*Citation:*

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this invariability we may derive from the observations the subsider of the A.P. in the tidal station with regard to the A.P. derived frc the marks in the 5 sluices, amounting to 165—80 = 85 mm. betwe 1700 and 1860.

The method by which the height of the water in the tidal stati was obtained and the possible causes of the subsidence of the zero on t rod added to the invariability of the 5 grooves in the sluices and hence a fairly large part of the ground of Amsterdam with regard the sea, render the idea very probable that this subsidence has a pu local character and that we are not entitled to derive any results wi regard to the subsidence of a larger part of the ground of Amsterdai

It has often been asked what the Amsterdam zero represen Our colleague Dr. VAN DIESEN has devoted to this subject a interesting study in which he has gathered from old documen everything which may help us to find how this zero has bee established. With certainty nothing can be derived from it. B the observations show: 1 that in 1700 the A.P. was 165 mm. ± 12 mm. above the mean sea level in the Y, 2 that the height of tl mean high water was $\frac{318}{2} = 159$ mm. ± 1 mm. above the same mea sea level, and we conclude thence that both in 1700 and 1860 th A.P. within the limits of the errors of observation agreed with th mean high water in the Y.

**Astronomy.** — "*On the masses and elements of Jupiter's satellite and the mass of the system* (continued), by Dr. W. DE SITTER (Communicated by Prof. J. C. KAPTEYN).

III. *The great inequalities.*

The values of these, derived from the heliometer-observations of 1891, 1901 and 1902, have been collected in Table III, togethe

TABLE III. GREAT INEQUALITIES.

| Authority | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| 1891 | 0°509 ± 0°018 | 1°021 ± 0°013 | 0°059 ± 0°007 |
| 1901 | 0·481 ± 47 | 1·089 ± 30 | 0·049 ± 20 |
| 1902 | 0 372 ± 34 | 1·171 ± 19 | 0·034 ± 12 |
| DAMOISEAU | 0·455 | 1·074 | 0·073 |
| SOUILLART's theory | 0·432 | 1·026 | 0·063 |
| Masses (C) | 0 430 ± ·020 | 0·988 ± ·017 | 0 064 ± 003 |

with their probable errors. The photographic determination of 1902 has been rejected for the reason which has already been explained.

From these values of $x_i$ have been derived the equations of condition, which will be given below.

The arguments of these inequalities are $l_i + v$, where

$$v = l_2 - 2l_3 = l_1 - 2l_2 + 180°.$$

Their periods are thus nearly the same as those of the equations of the centre, and in a short series of observations, such as those used here, the great inequalities are not well separated from the equations of the centre. This is the reason of the bad agreement of the results from the three series of observations.

In the eclipses the period of the great inequalities is the same for the three satellites, viz: 438 days [1]). The periods of the equations of the centre in the eclipses have between 10 and 19 times this length, and the two classes of unknowns are thus well separable by eclipse observations. Here however, there arises a new complication, which did not exist in the case of extra-eclipse observations. The periods of the inequalities of group II, which are between 406 and 486 days, are nearly the same as the period of the great inequalities, and therefore the reliability of the determination of $x_i$ from eclipse observations will depend in a large measure on the accuracy of our knowledge of the inequalities of group II. Thus *e. g.* with the masses $(C)$ the coefficient of the inequality in the longitude of satellite II, which has a period of 463 days, is 0°.038. This inequality is entirely neglected by DAMOISEAU (being proportional to $e_2$), and it is probable that his value of $x_2$ — which, according to the introduction to his tables, was derived directly from the observations — will be more or less affected by this circumstance. The same thing is true in a somewhat lesser degree of the corresponding terms in the longitudes of I and III.

The uncertainty which still reigns supreme with regard to the values of the great inequalities, is disappointing. We may hope that the reduction of the photometric eclipse observations of the Harvard observatory will contribute to diminishing this uncertainty.

## IV. *The Libration.*

The mean longitudes $l_1$, $l_2$, $l_3$, have been derived from the observations of 1891 (GILL, heliometer), 1892—93, 1893—94, 1894—95, 1895—96, 1897, 1898 (Helsingfors and Pulkowa, plates), 1901, 1902

[1]) See LAPLACE. *Mécanique Céleste*, Tome IV, Livre VIII, Chapitre II.

(COOKSON, heliometer) and 1904 (Cape, plates). The reduction has been carried out in Gron. Publ. 17. The masses (A) are the result of this discussion. The period of the libration being independent of $\varkappa'$, it is the same for the masses (B) as for (A). Also the transition from (B) to (C) does not affect this period. It is thus only necessary to investigate in how far the change from (A) to (C) affects the inequalities of group II, and what is the effect of this on the libration. This effect was found to be so small that a new determination of the libration appeared superfluous. The finally adopted libration is thus the same as in Gron. Publ. 17, viz:

$$\vartheta = 0^{\circ}{\cdot}158 \, sin \frac{T - 1895.09}{7.00},$$

where the time T is expressed in years.

The probable error of the period corresponding to the adopted probable errors of the masses ($C$) is $\pm$ 0.13.

The corrections to the mean longitudes on 1900 Jan. 0.0 also have been adopted unaltered from Gron. Publ. 17.

Table IV contains the observed corrections to the mean longitudes, with their probable errors as derived directly from the observations, and the residuals remaining after subtitution of the final values of the inequalities of group II and the libration. The last two columns contain the p.e. of the quantity $\Delta l_1 - 3\,\Delta l_2 + 2\,\Delta l_3$, and the residuals for this same quantity.

In determining the libration from extra-eclipse observations we find the mean longitudes for epochs, which approximately co-incide with the epoch of opposition, and which therefore are on the average separated by intervals of 400 days. This interval differs but little from the periods of the inequalities of group II. These latter thus present themselves as inequalities with apparent periods between 6 and 8 years, and are therefore not well separable from the libration. In the eclipses this difficulty does not exist.

The method of successive approximations, which has been used in Gron. Publ. 17, to derive from the observations the most probable values of the libration and of the inequalities of group II, need not be explained here. It must suffice to refer the reader to that publication (see also these Proceedings, June 1907). The residuals of Table IV are practically the same as those found in Gron. Publ. 17, and they also need not be considered in detail here. Those of the satellites I and III are not very satisfactory, as has been pointed out there. On this point also the results derived from extra-

TABLE IV. MEAN LONGITUDES AND LIBRATION.

| Series | $\Delta l_1$ | | | $\Delta l_2$ | | | $\Delta l_3$ | | | $\vartheta$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Observed correction | p. e. | Residual | Observed correction | p. e | Residual | Observed correction | p. e. | Residual | p. e. | Residual |
| 1891 | + 0°·100 | ± °·006 | — °·034 | + 0°·065 | ± °·003 | — °·007 | — 0°·031 | ± °·002 | — °·013 | ± °·012 | — °·039 |
| 1892—3 | + ·073 | ± 8 | — 22 | + ·051 | ± 5 | — 17 | — 023 | ± 3 | — 4 | ± 14 | + 21 |
| '93—4 | + ·128 | ± 14 | + 31 | + ·019 | ± 9 | — 26 | — ·034 | ± 5 | — 16 | ± 34 | + 76 |
| '94—5 | + ·131 | ± 11 | + 11 | — ·012 | ± 6 | + 9 | — ·029 | ± 3 | — 17 | ± 23 | — 49 |
| '95—6 | + ·152 | ± 6 | — 8 | — 026 | ± 4 | 0 | — 004 | ± 2 | + 13 | ± 14 | + 18 |
| '97 | + ·112 | ± 11 | — 64 | + ·019 | ± 9 | + 6 | — ·014 | ± 4 | + 9 | ± 30 | — 59 |
| '98 | + ·163 | ± 10 | + 2 | + ·120 | ± 5 | + 21 | — 002 | ± 3 | + 26 | ± 19 | — 10 |
| 1901 | + ·136 | ± 9 | + 23 | + 020 | ± 6 | — 1 | — ·037 | ± 4 | — 26 | ± 23 | — 24 |
| '02 | + ·134 | ± 7 | — 9 | — ·025 | ± 4 | + 2 | — ·023 | ± 3 | — 7 | ± 17 | — 29 |
| '04 | + 231 | ± 12 | + 64 | + ·063 | ± 7 | + 15 | — ·006 | ± 4 | + 26 | ± 27 | + 71 |

eclipse observations need confirmation from observations of eclipses.[1])

V. *Mean longitudes and mean motions.*

The corrections to the mean longitudes on 1900 Jan. 0.0 of the three inner satellites have been determined together with the libration, and the residuals have already been given in Table IV. For the fourth satellite the adopted correction is — 0°.030, and the residuals are given in Table V.

TABLE V $\Delta l_4$.

| Epoch | Observed correction | p. e. | Residual |
|---|---|---|---|
| 1891 | — 0°0248 | ± °0010 | + °0035 |
| 1901 | — ·0361 | ± 18 | — 58 |
| 1902 | — 0342 | ± 16 | — 37 |

If the corrections are added to the values adopted in computing the tabular places, and then referred to the first point of Aries by adding the adopted longitude of the point $O$, we find for 1900 Jan. 0, mean Greenwich noon, the values which are given below, sub I.

In the introduction to his tables DAMOISEAU states the mean longitudes for 1750 Jan. 0.5, mean time of Paris. If we consider these as being derived directly from the observations, they require a small correction, since DAMOISEAU has used the value 493ˢ.2 of the light-time, while in the reduction of the modern observations the value 498ˢ.46 was adopted. If DAMOISEAU had adopted this latter value, he would have found the same longitudes for an epoch which is 5ˢ.26 × Δ earlier, Δ being the mean distance of Jupiter. The observed mean longitudes, in order to correspond correctly to the tabular epoch, therefore require the correction [2]) :

$$+ \frac{5.26}{86400} \cdot \Delta \cdot n_i = + 0.000317\ n_i$$

1) It has also been pointed out in Gron. Publ. 17 that the series of extra-eclipse observations from which the libration was derived, not being made for this special purpose, does not in every respect fulfil the conditions necessary for a good determination of the libration.

2) In Gron. Publ. 17 I assumed, on the authority of COOKSON, Cape XII. 3, page 56, that MARTH's longitudes for 1750.0 were identical with DAMOISEAU's. This, however, they are not, MARTH having applied the correction for the change in the adopted constant of aberration with the wrong sign This was pointed out to me by Mr. BANACHIEWICZ.

Applying this correction, and carrying the longitudes forward to 1900 Jan. 0.0, Greenwich M. T., we find the values II below.

*Mean longitudes for* 1900 *Jan.* 0.0.

| | *I* (*modern*) | *II* (DAMOISEAU) |
|---|---|---|
| $l_1 =$ | 142°.604 ± 0°.010 | 142°.645 ± °.004 |
| $l_2 =$ | 99 .534 ± .007 | 99 .569 ± .006 |
| $l_3 =$ | 167 .999 ± .007 | 168 .028 ± .008 |
| $l_4 =$ | 234 .372 ± .002⁵ | 234 .360 ± .010. |

The estimated probable errors for DAMOISEAU do *not* contain the p. e. of the mean motions used for carrying the longitudes forward from 1750 to 1900. The uncertainty of DAMAUSEAU's mean motions has been estimated by the late Prof. OUDEMANS in these Proceedings (October 1906). He finds for the four mean motions, in units of the eighth decimal place:

± 73 ± 55 ± 37 ± 24

Comparing the values I and II we find the following corrections to DAMOISEAU's mean motions:

$$\delta n_1 = -0°.0000\,0075 \pm °.0000\,0020$$
$$\delta n_2 = -0\ .0000\,0064 \pm \quad 16^5$$
$$\delta n_3 = -0\ .0000\,0053 \pm \quad 20$$
$$\delta n_4 = +0\ .0000\,0022 \pm \quad 18$$

It is noticeable that these corrections are very nearly of the magnitude of the uncertainties estimated by OUDEMANS. If these corrections are applied, the resulting values do not satisfy the condition

$$n_1 - 3n_2 + 2n_3 = 0.$$

If, however, we apply the further corrections

$$\delta n_1 = -2 \qquad \delta n_2 = +3 \qquad \delta n_3 = -3$$

to the eighth decimal place, then the condition is rigorously satisfied. The mean motions thus derived are those finally adopted. They are

$$n_1 = 203°.4889\,9261 \qquad n_3 = 50°.3176\,4587$$
$$n_2 = 101\ .3747\,6145 \qquad n_4 = 21\ .5711\,0965$$

These are the mean motions relatively to the point Aries. If the sidereal mean motions are required, they must be diminished by 0°.0000 3822.

*VI. The mass of the system.*

The determination of the mass of the system of Jupiter by NEWCOMB [1]),

[1]) Astronomical papers of the American Ephemeris, Vol. 5, Part. 5.

which has now become a classic in astronomy, was based on observations of satellites, on perturbations in the motion of comets, and of the planets Themis, Polyhymnia and Saturn. It seems to me advisable to retain of these only the determinations from the three planets. Of the older observations of the satellites the uncertainty of the scale-value (which is increased threefold in the mass of the planet) is such that their weight, compared with the modern observations, and with the determinations from the perturbations of planets, is absolutely negligible. NEWCOMB has also, for this same reason, assigned a very small weight to these observations of the satellites.

The use of observations of comets seems to me very dangerous. It is very uncertain, if not improbable, that the observed centre of light should retain the same relative position with respect to the centre of gravity throughout one apparition of the comet, and *a fortiori* in different apparitions. NEWCOMB also points out that the results based on observations of comets are unreliable for this reason. Nevertheless he assigns a large weight to the determination by VON HAERDTL from WINNECKE's comet, on the ground that the normal places of this comet are so well represented by VON HAERDTL's results. It appears to me that this good representation does not diminish the stringency of the argument stated above, and in my opinion it is advisable to reject also this determination, together with those from other comets.

There remain the determinations from the three planets, which I adopt with the same weights assigned to them by NEWCOMB, and the modern observations of satellites, which were only made, or at least reduced, after NEWCOMB's discussion was published. For these latter the scale-value is determined in an entirely satisfactory manner by simultaneous observations of standard stars. Nevertheless I have assigned to these observations a relatively smaller weight than to the determinations from the planets, to allow for the possibility of small systematic errors in transferring the scale-value from the distance of the standard stars to the mutual distances of the satellites.

In my reduction of GILL's observations of 1891 I have included in the probable error of $\mathfrak{M}$ the effect of the uncertainty of the standard stars used for the determination of the scale-value. The probable errors stated by COOKSON do not include this uncertainty. The distances of the stars used by COOKSON are not so accurately known as of the stars used in 1891. I have for these reasons assigned a smaller weight to COOKSON's two determinations than to GILL's. The several determinations and their probable errors and adopted weights are given in Table VI.

TABLE VI. RECIPROCAL OF THE MASS OF THE SYSTEM.

| Authority | Observed values | Weight | Residual |
|---|---|---|---|
| KRÜGER, perturbations of Themis | 1047·54 ± 0 19 | 5 | + 0·14 |
| HILL, „ „ Saturn | 38 ± ·12 | 7 | — ·02 |
| NEWCOMB, „ „ Polyhymnia | 34 ± 06 | 20 | — ·06 |
| GILL—DE SITTER, Satellites, 1891 | ·50 ± ·06 | 10 | + ·10 |
| COOKSON, „ 1901 | ·46 ± 09 | 4 | + ·06 |
| COOKSON, „ 1902 | ·25 ± ·06 | 6 | — 15 |

The mean by weights is 1047.394 ± .026. The simple mean is 1047.412. The mean of the determinations from the planets alone is 1047.380, and the mean of the determinations from the satellites is 1047.417. The value which I propose to adopt is

$$\mathfrak{M} = \mathbf{1047.40 \pm 0.03}.$$

The probable error was derived from the residuals. The distribution of these residuals, each compared with its own probable error as stated by the observers, is in excellent agreement with the theoretical distribution according to the law of errors. The adopted p.e. can therefore be considered to be a trustworthy measure of the real accuracy

I may be allowed to state as my conviction that it will not be possible in the near future materially to improve the value here adopted. In order to attain from observations of satellites a smaller probable error than ± 0.03, or $^1/_{35000}$, the scale-value must be known within less than $^1/_{130000}$. It thus appears useless to attempt a new determination of the mass from observations of the satellites, until we are in the possession of means as well of fixing the distance of a pair of standard-stars with this accuracy, as of transferring the scale-value determined therefrom to other (smaller) distances without the possibility of systematic errors. Investigations of modern heliometers point to the conclusion that the transferring of the scale-value from a distance of, say, 7000″ to one of 700″ is still subject to uncertainties, which may reach an amount equivalent to an error of 0″.1 in the larger distance, and which therefore may amount to $^1/_{70000}$ of the scale-value. On the other hand it seems a high demand on our present observational means to fix a distance of about 2° of two stars with an uncertainty smaller than 0″.07 = 0ˢ.005 [1]).

1) The accuracy of the distance of the standard stars used in 1891 was ± $^1/_{60000}$. (See my dissertation, page 8).

Newcomb has already pointed out that oppositions of Polyhymnia, as favourable as the one used in his work, will not recur till the end of the twentieth century, and a similar statement is true for Themis. Hill has pointed out [1] that Jupiter produces in the motion of certain minor planets (those of the Hecuba type) perturbations of long periods, which amount to several degrees. Thus e. g. Freia is subject to a perturbation, whose geocentric amplitude is 12°.7 with a period of 121 years. The length of the period makes it impossible to derive an improved value of the mass by this method in the near future.

*Derivation of the final masses.*

The right-hand-members of the equations of condition, which have served to determine the corrections to the values ($B$) of the masses, have been derived, as explained above under I to IV, from:

I. the motions of the nodes $\theta_2$ and $\theta_3$ (those of $\theta_1$ and $\theta_4$ I leave out of consideration, as having too small weights),

II. the motion of the perijove $\tilde{\omega}_4$,

III. the great inequalities $x_1$, $x_2$, $x_3$,

IV. the period of the libration.

The equations are:

I.

$$\begin{array}{llllll} -.0266\,\delta\nu' & -.0030\,\delta\nu_1 & -.0001\,\delta\nu_2 & -\,0040\,\delta\nu_3 & -.0002\,\nu_4 & =-\,00010 \pm .00008 \\ -.0051 & -.0003 & -.0007 & 0 & -.0007 & =-.00041 \pm \quad 15 \end{array}$$

II.

$$+.00077\,\delta\nu' +.00004\,\delta\nu_1 +.00007\,\delta\nu_2 +.00082\,\delta\nu_3 -.00005\,\nu_4 = -.000036 \pm .000020$$

These three equations depend in part on the values of the elements in 1750, which were determined from eclipse-observations. It has already been pointed out above that practically the same results would be found from extra-eclipse observations alone.

III.

$$\begin{array}{llllllll} & & & 1891 & 1901 & 1902 & \text{Adopted} \\ -.003\,\delta\nu_1 & +\,403\,\delta\nu_2 & -.014\,\delta\nu_3 & =+\,080 & +\,051 & -.058 & +\,020 \pm 040 \\ +\,195 & -\,008 & +.816 & =+.019 & +\,087 & +\,169 & +\,050 \pm \;\;40 \\ -.001 & +.060 & -.006 & =-.004 & -\,014 & -.020 & -.009 \pm \;\;10 \end{array}$$

The probable errors of the separate determinations have been given in Table III. The p.e. of the adopted values were estimated according to the agreement of the separate values.

IV.

$$+2.40^5\,\delta\nu_1 +0.24^5\,\delta\nu_2 +1.35^5\,\delta\nu_3 = 0.000 \pm 0.18$$

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[1]) Collected works I, page 105.

If now we reduce all these equations to the same weight, so that the p. e. of their right-hand members becomes ± 0.10, we find, if also the signs of I are reversed:

| | | | | | | | *Res* | *Res.* Souill. |
|---|---|---|---|---|---|---|---|---|
| I. | $+33.3\delta\varkappa'$ | $+3.8\delta v_1$ | $+0.1\delta v_2$ | $+5.0\delta v_3$ | $+0.2 v_4$ | $= +0.12^5$ | +.02 | —.78 |
| | + 3.4 | +0.2 | $+0.4^5$ | 0 | +0 5 | = +0.27 | +.25 | +.16 |
| II. | $+ 3.8^5$ | +0.2 | $+0.3^5$ | +4.1 | $-0.2^5$ | = −0.18 | —.12 | —.82 |
| III. | | 0 | +0.6 | 0 | | = +0.05 | +.05 | $+.05^5$ |
| | | +0.5 | 0 | $+2.0^5$ | | = +0.12 | + 16 | $+.06^5$ |
| | | 0 | $+0.1^5$ | 0 | | = −0.09 | —.09 | —.09 |
| IV. | | +1.4 | $+0.1^5$ | +0.8 | | = 0.00 | .00 | +. 7 |

The finally adopted corrections are.

$\delta\varkappa' = + 0.005 \pm .0075$

$\delta v_1 = + 0.010 \pm .030$ $\qquad \delta v_2 = 0 \pm .050$

$\delta v_3 = - 0.020 \pm .020$ $\qquad v_4 = 0 \pm 0.25$

The corresponding values of the masses are.

$Jb^2 = 0.0214\,180 \pm .0001543$ $\qquad (b = 1$ for $d = 39''.0)$

$= 0.0000\,0000\,518169 \pm 3975$ $\qquad$ (astronomical units)

$m_1 = 0.0000\,260 \pm .0000\,012$

$m_2 = 0.0000\,231 \pm \quad 11$ $\qquad (C)$

$m_3 = 0\,0000\,804 \pm \quad 16$

$m_4 = 0.0000\,424\,751 \pm .0000\,106$

Substituting these corrections, there remain the residuals stated above. If Souillart's masses are substituted there remain the residuals given in the last column.

The equations II and III are contradicting each other II demands a negative value $\delta v_3$, III a positive value. On account of the bad agreement of the different determinations of $x_3$ I have assigned a very small weight to the equation III. It is to be noticed that the large negative correction $\delta v_3$ could have been partly avoided by assuming a large positive value of $v_4$, e. g. $v_4 = + 0.5$. Even then, however, it would not be possible to bring about a satisfactory agreement of II and III without spoiling the representation of I and IV.

The probable errors stated for the corrections $\delta\varkappa'$ and $\delta v_3$ as well as the values of these corrections themselves, depend largely on judgment[1]). In estimating the probable errors I have taken into account as accurately as I could the imperfections as well of the theory on which the left-hand members of the equations of condition depend as of the observations from which the right-hand members are derived. It has been my aim to estimate true *probable* errors, i. e. the masses $(C)$ are those which with our present knowledge of the system I consider the most probable, and I consider it equally probable that the deviation of the values $(C)$ from the truth is smaller than the stated p. e., as that it exceeds this quantity.

The above contains all that can be derived from modern extra-eclipse observations. The resulting values of the inclinations and nodes, and of the mass of the system, i. e. the groups A and C of unknowns, must be considered as final, so far as the observational data at present available go. The results for the other unknowns (those of group B) cannot be accepted as final until they are confirmed by the reduction of the photometric eclipse observations of the Harvard observatory. With regard to the inclinations and nodes, I have already pointed out in Cape XII. 3 (page 121) that a new determination about the year 1920 is desirable. For the determination of $m_4$ it will be necessary, as was pointed out by me in my dissertation, p. 82 and 85, to supplement the modern observations by a determination of $k_3$ and $k_2$ about 1790 from a re-reduction of old eclipses. Of these an amply sufficient number exists. Between the years 1772 and 1799 I have found in the literature of the epoch records of 63 eclipses of which the immersion and emersion have been observed by the same person, and about one third of these have been observed by more than one observer.

In order to derive entirely satisfactory results it will also be necessary to revise SOUILLART's analytical theory, as pointed out by me in Gron. Publ. 17, page 118.

The masses and elements derived in the above, though not to be considered as final, still doubtlessly are much nearer to the truth than those used in SOUILLART's theory. It therefore seemed desirable to introduce them into the expressions for the latitudes, longitudes and radii-vectores as given by that theory. To take account of the uncertainties of the masses I give the coefficients as functions of the small quantities $\varrho$ and $\lambda_i$, which are defined by

1) "The probable error arising from the uncertainty of such judgments must be included among the possible unavoidable sources of error." NEWCOMB, Astronomical Papers of the American Ephemeris, Vol. 5, Part 4, page 398.
[Note added in the English translation].

$$Jb^2 = (Jb^2)_0 (1 + \varrho)$$
$$m_i = (m_i)_0 (1 + \lambda_i),$$

where $(Jb^2)_0$ and $(m_i)_0$ represent the values ($C$). The squares and products of $\varrho$, $\lambda_1$, $\lambda_2$ and $\lambda_3$ will be neglected. These developments are based entirely on those of Gron. Publ. 17, and what was there said about their accuracy and reliability also applies here.

The semi-major-axes corresponding to the adopted mean motions and the adopted mass of the system have been computed by the formula [1]:

$$n_i^2 a_i^3 = f \frac{1 + m_i}{\mathfrak{M}(1 + \Sigma m_i)} \left(1 + \frac{Jb^2}{a_i^2}\right).$$

Their logarithms are

$$log\ a_1 = 7.450\ 1443 + 0\ 000\ 101\ \varrho$$
$$log\ a_2 = 7.651\ 8277 + \ \ 000\ 040\ \varrho$$
$$log\ a_3 = 7.854\ 6197 + \ \ 000\ 016\ \varrho$$
$$log\ a_4 = 8.099\ 8338 + \ .000\ 005\ \varrho$$

The values of the coefficients $\tau_{ij}$, which occur in the expressions for the equations of the centre, are

$$\tau_{21} = +0.0280 - 031\ \varrho + .027\ \lambda_1 - 002\ \lambda_2 + .055\ \lambda_3$$
$$\tau_{31} = -0.0053 - .003\ \varrho - .005\ \lambda_1 - .004\ \lambda_2 - .001\ \lambda_3$$
$$\tau_{41} = \ \ 0.0000$$

$$\tau_{12} = -0.0320 + .058\ \varrho + .027\ \lambda_1 - .011\ \lambda_2 - .061\ \lambda_3$$
$$\tau_{32} = -0.0447 + .022\ \varrho + .003\ \lambda_1 - .042\ \lambda_2 + .006\ \lambda_3$$
$$\tau_{42} = \ \ 0.0000$$

$$\tau_{13} = +0.0171 - .013\ \varrho + .002\ \lambda_1 + .014\ \lambda_2 + .015\ \lambda_3$$
$$\tau_{23} = +0.1619 - .098\ \varrho - .005\ \lambda_1 + .019\ \lambda_2 + .116\ \lambda_3 + .0019\ \lambda_4$$
$$\tau_{43} = -0.1173 + 112\ \varrho + .006\ \lambda_1 + 024\ \lambda_2 - .142\ \lambda_3 + .0163\ \lambda_4$$

$$\tau_{14} = +0.0016 - .002\ \varrho \qquad + .001\ \lambda_2 + .001\ \lambda_3 + .0014\ \lambda_4$$
$$\tau_{24} = +0.0139 - .018\ \varrho - .001\ \lambda_1 - .001\ \lambda_2 + .010\ \lambda_3 + .0112\ \lambda_4$$
$$\tau_{34} = +0.0828 - 072\ \varrho - .001\ \lambda_1 - .017\ \lambda_2 + .009\ \lambda_3 + .0726\ \lambda_4$$

The daily motions of the own perijoves (referred to the first point of Aries) are:

| | | $\varrho$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|---|---|---|---|---|---|---|
| $(\tilde{\omega}_1)$ | $+0^\circ.14703$ | $+^\circ.1295$ | $+^\circ.0070$ | $+^\circ.0166$ | $+^\circ.0007$ | $+^\circ.0001$ |
| $(\tilde{\omega}_2)$ | $+0.038955$ | $+.02590$ | $-00371$ | $+.00406$ | $+.01974$ | $+.00019$ |
| $(\tilde{\omega}_3)$ | $+0.007032$ | $+.00530$ | $+.00024$ | $+00100$ | | $+.00066$ |
| $(\tilde{\omega}_4)$ | $+0.001896$ | $+.00075$ | $+.00003$ | $+.00007$ | $+.00082$ | $-.00005$ |

[1]) It will be seen that I adopt here LAPLACE's definition of the mean distances. All other constant terms of the radius-vector will be included in $\rho_i = r_i/a_i$. These ratios $\rho_i$ must not be confounded with the small quantity $\varrho$ representing a possible correction to the adopted value of $Jb^2$.

The great inequalities are:

$$x_1 = 0^\circ.4303 - .0024\,\lambda_1 + .4228\,\lambda_2 - .0145\,\lambda_3$$
$$x_2 = 0.9875 + .1273 \quad - .0090 \quad + .8188$$
$$x_3 = 0.0636 - .0010 \quad + .0629 \quad - .0063$$

The coefficients of the inequalities of group II are:

$$\varkappa_{11} = \{-2.49 \quad - .04\ \varrho + .04\ \lambda_1 - 2.46\,\lambda_2 + .17\,\lambda_3\}\,e_1$$
$$\varkappa_{12} = \{+0.98 \quad - .19\ \varrho - .13\ \lambda_1 + .97\,\lambda_2 - .10\,\lambda_3\}\,e_2$$
$$\varkappa_{13} = \{+0.083 \quad - .03\ \varrho - .02\ \lambda_1 + .02\,\lambda_2\}\,e_3$$
$$\varkappa_{14} = \{+0.0062 - .003\,\varrho - .002\,\lambda_4\lambda_2 - .005\,\lambda_3 + .002\,\lambda_4\}\,e_4$$

$$\varkappa_{21} = \{+2.26 \quad - .05\ \varrho + 2.20\lambda_1 - .03\lambda_2 + .03\lambda_3\}\,e_1$$
$$\varkappa_{22} = \{+2.19 \quad + .16\ \varrho - .74\lambda_1 + .08\lambda_2 + 2.93\lambda_3\}\,e_2$$
$$\varkappa_{23} = \{-0.535 \quad - .27\ \varrho - .03\lambda_1 + .15\lambda_2 - .02\lambda_3\}\,e_3$$
$$\varkappa_{24} = \{-0.0368 + .005\varrho - (.002 - .017\lambda_4)\lambda_1 + .022\lambda_2 + .045\lambda_4\lambda_3 - .046\lambda_4\}\,e_4$$

$$\varkappa_{31} = \{-0.01 - .01\,\lambda_1\}\,e_1$$
$$\varkappa_{32} = \{-0.67 - .65\,\lambda_2\}\,e_2$$
$$\varkappa_{33} = \{+0.109 + .07\varrho - .01\lambda_1 + .07\lambda_2\}\,e_3$$
$$\varkappa_{34} = \{+0.0078 + (.002 + .009\lambda_4)\lambda_2 + .011\lambda_4\}\,e_4$$

The quantities determining the libration are:

$$Q_1 = +\{.003440 - .00022\lambda_1 - .00050\lambda_2 - .00142\lambda_3\}(1+\lambda_2)(1+\lambda_3)$$
$$Q_2 = -\{.005161 - .00021\lambda_1 - .00045\lambda_2 - .00135\lambda_3\}(1+\lambda_1)(1+\lambda_3)$$
$$Q_3 = +\{.000452 - .00002\lambda_3\}(1+\lambda_1)(1+\lambda_2)$$
$$\beta^2 = Q_1 - 3Q_2 + 2Q_3 \qquad \psi = \beta(t - t_0)$$
$$\vartheta = 0^\circ.158 \sin\psi$$
$$\vartheta_1 = +0.1735\,\vartheta \quad \vartheta_2 = -0.2603\,\vartheta \quad \vartheta_3 = +0.0228\,\vartheta$$

The position of the orbital planes of the satellites is in SOUILLART's theory referred to the orbit of Jupiter, of which the inclination and node[1]) referred to the ecliptic and mean equinox are (according to LEVERRIER, but with NEWCOMB's precession):

$$\varphi = 1^\circ\,18'\,31''.1 - 0''.2051\,\mathrm{T}$$
$$\vartheta = 99\ \ 26\ \ 36 \quad + 36.396\,\mathrm{T},$$

where T is the time counted in tropical years from 1900 Jan. 0.0 Greenwich M. T.

It is preferable, however, to refer the latitudes of the satellites to the mean equator of the planet. The inclination and node of this mean equator referred to the orbit (the node being counted "in the orbit") are:

$$\omega = 3^\circ\ \ 6'\,55''.1 + 0''.0243\,\mathrm{T}$$
$$\theta = 315\ \ 48\ \ 0 \quad + 50.158\ \mathrm{T}$$

The inclination and node of the mean equator referred to the ecliptic are thus:

[1]) Unless otherwise stated, node stands for ascending node.

$$= \quad 2^\circ 12' \ 8''7 + 0''.4231\ T$$
$$\mathfrak{N} = 336\ 24\ 24 \quad + 48\ .916\ \ T$$

The inclination and node of Jupiter's orbit referred to the mean equator are therefore (the node being counted "in the equator"):

$$\omega = \quad 3^\circ\ 6'\,55''.1 + \ 0''.0243\ T$$
$$\theta' = 135\ \ 46\ 44 \quad + 50\ .155\ \ T$$

The position of the orbital planes of the satellites — excluding periodic, but including secular perturbations — referred to the mean equator, are given by the formulas:

$$i_i \sin(\theta' - \Omega_i) = p_i = \Sigma_j \sigma_{ij} \gamma_j \sin \Gamma_j$$
$$i_i \cos(\theta' - \Omega_i) = q_i = \Sigma_j \sigma_{ij} \gamma_j \cos \Gamma_j + (1 - \mu_i)\,\omega$$

Referred to the orbit of Jupiter they are[1])

$$I_i \sin N_i = \Sigma_j \sigma_{ij} \gamma_j \sin \theta_j + \mu_i \omega \sin \theta$$
$$I_i \cos N_i = \Sigma_j \sigma_{ij} \gamma_j \cos \theta_j + \mu_i \omega \cos \theta$$

where we have [2])

$$\Gamma_i = 180^\circ + \theta - \theta_i$$

If the periodic perturbations are represented by $\delta p_i$, $\delta q_i$, $\delta s_i$, we have for the latitude of the satellite referred to the mean equator

$$\beta_i = (q_i + \delta q_i) \sin(v_i - \theta') + (p_i + \delta p_i) \cos(v_i - \theta')$$

and referred to the orbital plane of Jupiter

$$s_i = I_i \sin(v_i - N_i) + \delta s_i .$$

Here $v_i$ is the true orbit-longitude of the satellite. In both formulas quantities of the third order in the inclinations are neglected. The neglected terms in $\beta_i$ are thus of the order of magnitude of 0°.00002 and in $s_i$ of the order of 0°.01.

The values of the coefficients $\sigma_{ij}$ and $\mu_i$ are:

$\sigma_{21} = -\,0.019 + .012\,\varrho - .019\,\lambda_1$

$\sigma_{31} = -\,0.001 + .001\,\varrho - .001\,\lambda_1$

$\sigma_{41} = \quad 0.000$

$\sigma_{12} = +\,0.0203 - .020\,\varrho + .020\,\lambda_2$

$\sigma_{32} = -\,0.0347 + .028\,\varrho + .002\,\lambda_1 - .035\,\lambda_2 + .005\,\lambda_3 - .0005\,\lambda_4$

$\sigma_{42} = -\,0.0010 - .001\,\varrho - .001\,\lambda_2 + .001\,\lambda_3$

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[1]) Rigorously these formulas are true with reference to the *fixed* orbit of Jupiter, and a correction must be applied to derive the latitude referred to the moving orbit. It is, however, sufficiently accurate to use the same formulas for the latitude referred to the moving orbit, provided we take for $\omega$ and $\theta$ the inclination and node of the mean equator referred to this same moving orbit (as was done here). For the motion of the node $\theta$ referred to the moving orbit I adopted — 0''.0979 instead of — 0''.0710 (Souillart II page 166). This in the value which results if Souillart's final value of $b_4$ is used instead of the approximate value used by Souillart himself.

[2]) The meaning of $\Gamma_i$ is thus here slightly different from what it was in the subordinate investigation I.

$$\sigma_{13} = +\,0.0056 - .013\,\varrho + .003\,\lambda_2 + .010\,\lambda_3 - .0001\,\lambda_4$$
$$\sigma_{23} = +\,0.1488 + .132\,\varrho - .011\,\lambda_1 + .005\,\lambda_2 + .125\,\lambda_3 + .0026\,\lambda_4$$
$$\sigma_{43} = -\,0.1772 + .176\,\varrho + .008\,\lambda_1 + .028\,\lambda_2 - .211\,\lambda_3 + .0282\,\lambda_4$$

$$\sigma_{14} = -\,0.0018 - .003\,\varrho + .001\,\lambda_3 + .0018\,\lambda_4$$
$$\sigma_{24} = +\,0.0183 - .034\,\varrho - .002\,\lambda_1 - .002\,\lambda_2 + .017\,\lambda_3 + .0207\,\lambda_4$$
$$\sigma_{34} = +\,0.1203 - .110\,\varrho - .005\,\lambda_1 - .016\,\lambda_2 + .021\,\lambda_3 + .1064\,\lambda_4$$

$$\mu_1 = 0.99944 + .0009\,\varrho - .0002\,\lambda_3 - .0002\,\lambda_4$$
$$\mu_2 = 0.99428 + .0095\,\varrho + .0002\,\lambda_1 + .0001\,\lambda_2 - .0022\,\lambda_3 - .0023\,\lambda_4$$
$$\mu_3 = 0.97271 + .0294\,\varrho + .0012\,\lambda_1 + .0040\,\lambda_2 - .0010\,\lambda_3 - .0088\,\lambda_4$$
$$\mu_4 = 0.86245 + .0555\,\varrho + .0018\,\lambda_1 + .0045\,\lambda_2 + .0503\,\lambda_3 - .0056\,\lambda_4$$

The daily motions of the nodes $\theta_i$ are:

$$(\theta_1)\quad -0^\circ.13614 - .1327\,\rho - 0023\,\lambda_2 - .0010\,\lambda_3 - 00008\,\lambda_4$$
$$(\theta_2)\quad -0\ 032335 - 02602\,\rho - 00198\,\lambda_1 - .00013\,\lambda_2 - .00399\,\lambda_3 - 000191\,\lambda_4$$
$$(\theta_3)\quad -0.006854 - .00493 - 00021\,\lambda_1 - .00071\,\lambda_2 - 00004\,\lambda_3 - 000695\,\lambda_4$$
$$(\theta_4)\quad -0\ 001772 - .00077\,\rho - .00003\,\lambda_1 - 00007\,\lambda_2 - 00075\,\lambda_3 + .000098\,\lambda_4$$

and for the angles $\Gamma_i$ we have:

$$\frac{d\Gamma_i}{dt} = 0^\circ.000038 - \frac{d\theta_i}{dt}.$$

The quantities $p_i$ are thus:

| | | | | |
|---|---|---|---|---|
| $p_1 =$ | $+0\ 02720\ \sin \Gamma_1$ | $+0.00951\ \sin \Gamma_2$ | $+0.00103\ \sin \Gamma_3$ | $-0.00046\ \sin \Gamma_4$ |
| $p_2 =$ | $-\ 00052$ | $+\ .46830$ | $+\ .02734$ | $+\ .00464$ |
| $p_3 =$ | $-\ .00003$ | $-\ .01625$ | $+\ .18390$ | $+\ 03051$ |
| $p_4 =$ | $.00000$ | $-\ 00047$ | $-\ .03259$ | $+\ .25360$ |

In $q_i$ we have the same coefficients, and again in $I_i \sin N_i$ and $I_i \cos N_i$. The constant terms $(1-\mu_i)\,\omega$ of $q_i$ and the coefficients of $\sin\theta$ and $\cos\theta$ in $I_i \sin N_i$ and $I_i \cos N_i$ respectively are:

$$(1-\mu_1)\,\omega = 0^\circ.00174 \qquad \mu_1\,\omega = 3^\circ.1136$$
$$(1-\mu_2)\,\omega = 0.01792 \qquad \mu_2\,\omega = 3.0974$$
$$(1-\mu_3)\,\omega = 0.08502 \qquad \mu_3\,\omega = 3.0303$$
$$(1-\mu_4)\,\omega = 0\ 42851 \qquad \mu_4\,\omega = 2\ 6868$$

The position of the true equator referred to the mean equator is defined by its inclination $\omega_1$ and node $\psi_1$, which are determined by the formulas

$$\omega_1 \sin(\theta' - \psi_1) = \Sigma_j\, \sigma_{oj}\, \gamma_j \sin \Gamma_j$$
$$\omega_1 \cos(\theta' - \psi_1) = \Sigma_j\, \sigma_{oj}\, \gamma_j \cos \Gamma_j.$$

The inclination $\Omega$ and node $\Psi$ of the true equator referred to the orbit of Jupiter are then:

$$\Omega \sin \Psi = \Sigma_j\, \sigma_{oj}\, \gamma_j \sin \theta_j + \omega \sin \theta$$
$$\Omega \cos \Psi = \Sigma_j\, \sigma_{oj}\, \gamma_j \cos \theta_j + \omega \cos \theta,$$

where we have:

$$\sigma_{01} = -0.00097\,(1+\lambda_1) \qquad \sigma_{01}\gamma_1 = -0^\circ.00003$$
$$\sigma_{02} = -0.00094\,(1+\lambda_2) \qquad \sigma_{02}\gamma_2 = -0.00044$$
$$\sigma_{03} = -0.00441\,(1+\lambda_3) \qquad \sigma_{03}\gamma_3 = -0.00081$$
$$\sigma_{04} = -0.00363\,(1+\lambda_4) \qquad \sigma_{04}\gamma_4 = -0.00092$$

Before giving the expressions for the perturbations I will first state the values of the arguments. For brevity I put

$$\tau = l_2 - l_3 \qquad v = l_2 - 2l_3 \qquad \varphi_i = v + \tilde{\omega}_i$$

$L =$ the mean longitude of Jupiter
$M =$ ,, ,, anomaly ,, ,,
$W = 5l^V - 2l^{IV} - 16^\circ 31'$ } in LEVERRIER's notation.
$W_1 = l^{IV} - 2l^V - 1\ 30$ }
$V = 2L - 2\theta' + 180^\circ$
$V' = 2L - \theta.$

The values of the arguments then are, if $t$ is the time counted in days from 1900 Jan. 0, Mean Greenwich Noon (J. D. 2415020):

$$l_1 = 142^\circ.604 + 203^\circ.4889\,9261\,t$$
$$l_2 = 99.534 + 101.3747\,6145\,t$$
$$l_3 = 167.999 + 50.3176\,4587\,t$$
$$l_4 = 234.372 + 21.5711\,0965\,t$$

$$\tau = 291^\circ.535 + 51^\circ.0571166\,t \qquad v = 123^\circ.5 + 0^\circ.73947\,t$$

$$\psi = 252^\circ.4 + 0^\circ.14081\,t$$

$$\tilde{\omega}_1 = 155^\circ.5 + 0^\circ.14703\,t \qquad \varphi_1 = 279^\circ.0 + 0^\circ.88650\,t$$
$$\tilde{\omega}_2 = 62.7 + 0.03896\,t \qquad \varphi_2 = 186.2 + 0.77843\,t$$
$$\tilde{\omega}_3 = 338.3 + 0.00703\,t \qquad \varphi_3 = 101.8 + 0.74650\,t$$
$$\tilde{\omega}_4 = 283.15 + 0.001896\,t \qquad \varphi_4 = 46.7 + 0.74137\,t$$

$$\Gamma_1 = 75^\circ.6 + 0^\circ.136\,18\,t \qquad \theta_1 = 60^\circ.2 - 0^\circ.13614\,t$$
$$\Gamma_2 = 202.64 + 0.032\,373\,t \qquad \theta_2 = 293.16 - 0.032335\,t$$
$$\Gamma_3 = 176.09 + 0.006\,892\,t \qquad \theta_3 = 319.71 - 0.006854\,t$$
$$\Gamma_4 = 123.84 + 0.001\,810\,t \qquad \theta_4 = 11.96 - 0.001772\,t$$

$$\theta = 315^\circ.800 + 0^\circ.000\,0381\,t$$
$$\theta' = 135.779 + 0.000\,0381\,t$$

$$L = 238^\circ.0 + 0^\circ.08313\,t \qquad M = 225^\circ.3 + 0.08308\,t$$
$$W = 117.9 + 0.00112\,t \qquad W_1 = 64.2 + 0.01617\,t$$
$$V = 24.5 + 0.16608\,t \qquad V' = 160.3 + 0.16612\,t$$

The periodic perturbations in the latitudes are of the form:

$$\delta p_i = \varkappa \sin \alpha \qquad \delta s_i = \varkappa \sin (v_i - \alpha - \theta')$$
$$\delta q_i = \varkappa \cos \alpha$$

All coefficients being very small, we may in the arguments replace $v_i$ by $l_i$, and neglect the difference of $\theta'$ and $180° + \theta$. The coefficients and arguments are:

| | coefficient | argument $\delta p_i, \delta q_i$ | argument $\delta s_i$ |
|---|---|---|---|
| Sat. I | $+ 0°.00042$ | $\Gamma_2 - 2v - 2\theta'$ | $l_1 + 2v + \theta_2$ |
| | $+ 0.00025$ | $V$ | $l_1 - V'$ |
| Sat. II | $- 0\ 00099$ | $\Gamma_2 - 2v - 2\theta'$ | $l_2 + 2v + \theta_2$ |
| | $+ 0.00010$ | $V + \Gamma_2$ | $l_2 + \theta_2 - 2L$ |
| | $+ 0.00078$ | $V$ | $l_2 - V'$ |
| Sat. III | $+ 0.00010$ | $V + \Gamma_3$ | $l_3 + \theta_3 - 2L$ |
| | $+ 0\ 00177$ | $V$ | $l_3 - V'$ |
| Sat. IV | $+ 0.00032$ | $V + \Gamma_4$ | $l_4 + \theta_4 - 2L$ |
| | $+ 0\ 00380$ | $V$ | $l_4 - V'$ |

The expressions for the longitudes and radii-vectores are given below. The inequalities are arranged in three groups, according to the periods, as explained in the beginning of this paper. Inequalities which are smaller than 1" in longitude and 0.000005 in radius-vector have been neglected. The developments in powers of the small quantities $\varrho$ and $\lambda_i$ of the great inequalities (arguments $4\tau$, $2\tau$ and $\tau$ for the satellites I, II, and III respectively), of the inequalities of group II and of the libration have already been given above, and only the values of the coefficients are repeated here. The more important of the smaller inequalities are here also given as functions of $\varrho$ and $\lambda_i$. Where no development is given the coefficients were taken from SOUILLART's theory, corrected for the adopted values of the excentricities (and inclinations) but not for the masses. The multipliers of $\varrho$ and $\lambda_i$ are given in units of the last decimal place of the coefficients to which they belong.

The true orbit-longitudes are:

$$v_1 = l_1 + 0°.0276 \sin\psi + \delta v_1$$
$$v_2 = l_2 - 0\ 0411 \sin\psi + \delta v_2$$
$$v_3 = l_3 + 0\ 0036 \sin\psi + \delta v_3$$
$$v_4 = l_4 + \delta v_4$$

The radii-vectores are:

$$r_i = a_i \varrho_i$$

$$\varrho_1 = 1.000\ 0066 + \delta\varrho_1$$
$$\varrho_2 = 1.00\,1\ 0549 + 000\ 014\ \lambda_1 + 000\ 084\ \lambda_3 + \delta\varrho_2$$
$$\varrho_3 = 1.000\ 0155 + 000\ 009\ \lambda_1 + 000\ 011\ \lambda_2 - .000\ 002\ \lambda_4 + \delta\varrho_3$$
$$\varrho_4 = 1\ 000\ 0755 + 000\ 008\ \lambda_1 + .000\ 008\ \lambda_2 + .000\ 034\ \lambda_3 + \delta\varrho_4$$

The inequalities $\delta v_i$ and $\delta \varrho_i$ are:

*Ia. Equations of the centre.*

$$\delta v_i = a_{i1} \sin (l_i - \tilde{\omega}_1) + a_{i2} \sin (l_i - \tilde{\omega}_2) + a_{i3} \sin (l_i - \tilde{\omega}_3) + a_{i4} \sin (l_i - \tilde{\omega}_4).$$

| | | | |
|---|---|---|---|
| $a_{11} = + 0^\circ\!.0062$ | $a_{12} + - 0^\circ\!.0011$ | $a_{13} = + 0^\circ\!.0030$ | $a_{14} = + 0^\circ\!.0014$ |
| $a_{21} = - 0.0002$ | $a_{22} = + 0.0344$ | $a_{23} = + 0.0281$ | $a_{24} = + 0.0118$ |
| $a_{31} = 0.0000$ | $a_{32} = - 0.0015$ | $a_{33} = + 0.1736$ | $a_{34} = + 0.0706$ |
| $a_{41} = 0.0000$ | $a_{42} = 0.0000$ | $a_{43} = - 0.0204$ | $a_{44} = + 0.8528$ |

$$\delta \varrho_i = a'_{i1} \cos (l_i - \tilde{\omega}_1) + a'_{i2} \cos (l_i - \tilde{\omega}_2) + a'_{i3} \cos (l_i - \tilde{\omega}_3) + a'_{i4} \cos (l_i - \tilde{\omega}_4)$$

| | | | |
|---|---|---|---|
| $a'_{11} = -.000054$ | $a'_{12} = +.000010$ | $a'_{13} = -.000026$ | $a'_{14} = -.000012$ |
| $a'_{21} = +.000002$ | $a'_{22} = -.000300$ | $a'_{23} = -.000245$ | $a'_{24} = -.000103$ |
| $a'_{31} = .000000$ | $a'_{32} = +.000013$ | $a'_{33} = -.001516$ | $a'_{34} = -.000616$ |
| $a'_{41} = .000000$ | $a'_{42} = .000000$ | $a'_{43} = +.000178$ | $a'_{44} = -.007445$ |

The inequalities of the groups I*b* and I*c* are of the form:

$$\delta v_i = \varkappa \sin \alpha \qquad \delta \varrho_i = \varkappa' \cos \alpha.$$

They are:

| *Argument* | *coefficient in* $\delta v_i$ | *coefficient in* $\delta \varrho_i$ | |
|---|---|---|---|
| *Satellite I.* | | | |
| $2\tau$ | $+0^\circ\!.0034\,(1+\lambda_2)$ | $-.000\,017\,(1+\lambda_2)$ | |
| $3\tau$ | $+0.0016\,(1+\lambda_3)$ | $-.000\,011\,(1+\lambda_3)$ | |
| $4\tau$ | $+0.4303$ | $-.003\,755$ | ($x_1$, *see above*) |
| $8\tau$ | $+0.0014+23\lambda_2$ | $-.000\,012-20\lambda_2$ | |
| *Satellite II.* | | | |
| $\tau$ | $-0^\circ\!.0123\,(1+\lambda_3)$ | $+.000\,061\,(1+\lambda_3)$ | |
| $2\tau$ | $+0.9875$ | $-.008\,617$ | ($x_2$, *see above*) |
| $3\tau$ | $+0.0052\,(1+\lambda_3)$ | $-.000\,058\,(1+\lambda_3)$ | |
| $4\tau$ | $+0.0051+1\lambda_1-1\lambda_2+109\lambda_3$ | $-.000\,034+8\lambda_1+1\lambda_2-83\lambda_3$ | |
| $5\tau$ | $+0.0004\,(1+\lambda_3)$ | $-.000\,006\,(1+\lambda_3)$ | |
| $6\tau$ | $+0.0005+3\lambda_1+2\lambda_3$ | $-.000\,008-5\lambda_1-2\lambda_3$ | |
| $l_2-l_4$ | $-0.0006\,(1+\lambda_4)$ | $+.000\,004\,(1+\lambda_4)$ | |
| $2(l_2-l_4)$ | $+0.0005\,(1+\lambda_4)$ | $-.000\,006\,(1+\lambda_4)$ | |
| $\tau+\varphi_3$ | $-0.0005$ | $+.000\,002$ | |
| $2\tau+\varphi_2$ | $-0.0003$ | $+.000\,006$ | |
| $2\tau+\varphi_3$ | $+0.0026$ | $-.000\,021$ | |
| $2\tau+\varphi_4$ | $+0.0010$ | $-.000\,008$ | |
| $l_1-\tilde{\omega}_2$ | $-0.0004$ | $+.000\,003$ | |
| $l_1-\tilde{\omega}_3$ | $-0.0004$ | $+.000\,002$ | |

| Argument | coefficient in $\delta v_i$ | coefficient in $\delta \varrho_i$ | |
|---|---|---|---|
| *Satellite III.* | | | |
| $\tau$ | $-0\overset{\circ}{.}0636$ | $+.000\,555$ | ($x_3$, *see above*) |
| $2\tau$ | $-0.0011\,(1+\lambda_2)$ | $+.000\,015\,(1+\lambda_2)$ | |
| $3\tau$ | $-0.0008-6\lambda_1-2\lambda_2$ | $-.000\,006-11\lambda_1+4\lambda_2$ | |
| $l_3-l_4$ | $-0.0041\,(1+\lambda_4)$ | $+.000\,022\,(1+\lambda_4)$ | |
| $2(l_3-l_4)$ | $+0.0138\,(1+\lambda_4)$ | $-.000\,132\,(1+\lambda_4)$ | |
| $3(l_3-l_4)$ | $+0.0010\,(1+\lambda_4)$ | $-.000\,012\,(1+\lambda_4)$ | |
| $\tau+\varphi_3$ | $-0.0008$ | $+.000\,007$ | |
| $\tau+\varphi_4$ | $-0.0003$ | $+.000\,003$ | |
| $l_3-2l_4+\tilde{\omega}_3$ | $+0.0004$ | $-.000\,000$ | |
| $l_3-2l_4+\tilde{\omega}_4$ | $-0.0004$ | $+.000\,001$ | |
| $2l_3-3l_4+\tilde{\omega}_4$ | $-0.0004$ | $+.000\,003$ | |
| $l_3-2L+\tilde{\omega}_3$ | $+0.0006$ | $-.000\,005$ | |
| *Satellite IV.* | | | |
| $l_4-l_1$ | $-0\overset{\circ}{.}0003\,(1+\lambda_1)$ | $+.000\,005\,(1+\lambda_1)$ | |
| $l_4-l_2$ | $-0.0005\,(1+\lambda_2)$ | $+.000\,008\,(1+\lambda_2)$ | |
| $l_3-l_4$ | $-0.0023\,(1+\lambda_3)$ | $+.000\,101\,(1+\lambda_3)$ | |
| $2(l_3-l_4)$ | $-0.0012\,(1+\lambda_3)$ | $+.000\,018\,(1+\lambda_3)$ | |
| $2l_4-2L$ | $+0.0012$ | $-.000\,015$ | |
| $l_3-2l_4+\tilde{\omega}_3$ | $-0.0006$ | $+.000\,002$ | |
| $l_3-2l_4+\tilde{\omega}_4$ | $+0.0007$ | $-.000\,006$ | |
| $l_4-2L+\tilde{\omega}_4$ | $+0.0064$ | $-.000\,056$ | |
| $2l_4-2\tilde{\omega}_4$ | $+0.0040$ | $-.000\,028$ | |

*Inequalities of group II.* (The expressions as functions of $\varrho$ and $\lambda_i$ have already been given above).

| Argument | Coefficients in $\delta v_i$ Sat. I | Sat. II | Sat. III |
|---|---|---|---|
| $\varphi_1$ | $-0\overset{\circ}{.}0077$ | $+0\overset{\circ}{.}0070$ | $-0\overset{\circ}{.}0000$ |
| $\varphi_2$ | $+0.0169$ | $+0.0377$ | $-0.0115$ |
| $\varphi_3$ | $+0.0072$ | $-0.0464$ | $+0.0095$ |
| $\varphi_4$ | $+0.0026$ | $-0.0157$ | $+0.0033$ |

In the radii-vectores these inequalities can be neglected, with the exception of the following term in Satellite II:

$$\delta\varrho_2 = +\,.000\,006\,\cos\varphi_3.$$

*Inequalities of group III.* These also are negligible in the radii-vectores. The largest of them is:

$$\delta\varrho_3 = +\,.000\,001\,\cos(\tilde{\omega}_3-\tilde{\omega}_4).$$

In the longitudes we have:

| *Argument* | *Coefficients in* $\delta v_i$ | | | |
|---|---|---|---|---|
| | *Sat I* | *Sat. II* | *Sat. III* | *Sat IV* |
| $M$ | + 0°.0006 | — 0°.0102 | — 0°.0135 | — 0°.0320 |
| $W$ | | — 0.0008 | — 0.0012 | — 0.0029 |
| $W_1$ | | — 0.0001 | — 0.0001 | — 0.0003 |
| $\Gamma_2$ | + 0.0099 | + 0.0028 | + 0.0001 | |
| $\Gamma_3$ | | + 0.0016 | + 0.0024 | — 0.0018 |
| $\Gamma_4$ | — 0.0001 | + 0.0019 | + 0.0029 | + 0.0010 |
| $\Gamma_2-\Gamma_3$ | — 0.0027 | — 0.0011 | — 0.0005 | |
| $\Gamma_2-\Gamma_4$ | — 0.0005 | — 0.0002 | — 0.0001 | |
| $\Gamma_3-\Gamma_4$ | + 0.0011 | — 0.0005 | — 0.0013 | — 0.0010 |
| $\tilde{\omega}_3-\tilde{\omega}_4$ | — 0.0005 | + 0.0002 | + 0.0009 | — 0.0011 |

It should be kept in mind that all the above developments are based on SOUILLART's analytical theory. New values of the masses and elements were introduced into his formulas, and a few numerical mistakes were corrected, but the analytical formulas were not altered. The only exception is the expression for the period of the libration, which was computed to terms of the third order in the masses inclusive, while SOUILLART rested content with those of the second order (See Gron. Publ. 17, art. 18).

**Physics.** — *"An auto-collimating spectral apparatus of great luminous intensity"*, by Prof. H. E. J. G. DU BOIS, G. J. ELIAS and F. LÖWE. (Communication from the Bosscha-Laboratory).

In optical work an illuminator is often wanted, which combines great brightness with monochromatic purity of the light, if possible of the order 0,1 $\mu\mu$. In spite of the almost boundless variety of available spectral apparatus [1]), such an appliance is wanting. For this purpose WÜLFING [2]) it is true, constructed a monochromator and investigated the luminosities obtainable by means of different sources of light, but its aperture was only $^1/_8$, the dispersion also rather slight. An auto-collimator lately described by FABRY and JOBIN [3]) has an aperture of only $^1/_{10}$. Recently one of us has described three spectral apparatus with constant deviation (parallel or at right angle) [4]);

1) H. KAYSER, Handb. d. Spectroscopie 1, p.p. 489 et seq gives a survey leading up to 1900.

2) E. A. WÜLFING, N. Jahrb. f. Mineralogie Beil. 12, p. 343, 1898. — C. LEISS, Zeitschr. für Instr. kunde 13 p. 209, 1898. S. NAKAMURA, Ann. d. Phys. (4) 20 p. 811, 1906.

3) CH. FABRY & A. JOBIN, Journ. de Phys. (4) 3 p. 202, 1904.

4) F. LÖWE, Zeitschr. f. Instr. Kunde 26, p. 330, 1906 and 27, p. 271, 1907.