Mathematics. — "*On algebraic twisted curves on scrolls of order n with (n—1)-fold right line.*" By Prof. JAN DE VRIES.

1. If we intersect a cubic scroll Φ^3 by a pencil of planes having a generatrix a of Φ^3 as axis, we get a system of conics ϱ^2, all passing through the point O, where a meets the double right line d. If we take a (p, q)-correspondence between this pencil of planes and the pencil of planes with axis d, then in this way to each ϱ^2 are assigned p right lines r of Φ^3 and to each right line r evidently q conics ϱ^2. The locus of the points of intersection of the lines r and ϱ^2 corresponding to each other is a twisted curve of order $m = p + q$; for the points of the rational cubic curve which Φ^3 determines on an arbitrary plane are arranged in a (p, q)-correspondence, of which each coincidence is the point of intersection of a ϱ^2 with a right line r corresponding to it.

The twisted curve ϱ^m has the right lines r as q-fold secants, whilst it is intersected by each of the ∞^2 conics of Φ^3 in p points.

2. If Φ^3 is represented by central projection out of O on a plane τ cutting a and d in A and D, then the systems (r) and (ϱ^2) are transformed into the pencils (D) and (A) which are now likewise arranged in a (p, q)-correspondence. The curve c^m generated in this way has in D a p-fold point, in A a q-fold one. But it has moreover a q-fold point in the point of intersection B of the right line b of Φ^3, which still passes through O, for b is q-fold secant of ϱ^m. From this ensues that the correspondence (p, q) in τ cannot be taken arbitrarily.

The curve ϱ^m is completely determined by its central projection c^m. For, the cone projecting c^m out of O has a p-fold edge along d and q-fold edges along a and b, so its section with Φ^3 consists of $2p + 2q = 2m$ right lines and a twisted curve of order m having p points in common with d and q points with a.

As the singular points of c^m are equivalent to $\frac{1}{2}p\,(p-1) + q\,(q-1)$ nodes, the genus of c^m is indicated by

$$g = \frac{1}{2}\,(p+q-1)\,(p+q-2) - \frac{1}{2}\,p\,(p-1) - q\,(q-1) =$$

$$= (p-1)\,(q-1) - \frac{1}{2}q\,(q-1),$$

or by

$$g = (m-1)\,(q-1) - \frac{3}{2}\,q\,(q-1).$$

This is at the same time the genus of ϱ^m. It is evident that p may not be smaller than $(\frac{1}{2}q + 1)$. For the smallest values of p and q we have

m	p	q	g
2	1	1	0
3	2	1	0
4	3	1	0
4	2	2	0
5	4	1	0
5	3	2	1
6	5	1	0
6	4	2	2
6	3	3	1

The above considerations may be extended by taking instead of scroll Φ^3 a scroll Φ^n with $(n-1)$-fold right line d. Out of a point O of d now start $(n-1)$ right lines $a_1, a_2, \ldots a_{n-1}$. A (p, q)-correspondence between the pencils of planes (a_1) and (d) determines again a twisted curve of order $p + q = m$, having as central projection a c^m with p-fold point D and q-fold points in $A_1, A_2, \ldots A_{n-1}$; and inversely ϱ^m is again entirely determined by c^m. For the genus of ϱ^m (and c^m) we now find

$$g = \frac{1}{2}(p + q - 1)(p + q - 2) - \frac{1}{2}p(p-1) - \frac{1}{2}q(q-1)(n-1)$$

or

$$g = (q-1)(m-1) - \frac{1}{2}q(q-1)n.$$

To obtain general twisted curves we shall not be allowed to take p larger than 4, q larger than 3. For $n = 2$ we find evidently the well known considerations concerning curves on an hyperboloid.

5. If we substitute in τ for the curve c^n a curve passing p times through D, q times through A_k and moreover cutting the right line DA_k in s points, then this curve is evidently the central projection of a curve on Φ^n, having a multiple point in O. For, each of the $(n-1)$ tangential planes in O will now contain s right lines, touching the twisted curve in O.