (\ )_2} = \text{etc.}$$

Farthermore the formulae of reduction hold :

$$\left.\begin{aligned} \overset{(2k)}{I_1} &= (k+n-2)_{n-1} \overset{(2)}{T_1} + (k+n-3)_{n-1} \overset{(2)}{T_2} + \ldots + (k)_{n-1}\, \overset{(2)}{T_{-1}} \\ \overset{(2k+1)}{I_1} &= (k+n-1)_{n-1} \overset{(1)}{I_1} + (k+n-2)_{n-1} \overset{(1)}{I_2} + \ldots + (k)_{n-1}\, \overset{(1)}{I_{-1}} \end{aligned}\right\}, \ldots (1)$$

which enable us to calculate the number of the parts of different kinds, into which a block of $(2k)^n$ or $(2k+1)^n$ measure-polytopes $\overset{(2)}{M_n}$ can be cut up.

As an example, which gives something to calculate, we consider the case of the middle section perpendicular to the diagonal of a block of 10^{10} measure-polytopes $\overset{(2)}{M_{10}}$. We then find in connection with the relations

$$\frac{T_1}{1}=\frac{T_2}{502}=\frac{T_3}{14608}=\frac{T_4}{88234}=\frac{T_5}{156190}=\frac{R}{9\,!},$$

where R represents the rhombotope that is the sum of the nine forms

$$T_1\,,\quad T_2\,.\quad T_3\,,\quad T_4\,,\quad T_5\,,\quad T_{-4}\,,\quad T_{-3}\,,\quad T_{-2}\,,\quad T_{-1}\,,$$

starting from

$$\frac{1}{2}M_{10}^{(20)}=T_5^{(20)}=I_1^{(100)}-10\,I_1^{(80)}+45\,I_1^{(60)}-120\,I_1^{(40)}+210\,I_1^{(20)},$$

by applying

$$I_1^{(20k)}=(10\,k+8)_9\,T_1^{(2)}+(10\,k+7)_9\,T_2^{(2)}+\ldots.+(10\,k)_9\,T_{-1}^{(2)}$$

for $k=5,4,3,2,1$ after some calculation the result

$$394713550\,(T_1^{(2)}+T_{-1}^{(2)})+410820025\,(T_2^{(2)}+T_{-2}^{(2)})$$

$$+\,422709100\,(T_3^{(2)}+T_{-3}^{(2)})+430000450\,(T_4^{(2)}+T_{-4}^{(2)})$$

$$+\,432457640\,T_5^{(2)},$$

which after substitution of the relations given above leads back to the identity

$$T_5^{(20)}=10^9\,T_5^{(2)}.$$

Physiology. — *"The electric response of the eye to stimulation by light at various intensities".* By W. EINTHOVEN and W. A. JOLLY. (Communication from the Physiological Laboratory of Leiden).

Although the electrical response of the eye to stimulation by light, which was discovered by HOLMGREN has since been studied by numerous observers, there has not so far been undertaken a systematic investigation of the electromotive changes which are caused by stimuli of very varying strength. Such an investigation, however, can as we hope to show, contribute not a little to our comprehension of the retinal processes.

We have in our work employed exclusively isolated frogs' eyes. We have been enabled on the one hand by means of the string galvanometer, which for the retinal currents may be regarded as the most sensitive instrument available, to record and measure very weak electromotive forces, such as are evoked by light of extremely low intensity; on the other hand we have tried by a suitable system of lenses to concentrate light of as great intensity as possible