

*Citation:*

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In order to prepare the urea derivative, the cinnamide is dissolved in eight times its weight of 96 pCt. alcohol, and when cooled to the temperature of the room the hypochlorite solution, prepared according to GRAEBE<sup>1)</sup>, is slowly dropped in, the free alkali being neutralised with 2N hydrochloric acid immediately before use. For every 2 mols. of amide, 1 mol. of potassium hypochlorite should be added. The liquid gets warm, and very soon a crystalline mass composed of very slender needles is deposited. After a few hours the mass is collected at the pump; this does not go very readily on account of the fine state of division. The yellowish mass is treated with hot alcohol and then washed with water. A fairly pure urea derivative is thus obtained (m. p. about 218). By recrystallisation once or twice from glacial acetic acid it is obtained pure in needles (m. p. 225—226).

0,1733	0,0894	0,4682		
0,1654	0,1863	0,4467		
0,1654	13,9	765		
	Found	73,68	5,78	
		pCt. C	pCt. H	9,70 pCt. N
		73,66	5,85	

Theory  $C_{18}H_{16}N_2O_2$ : 73,95 pCt. C 5,51 pCt. H 9,59 pCt. N

The compound is insoluble at a low temperature in water, ligroin, alcohol, methyl alcohol, ether, carbon disulphide and benzene; at the boiling temperature slightly soluble in alcohol and benzene and freely so in glacial acetic acid, chloroform and acetone. It is insoluble in alkalis or acids.

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**Astronomy.** *“Mutual occultations and eclipses of the satellites of Jupiter in 1908.* By Prof. J. A. C. OUDEMANS.

(Communicated in the meeting of September 29, 1906).

N.B. In the present communication the four satellites of Jupiter, known since 1608, have been denoted by I, II, III and IV in accordance with their mean distances from the planet. The further letters *n* and *f* indicate whether the satellite is *near* or *far*, i.e. whether it is in that half of the orbit which is nearest to or furthest from the Earth. The joventric longitudes as well as the geocentric amplitudes are counted in “signs” and “degrees”, the latter beginning from the superior

<sup>1)</sup> Ber. 35, 2753.

geocentric conjunction. Eastern elongation, denoted by *e. e.*, has an amplitude of  $3^\circ$ , western elongation, *w. e.*, one of  $9^s$ .

Not to interrupt the text unnecessarily, all particulars have found a place at the end of the paper.

### FIRST PART. OCCULTATIONS.

In the numbers 3846 and 3857 of the *Astronomische Nachrichten* we find two communications relative to observations of the occultation of one satellite of Jupiter by another. The first (1) is by Mr. PH. FAUTH at Landstuhl, dated 8 December 1902, with postscripts of 29 December 1902 and 14 January 1903. The other (2) by Mr. A. A. NIJLAND at Utrecht, dated 27 February 1903.

FAUTH notes in addition that HOUZEAU, in his *Vademecum*, p. 666 mentions a couple of similar observations (3), and further that STANLEY WILLIAMS, on the 27<sup>th</sup> March 1885 at 12<sup>h</sup> 20<sup>m</sup>, saw the third satellite pass the first in such a way that the two satellites combined had a pear-shaped appearance. (4)

The satellites of Jupiter move in orbits but little inclined to the plane of Jupiter's equator. LAPLACE assumed a fixed plane for each satellite; the plane of the satellite's orbit has a constant inclination on this fixed plane, whereas the line of intersection, the line of the nodes, has a slow retrograde motion. The inclinations of the fixed planes on the plane of Jupiter's equator amount only to a few minutes; their intersection with the plane of Jupiter's orbit is identical with the line of the nodes of the equator. The value generally adopted for the inclination of the latter plane on the orbit of Jupiter is  $3^\circ 4'$ , whereas the longitude of the ascending node, which therefore is also that of the fixed planes, is at present about  $315\frac{1}{2}^\circ$ .

In order to be able to assign the time at which, as seen from the Earth, an occultation of one satellite by another is possible, it is necessary to know the longitude of the ascending node and the inclination of the mean fixed plane on the orbit of the Earth. At the time that the mean fixed plane, prolonged, passes through the Earth, occultations of one satellite by another may be observed. As Jupiter completes a revolution around the sun in nearly 12 years, these times will succeed each other after periods of six years. Jupiter will pass alternately through the ascending and the descending node of the plane which passes through the centre of the sun parallel to the mean fixed plane.

It follows that, as occultations of one satellite by another have been observed in 1902, we must expect that these phenomena will be again visible in 1908 (5).

To facilitate these observations I thought it desirable to calculate in advance the conjunctions of any two satellites for the most favourable part of 1908.

We have to consider that while formerly the orbits of the satellites were determined by repeatedly measuring the distances and their angles of position relatively to the planet, this method is now replaced by the measurement of the distances and the angles of position of the satellites relative to each other (especially with the heliometer) (6). For observations during a moderate interval the periodic times of the satellites may be assumed to be accurately known. Admitting this, if, leaving out of consideration KEPLER'S third law, we introduce the major axis of each satellite as an unknown quantity, the total number of such unknowns will be six for each orbit at a determined time. If, as was done by BESSEL at Königsberg in 1834—39, and by SCHUR at Göttingen in 1874—1880, the distance and the angle of position between the planet and the satellite are measured, we get two equations with six unknown quantities. If however we measure the distance and the angle of position of two satellites relative to each other, the number of unknown quantities in these equations is doubled and thus becomes 12. If finally all the combinations two by two, are observed, as was done by GILL and FINLAY at the Observatory of the Cape, we get a great number of equations with a total of 24 unknown quantities. These equations must then be solved by the method of least squares. This number becomes 29 if we add the masses of the satellites, (only to be found by the perturbations caused by one satellite in the motion of the others,) and the compression of Jupiter (7), given by the retrogradation of the lines of the Nodes on the fixed planes.

Now the observation of an occultation, even of a conjunction without an occultation, can be made by everybody possessing a telescope of sufficient power. Such an observation also furnishes two equations between the unknown quantities, at least if, for a non central occultation or a simple conjunction, the difference in latitude is measured at the filar micrometer. This consideration engaged me to compute in advance the time of these conjunctions for the most favourable part of 1908. If by experience we find that this preliminary work leads to valuable results, it might be worth while to continue it for some future period, for instance for 1914.

For the moment at which the mean fixed plane passes through the centre of the Earth, I find, 1908 July 8, 19<sup>h</sup>.6 Mean Time at Greenwich, (5).

This date, it is to be regretted, is very unfavourable. For on that

day Jupiter culminates at Greenwich at  $2^{\text{h}}10^{\text{n}}$  M. T., its declination being  $16^{\circ}48'5$  North, whereas the Sun's declination is  $22^{\circ}30'$  North. From these data I find for the 8<sup>th</sup> of July, for Utrecht, duly making allowance for refraction :

Setting of the upper limb of the sun at  $8^{\text{h}}20^{\text{m}}$  mean time,  
 „ „ Jupiter „ 9 44 .5 „ „ .

So there is but a poor chance for an observation of the computed occultation at Utrecht. For southern observatories it is somewhat better. At the Cape for instance, we have :

Sunset at  $5^{\text{h}}5^{\text{m}}$  mean time,  
 Setting of Jupiter „ 7 25 „ „ .

We thus find that on July 8, 1908, at Utrecht, the setting of the sun precedes that of Jupiter by  $1^{\text{h}}24^{\text{m}}.5$ ; at the Cape by  $2^{\text{h}}20^{\text{n}}$ .

We have computed all the conjunctions of the satellites of Jupiter which will occur between 31 May and 20 July 1908. In what follows a short account is given of the way which led to our results.

In the *Nautical Almanac* are given the *Geocentric Superior Conjunctions*; in the Almanac of 1908 they will be found on pp. 504, 505.

To begin with, a separate drawing was made of the four orbits, which were supposed to be circular, for each interval of two periods of I (about  $85^{\text{h}}$ ). On these orbits we plotted the positions of the satellites for each second hour, making use of divided pasteboard arcs. The number of hours elapsed since the moment chosen as a starting-point were noted for each position. The equation of the centre etc. was neglected.

The scale of this drawing gave  $4''$  to 1 mm. The radii, of the orbits therefore were: for I 27.9 mm.; for II 44.45 mm.; for III 70.9 mm. and for IV 124.7 mm.

The direction from the Zero of I to the common centre of all the circles showed the direction towards the Earth. Knowing this, we could easily find for each of the six possible combinations of two of the satellites, those equal hour numbers, the connecting line of which is parallel to this direction.

These connecting lines show the approximate times at which, as seen from the Earth, one of the satellites is in conjunction with another. The want of parallelism of the real lines joining the Earth with the satellites, in different parts of their orbits, may safely be disregarded. The plate annexed to this paper represents, reduced to half the scale, the drawing for the period of 85 hours, following 12 July 1908,  $11^{\text{h}}2^{\text{m}}.3$  M. T. Greenwich.

The dotted lines indicate the lines connecting the equal numbers.

Each of them represents a conjunction of two satellites. The corresponding hours read off from the figure are :

6 <sup>u</sup> .2	:	IV <sub>f</sub>	occulted	by	III <sub>n</sub> ,
21 .8	:	IV <sub>f</sub>	„	„	I <sub>n</sub> ,
25 .0	:	IV <sub>f</sub>	„	„	II <sub>n</sub> ,
35 .0	:	I <sub>f</sub>	„	„	II <sub>n</sub> ,
66 .25	:	II <sub>f</sub>	„	„	I <sub>n</sub> ,
71 .0	:	III <sub>f</sub>	„	„	I <sub>n</sub> ,

They were added to the instant which must be regarded as the startingpoint for this figure. The instants of the conjunctions were next converted into civil time of Paris by the addition of 12<sup>h</sup>9<sup>m</sup>21<sup>s</sup>. The elongation and the latitude of both the satellites, expressed in radii of Jupiter, were then computed by the aid of the *Tables éclipitiques* of DAMOISEAU, 2nd part. (8). In the case that the elongations did not perfectly agree, a slight computation led to a more accurate result for the time of conjunction (9).

In the case that the two satellites moved apparently in opposite directions, (which happens if the one is in the further part of its orbit, the other in the nearer part), the correction to the adopted time was mostly insignificant.

If, on the contrary, they moved the same way (which happens if both are "far" or if both are "near", so that the one has to overtake the other) the correction amounted sometimes to an hour or more. In every case, in which the correction exceeded 20 minutes, the computation was repeated with the corrected time. Further below will be found the list of the results. From May 31 to July 19, i.e. during a period of fifty days, there occur 72 conjunctions. It is to be regretted that at a determined place of observation but very few of them will be visible. For only those conjunctions are visible which occur between sunset and the setting of Jupiter. For Utrecht we have, in mean time :

		Setting of the upper limb of the Sun	Setting of Jupiter	Difference
1908	June 1	8 <sup>h</sup> 10 <sup>m</sup>	11 <sup>h</sup> 54 <sup>m</sup>	3 <sup>h</sup> 44 <sup>m</sup>
	„ 11	8 20 .5	11 19	2 58 .5
	„ 21	8 24	10 44	2 20
	July 1	8 24	10 9	1 45
	„ 11	8 18	9 34	1 16
	„ 21	8 7 .5	8 59	0 51 .5

For the Cape of Good Hope :

1908 June 1	4 <sup>h</sup> 59 <sup>m</sup>	9 <sup>h</sup> 18 <sup>m</sup>	4 <sup>h</sup> 19 <sup>m</sup>
„ 11	4 57 .5	8 46 .5	3 49
„ 21	4 58	8 16	3 18
July 1	5 2	7 46	2 44
„ 11	5 6 .5	7 16	2 9 .5
„ 21	5 13	6 47	1 34

The circumstances are thus seen to be considerably more favourable for a southern than for a northern observatory.

Several of the occultations will not be visible because the common elongation falls short of unity i. e. of the radius of Jupiter. This is the case of Nos. 8, 9, 12, 13, 15, 16, 20, 23, 39 and 64. In the first eight of these cases and in the last one the planet stands between the two satellites. In case No. 39 both the satellites I and IV are covered by the planet <sup>1)</sup>.

For other conjunctions it may happen that one of the satellites is invisible because of its being in the shadow of the planet. Such cases are :

(N <sup>o</sup> . 21), June 13	9 <sup>h</sup> 28 <sup>m</sup>	M. T. Grw.,	II eclipsed,
(N <sup>o</sup> . 31), „ 20	12 51	„ „	II „ ,
(N <sup>o</sup> . 51), July 4	18 15 .7	„ „	II „ ,
(N <sup>o</sup> . 65), „ 13	12 3	„ „	IV „ .

If the satellite which at the conjunction is nearest to the Earth is eclipsed by the planet's shadow, it might, as seen from our standpoint, project itself wholly or partially as a black spot on the other satellite. The case however has not presented itself in our computations.

Possibly the last of the conjunctions just mentioned may really be visible; for according to the *N. Almanac*, the reappearance of IV from the shadow of the planet takes place at 12<sup>h</sup>7<sup>m</sup>15<sup>s</sup> M. T. Greenwich and the predicted eclipses of this satellite are occasionally a few minutes in error. A few minutes later, according to the *N. Almanac* at 12<sup>h</sup>16<sup>m</sup>, II enters the disc of Jupiter.

<sup>1)</sup> According to the Nautical Almanac we have for this night (M. T. of Greenwich):

IV.	Occultation Disappearance	10 <sup>h</sup> 19 <sup>m</sup> ,
I.	Occultation Disappearance	11 20,
I.	Eclipse Reappearance	14 26 27 <sup>s</sup> ,
IV.	Occultation Reappearance	15 13,
IV.	Eclipse Disappearance	18 5 6 ,
IV.	Eclipse Reappearance	22 52 2 .

## N O T E S.

(1) The article of FAUTH, abridged, runs thus :

— — — Ausser den in HOUSSEAU, Vademecum p. 666 aufgeführten Beobachtungen, (vid. below Note 3), kenne ich aus neuerer Zeit nur einen Fall: STANLEY WILLIAMS sah am 27 März 1885 an einem 7 cm. Rohre mit 102-facher Vergrößerung um 12<sup>h</sup> 20<sup>m</sup> den III Trabanten vor dem I, wobei beide ein birnformiges Objekt bildeten.

— — — In fünf Wochen konnte ich drei Bedeckungen verfolgen, wobei anzunehmen ist, dass mir durch schlechte Witterung etwa 10 andere Gelegenheiten entgangen sein mögen, unter denen sicher einige Bedeckungen vorkommen. Nach meiner Erfahrung können Konjunktionen der Jupitermonde unter sich weit genauer beobachtet werden als Bedeckungen durch Jupiter oder Vorübergänge vor ihm. Somit möchten die hier angegebenen Beispiele Anlass bieten, in den späteren Oppositionen Jupiters den durchaus nicht seltenen Bedeckungen oder wenigstens Berührungen und sehr nahen Konjunktionen der Trabanten unter sich mehr Aufmerksamkeit zu schenken, zumal schon kleine Instrumente zur Wahrnehmung der Phasen einer event. Bedeckung genügen. Die Beobachtungen der letzten Zeit sind :

1. Oct. 7; II bedeckt I; die S. Ränder berühren sich und I ragt im N. etwas hervor. Konj. um 9<sup>h</sup> 16<sup>m</sup> M. E. Z. <sup>1)</sup>)

2. Oct. 23; II bedeckt III so, dass die Mitte von II nördlich am N. Rand von III vorbeigeht; Konjunktion um 8<sup>h</sup> 7<sup>m</sup> 3<sup>s</sup>,5.

3. Nov. 10; III bedeckt I so, dass der S. Rand von III die Mitte von I streift (gute Luft); Konjunktion um 7<sup>h</sup> 33<sup>m</sup> 20<sup>s</sup>.

Instrument: 178 mm., Vergrößerung 178 fach.

Landstuhl, 1902 Dez. 8.

P.S. vom 29 Dezember. Am Abend des 24 Dezember gelang nochmals die Beobachtung einer Bedeckung, bei welcher I über IV hinwegzog. Aus je fünf vor- und nachher notierten Zeitmomenten folgen als Mittelwerte 6<sup>h</sup> 24<sup>m</sup>,25, 24<sup>m</sup>,625, 24<sup>m</sup>,50, 24<sup>m</sup>,625 und 24<sup>m</sup>,50. Die Konjunktion fand also statt 6<sup>h</sup> 24<sup>m</sup> 30<sup>s</sup>.

Der Uhrstand war um 3<sup>h</sup> mit dem Zeitsignal verglichen worden. IV stand ein wenig südlicher als I, vielleicht um ein Viertel seines Durchmessers. Die weitaus interessantere Konjunktion zwischen II und IV am 25 Dezember blieb gegenstandslos, weil IV um etwa zwei Durchmesser vorüberging.

P.S. vom 14 Januar (1903). Heute Abend, am 14 Januar, bewegte

<sup>1)</sup> i. e. Mittlere Europäische Zeit, 1<sup>h</sup> later than Greenwich-time.



sich der Trabant III über II hinweg. Die sehr schlechte Luft liess nur den ersten Kontakt auf etwa  $6^h 2^m$  feststellen. Um  $6^h 18^m$  mochten sich beide Komponenten so weit getrennt haben, dass dies in einem weniger schlechten Augenblick bemerkt wurde; um  $6^h 32^m$ , dem nächsten blickweisen Auftauchen der beiden Lichtpunkte, waren diese um etwa einen Durchmesser von einander entfernt. Die Bedeckung war fast genau central.

PH. F.

(2) Mr. NIJLAND writes in N<sup>o</sup>. 3857 of the *Astronomische Nachrichten*:  
 — — — Am 15 Juli 1902 fand eine Konjunktion der Trabanten II und III statt, welche ich bei guter Luft am Refraktor (Brennweite 319 cm., Oeffnung 26 cm.) mit Vergr. 248 beobachten konnte. Es wurde III nahezu central von II bedeckt. Einige Minuten lang blieb eine feine schwarze Linie zwischen den beiden Scheibchen sichtbar, welche um  $14^h 10^m 11^s$  M.Z. Utrecht verschwand and um  $14^h 20^m 31^s$  wieder erschien; die Konjunktion musz also um  $14^h 15^m 21^s$  stattgefunden haben. Dass diese Trennungslinie vor und nach der Konjunktion immer dieselbe Richtung hatte, und zwar scheinbar senkrecht auf der Bahnebene der Trabanten stand, mag als Beweis dafür gelten, dass der Vorübergang wirklich nahezu central gewesen ist. Dann lässt sich aber aus dieser centralen Passage die Summe der Durchmesser der Monde II und III mit erheblicher Genauigkeit bestimmen.

Nehme ich für die mittlere Entfernung  $\psi - \odot$  die Halbmesser der Bahnen gleich  $177''{,}8$  und  $283''{,}6$ , so finde ich für die relative Bewegung von II und III zur Beobachtungszeit  $13''{,}86$  pro Stunde. Aus der beobachteten Zeitdauer von  $10^m 20^s = 0^h 172$  folgt dann für die Summe der beiden Durchmesser,  $2''{,}38$ . Wird (siehe die Angaben von DOUGLASS, Astr. Nachr. 3500) für das Verhältniss der Durchmesser von II und III  $\frac{4}{11}$  angenommen, so finde ich, in vorzüglicher Uebereinstimmung mit den a. a. O. genannten Werten, für den Durchmesser von II  $0''{,}87$  und von III  $1''{,}51$  (in mittl. Entf.).

Utrecht, 1903 Febr., 27.

A. A. NIJLAND.

Remark. As from the observed instants I derived a result slightly different from that of Mr. NIJLAND, this gentleman allowed me to consult his reduction of the observation. It appeared that, in order to find the amplitudes, he had combined the preceding geocentric superior conjunction with the following transit, from the ingress and egress of which the inferior conjunction could be derived. A slight error had however been committed in the computation. After correction the relative motion of the two satellites was found to be  $13''{,}786$  and the sum of the diameters  $2''{,}374$ . Moreover their proportion was, evidently erroneously and

against the real intention, put at 4 to 11 instead of at 4 to 7. We thus get for the diameters 0".863 and 1".511, which is still in good agreement with the result of Mr. NIJLAND. As values have been assumed for the radii of the orbits which hold for the mean distance of Jupiter from the sun, these values need no further reduction.

(3) We find in HOUZEAU, *Vademecum* (Bruxelles, 1882), p. 666 :  
On rapporte une occultation du satellite II par le satellite III, observée à Sommerfeld, près de Leipzig, par C. ARNOLDT, le 1<sup>er</sup> novembre 1693, (WHISTON, *The longitude discovered by the eclipses*, 8°, London, 1738), et une autre du satellite IV, également par le III<sup>me</sup>, vue par LUTHMER à Hanovre, le 30 octobre 1822 (*Nature*, 4°, London; vol. XVII, 1877, p. 148).

1st Remark. The little book of WHISTON here quoted is in the library of the University at Utrecht, Division P, 8<sup>vo</sup>, number 602. We have turned over the leaves several times, but have not found any mention of the observation of C. ARNOLDT. It is true that the author, in § XVIII, recommends the observation of the mutual occultations of the satellites. He remarks that, if at such an occultation they have opposite motions, the relative velocity is "doubled". He mentions the complaint of DERHAM <sup>1)</sup>, that the strong light of Jupiter renders the observation of these occultations rather difficult. He remarks that, the interval being equal, their number must be one and a half time as large as that of the eclipses. Again he mentions that LYNN is the first who, in the *Philosophical Transactions* N<sup>o</sup>. 393, has proposed to apply these conjunctions to the determination of the longitude, seeing that they can often be observed with an accuracy of less than half a minute <sup>2)</sup>. But I do not find the observation of a single occultation nor its prediction.

It needs hardly be said that the conjunctions, visible from places, the difference in longitude of which is to be determined, are too rare to be of much importance for the purpose. In accuracy of observation they are at all events surpassed by occultations of stars. But they may well be compared with the eclipses of the satellites of Jupiter and are indeed superior to them in this respect that they yield a result in a few minutes which is independent of the optical power of the telescope. For the eclipses this is only true in the case of the combination of a disappearance with a reappearance.

2nd Remark. The original account of the observation of LUTHMER was communicated by him to BODE who inserted it in the (Berliner) *Astronomisches Jahrbuch* für 1826, p. 224 :

"Am 30 Oct. Ab. 6<sup>u</sup> 55' Bedeckung des vierten ¼ Trabanten vom dritten."

<sup>1)</sup> POGGENDORFF's *Biographisches Wörterbuch*, (article W. DERHAM) gives no reference to the passage where this complaint is to be found, nor even to any paper on the observation of the satellites of Jupiter.

<sup>2)</sup> At least if there were no undulation of the images. See at the end of note 4

If we assume  $9^{\circ}42' = 38^m48^s$  East of Greenwich for the longitude of Hannover, this is  $\approx 6^h 16^m 12^s$  M. T. of Greenwich, at least supposing that at that time it was already usual to give the observations expressed in mean time.

In *Nature*, XVII (Nov. 1877—April 1878) p. 149 (not 148) we find in "Our Astronomical Column":

"JUPITER'S SATELLITES. — Amongst the recorded phenomena connected with the motions of the satellites of Jupiter are several notices of observed occultations of one satellite by another, and of small stars by one or other of the satellites. <sup>1)</sup> The following cases may be mentioned: — On the night of November 1, 1693, CHRISTOPH ARNOLDT, of Sommerfeld, near Leipzig, observed an occultation of the second satellite by the third at  $10^h 47^m$  apparent time. On October 30, 1822, LUTHER, of Hannover, witnessed an occultation of the fourth satellite by the third at  $6^h 55^m$  mean time.

It thus appears that the editor of *Nature* also took it for granted that the statement must be understood to have been made in mean time.

(4) I did not succeed in finding the account of this observation of STANLEY WILLIAMS in any of the journals accessible to me, and therefore applied to the author, who lives at Hove near Brighton, for particulars about the place of its publication.

He kindly replied on the 7<sup>th</sup> instant, that the details of his observation of 27 March 1885 were published both in the 41<sup>th</sup> volume of the "*English Mechanic*" and in the volume for 1885 of the German Journal "*Sirius*".

He had moreover the courtesy of communicating to me the original account of the observation in question. From this account the following passages may be quoted:

*Occultation of satellite I by satellite III.*

1885 March 27, . . . .  $2\frac{3}{4}$  inch refractor. Power 102.

$11^h55^m$  (Greenwich mean time). They are now only *just* free from contact.  $\text{III} \bigcirc \text{I}$ , like an elongated star with little more than a black line between the components.

$12^h00^m$  to  $12^h04^m$ . After steady gazing I cannot see any certain separation between the satellites, and therefore with this instrument and power first contact must have occurred about  $12^h02^m$ . Definition is very bad, however, and in a larger telescope there probably might still be a small separation between the limbs.

<sup>1)</sup> It is to be regretted that these "several notices of observed occultations of one satellite by another" are not more fully quoted.

12<sup>h</sup>10<sup>m</sup>. They now appear as one elongated satellite. At times a trace of the notches is apparent.

12<sup>h</sup>20<sup>m</sup>. The elongation is now very nearly at right angles to the direction of the motion of the satellites, and is so slight as to be scarcely noticeable in this bad and unsteady definition. I think from the smallness of the elongation that nearly half satellite I must be concealed behind III. In this bad definition it is not possible to say which satellite is in front of the other from the appearance alone.

In his letter Mr. STANLEY WILLIAMS mentions the remarkable fact that he too observed on 15 July 1902 the same conjunction which has been described by NIJLAND. His instrument was a reflector of 6½ inch, with a power of 225. The following are the particulars as communicated :

1902 July 15, 13<sup>h</sup>45<sup>m</sup>.2. Satellites II and III are in contact. The one will occult the other. See diagram III ○ II.

13<sup>h</sup>52<sup>m</sup>. The satellites form one disc, which has the slightest possible elongation in a north and south direction. Owing to confused seeing this disc always appeared more or less fuzzy, and it is impossible from the appearance alone to say, which satellite is occulting the other.

13<sup>h</sup>56<sup>m</sup>. The combined disc is considerably elongated now.

14<sup>h</sup>02<sup>m</sup>.2. Satellites II and III in contact as in diagram adjoining II ○ III.

14<sup>h</sup>04<sup>m</sup>. Satellites clearly separated. The occultation must have been nearly central. II is a little more south now relative to III, than it was before occultation. Possibly the slight elongation noted at 13<sup>h</sup>52<sup>m</sup> was not real.

The above times are Greenwich mean times. Satellite III was on the farther side of its orbit moving east, II on the near side moving west. As the disc of III is larger than that of II, the phenomenon should be described as a transit of II over or across III, rather than an occultation of one satellite by the other.

The arithmetical mean of 13<sup>h</sup>45<sup>m</sup>.2 and 14<sup>h</sup>02<sup>m</sup>.2 is 13<sup>h</sup>53<sup>m</sup>.7, which is 1<sup>m</sup>.1 earlier than NIJLAND's observation.

(5) For the numbers which follow we refer to KAISER'S "*Sterrenhemel*", 4th Edition, p. 707 and following.

In the 4<sup>th</sup> Vol. of his *Mécanique Céleste*, p. 62, TISSERAND, following SOUILLART, adopts inclinations for the orbits of III and IV, which

respectively exceed those given in the "*Sterrenhemel*" by  $+4''$  and  $-8''$ .

According to LEVERRIER we have, for the orbit of Jupiter in 1908,0:

$$\text{Ascending Node} = 99^{\circ}31'56'',$$

$$\text{Inclination} = 1\ 18\ 29.$$

The fixed plane of the first satellite coincides with the plane of Jupiter's equator: the longitude of the ascending node on the plane of Jupiter's orbit, for the beginning of 1908 is therefore  $315^{\circ}33'35''$ , the inclination  $3^{\circ}4'9''$ .

Furthermore we have for the four fixed planes relative to the plane of Jupiter's orbit:

	Long. asc. node	Inclination	
I	$315^{\circ}33'35''$	$3^{\circ}4'3''$	} Epoch 1908.0.
II	315 33 35	3 3 4	
III	315 33 35	2 59 11	
IV	315 33 35	2 39 57	

For the mean fixed plane of the three first satellites we thus find: longitude of ascending node on the plane of Jupiter's orbit at the beginning of 1908:  $315^{\circ}33'35''$ , inclination  $3^{\circ}2'6''$ .

Moreover we have for the respective fixed planes in 1908, according to TISSERAND:

	long. asc. node	Change in 1000 days	Inclination
II	$122^{\circ}293$	$-33^{\circ}031$	$0^{\circ}28'9''$
III	26 173	$-6\cdot955$	0 10 44
IV	238 982	$-1\cdot856$	0 13 51

The effect of these inclinations, however, is but trifling. At the distance of  $90^{\circ}$  from the node they produce only deviations

for II of  $1''\cdot46$ ,

„ III „  $0''\cdot89$ ,

„ IV „  $2\cdot01$ .

The determination of the position of the fixed planes, as also that of the planes of the orbits of the satellites relative to these, will be much improved by the measurements which DE SITTER at Groningen is making on photographic plates. Eventual observations of conjunctions of the satellites, rather even of occultations, will contribute their part in this determination and will furnish a test for the adopted values.

In the meeting of our section of last March a provisional account of these measures by DE SITTER was communicated by Messrs J. C. KAPTEYN and E. F. VAN DE SANDE BAKHUYZEN<sup>1)</sup>:

Our computations were then already too far advanced to keep them back altogether; but we hope that by the side of these measures they still may have their use, for this reason that conjunctions and mutual occultations of the satellites may well be observed at several observatories which are not equipped for taking photographs.

From the preceding numbers we find for the position of the fixed plane relative to the ecliptic (for 1908,0).

$$\text{Ascending Node } 336^{\circ}48'23'' = \Omega,$$

$$\text{Inclination } 2\ 717 = I,$$

Now, if  $R_{\text{J}}$ ,  $L_{\text{J}}$  and  $\beta$  represent the radius vector, the longitude and the latitude of Jupiter;  $R_{\text{E}}$ ,  $L_{\text{E}}$  the radius vector and the longitude of the Earth, (those given in the N. Almanac after correction for aberration), the condition that the fixed plane must pass through the Earth is expressed by:

$$R_{\text{J}} \cos \beta \sin (L_{\text{J}} - \Omega) - R_{\text{J}} \sin \beta \cot I = R_{\text{E}} \sin (L_{\text{E}} - \Omega),$$

which is satisfied July 8, 1908 at 19<sup>h</sup>38<sup>m</sup>.3. For at that moment

$$\begin{array}{ll} \log R_{\text{J}} = 0.728527 & \log R_{\text{E}} = 0.007179 \\ L_{\text{J}} = 141^{\circ}23' 9''.0 & L_{\text{E}} = 286^{\circ}40' 3''.5 \\ \beta = + 0\ 52\ 26\ .73 & \Omega = 336\ 48\ 52\ .0 \end{array}$$

so that our equation becomes

$$1.423706 - 2.204190 = -0.780484$$

Similarly we find for the instant at which the same plane passes through the centre of the sun:

$$25 \text{ April } 1908 \text{ at } 18^{\text{u}}.5 \text{ M. T. Grw.}$$

On both sides of this latter epoch there exists the possibility of an eclipse of one satellite by another, at the time of the heliocentric conjunctions. We hope to treat this subject in the second part of this communication.

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<sup>1)</sup> This provisional account may be considered as a sequel to the thesis of Mr. DE SITTER. This thesis, maintained by him at Groningen on 17 May 1901, bears the title: *Discussion of Heliometer-observations of Jupiter's satellites made by* sir DAVID GILL K.C.B. *and* W. H. FINLAY M.A. Further particulars will be given in the *Annals of the Royal Observatory at the Cape of Good Hope*.

(6) In 1833—39 BESSEL, at the Heliometer, measured not only distances of all the satellites from both limbs of the planet, but also angles of position of the centre of the planet to III and IV.

His heliometer was the first big instrument of the sort made in the establishment of FRAUNHOFER; the objective had an aperture of 70·2 Par. lines and a focal distance of 1131·4 Par. lines = 7 feet 10 inches 3·4 lines, Paris measure, (15·84 and 255·22 c.M.). The mean error of a single observation of distance (which properly was the mean of eight pointings) appeared to be

for I	$\pm 0''26$ ,	for the mean distance resulting from all the measures,	$\pm 0''055$
„ II	$\pm 0''24$ ,	„ „ „ „ „ „ „ „ „	$\pm 0''067$
„ III	$\pm 0''31$ ,	„ „ „ „ „ „ „ „ „	$\pm 0''042$
„ IV	$\pm 0''43$ ,	„ „ „ „ „ „ „ „ „	$\pm 0''045$
Mean.	$\pm 0''31$ ,	„ „ „ „ „ „ „ „ „	$\pm 0''052$

SCHUR, at Göttingen, used the heliometers which were made by MERZ at München for the observation of the transits of Venus in 1874 and 1882. The aperture of the objectives of these instruments was 34 Par. lines, something less than half that of the heliometer of Königsberg; the focal distance was  $3\frac{1}{2}$  feet (113·7 c.M.).

At these heliometers the reading, instead of being made on the drums of two micrometers, was made by a microscope at right angles to two scales fitted to the two halves of the objective. As however in this way more time was required than for reading the drums of a micrometer of BESSEL's instrument, SCHUR, instead of taking the mean of eight pointings, was content with the mean of four pointings, which also make a complete measurement.

The mean errors of each observation obtained by SCHUR for a complete set of four measures was:

for I	$\pm 0''34$ ,
„ II	$\pm 0''44$ ,
„ III	$\pm 0''37$ ,
„ IV	$\pm 0''42$ ,
Mean:	$\pm 0''39$ ,

a result, which, taking into account the shorter focal distance, may be considered fairly good. BESSEL as well as SCHUR aimed not so much at the determination of the position of the orbits of the satellites as at that of the mass of Jupiter.

SCHUR improved in different respects the reduction of the observations of the measures made by BESSEL. In consequence, the mean errors of the single determinations of BESSEL were considerably lessened. The numbers quoted just now, became:

$$\begin{aligned} \text{for } I &\pm 0''\cdot21, \\ \text{,, } II &\pm 0\cdot10, \\ \text{,, } III &\pm 0\cdot26, \\ \text{,, } IV &\pm 0\cdot30, \\ \hline \text{Mean: } &\pm 0''\cdot24. \end{aligned}$$

As has been mentioned already, GILL and FINLAY, acting on a suggestion formerly made by OTTO STRUVE<sup>1)</sup>, did not measure the distances and the angles of position of the satellites relative to the centre of the planet, but relative to each other. (The instrument at their disposal, a heliometer of REPSOLD, aperture  $7\frac{1}{2}$  inch = 19·05 cm., focal distance somewhat over 2 Meter, far surpassed in perfection all the instruments used up to that time). These observations can be made with much more precision. The drawback is that the formation of the equations of condition and their solution become more complex and absorb much more time. Both the gentlemen named and Mr. de SITTER have not been deterred by this consideration. They found  $\pm 0''\cdot087$ , a number considerably less than that of BESSEL, for the probable error of the measurement of a single distance. Mr. de SITTER even finds that the probable error of the mean distances (the real unknown quantities) does not exceed  $\pm 0''\cdot020$  or  $\pm 0''\cdot021$ .

(7) It may be remarked that Mr. de SITTER found it expedient to alter the choice of the unknown quantities. He retained for each satellite: the longitude in the orbit, the inclination and the ascending node relative to an adopted position of the fixed plane, but not the eccentricity nor the position of the perijovium and the mass. There thus remained as unknown quantities only three elements of each satellite. On the other hand he introduced corrections of the coefficients of the perturbations or rather of the periodic terms, which afterwards must lead to the knowledge of the mass of the satellites, to that of the eccentricities and of the position of the

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<sup>1)</sup> *Vide* the first report of HERMANN STRUVE, in the first supplementary vol. of the Pulkowa observations, 1st page at the bottom.



apsides. He further introduced two unknown quantities, *viz.* the constant errors which might vitiate the observations of the two observers GILL and FINLAY. He thus also obtained a total of 29 unknown quantities. It need not be said that the solution of about 400 equations with so many unknown quantities, is an enormous labour. Still, owing to the help of some other computers, this labour has been brought to a happy issue.

We must not enter here into further particulars about this important work, though we did not feel justified in omitting to mention it altogether. I will only remark that it is not sufficient to determine the position of the planes of the orbits of the satellites for one epoch; for as was already remarked the position of these planes changes continually. It seems that these changes may be sufficiently represented by assuming a regular retrogradation of the line of intersection with a fixed plane, the inclination remaining the same.

The main cause of this retrogradation is the polar compression of Jupiter. It is desirable however to establish the amount of this retrogradation by the observations, and to derive afterwards the compression by means of this amount. Consequently the position of the planes of the orbits has to be determined for different epochs. In this respect too Mr. DE SITTER has done good work, *vide* the communication already mentioned, presented in the meeting of last March by Messrs KAPTEYN and E. F. VAN DE SANDE BAKHUYZEN.

(8) The same volume, which contains the ecliptic tables of DAMOISEAU, contains also in a second part (not mentioned on the title) tables "*pour trouver les configurations des satellites de Jupiter.*"

We have contemplated whether it would not be desirable not to use these tables, unmodified, for our computations. We have therefore taken note of the investigations of SOUILLART, ADAMS, MARTH, GILL, FINLAY, and de SITTER, but it appeared that such a course would aggravate our labour very considerably. We would have had to determine new elements for all the satellites and to compute new tables. This would have caused considerable retardation, unnecessary for our purpose, which was no other than to prepare astronomers for the observation of the conjunctions visible in 1908.

We therefore have based our computations on the tables of DAMOISEAU, but we have first examined in how far they represent the observed conjunctions. The following summary shows not only the difference between the observation and the tables in the elongations  $x$  and  $x'$ , of the two satellites, expressed in radii of Jupiter, but also their difference in time.

Observer	Date	Occul- tation of	by	Error		Relative hourly motion	Corr. of Table	$y'-y$
				r	R			
Fauth	1902 Oct 7	II <sub>f</sub>	I <sub>n</sub>	0,025	1,278	+ 1,2	+ 0,04	
»	» » 23	III <sub>f</sub>	II <sub>n</sub>	0,02	1,130	+ 1,1	+ 0,08	
»	» Nov. 10	I <sub>f</sub>	III <sub>n</sub>	0,00	0,883	0,0	+ 0,13	
»	» Dec. 24	IV <sub>f</sub>	I <sub>n</sub>	0,10	1,089	- 5,5	+ 0,00 <sup>s</sup>	
»	1903 Jan. 14	II <sub>n</sub>	III <sub>n</sub>	0,11	0,344	-19,2	- 0,05	
Njland	1902 July 15	III <sub>f</sub>	II <sub>n</sub>	0,08	0,751	+ 6,4	- 0,01	
Stanley Williams	» » »	»	»	0,07	0,751	+ 5,3	- 0,01	
»	1885 March 27	I <sub>n</sub>	III <sub>n</sub>	0,00	0,292	0,0	+ 0,01	

The observation of LUTHER in Hannover, of Octob. 30, 1822 is not contained in this table. Its calculation yields the result :

	Jovic. Long.,	Amplitude	$x$	$y$ en $y'$
III	10 <sup>s</sup> 26°·77	8 <sup>s</sup> ,22°·25	- 15·21	+ 0·18
IV	9 6·74	7 2·22	- 14·40	- 0·74
		Difference	+ 0·81	+ 0·92

So there is a difference in the amplitudes, of  $0·81 = 0·81 \times 18''·37 = 14''·9$ , in the latitudes, of  $0·92 = 16''·9$ . Probably the observation has been made with an unsatisfactory instrument, for it is impossible to suppose an error of this amount in the tables of DAMOISEAU for 1822. The difference in sign of the latitudes  $y$  and  $y'$  is explained by the fact that the longitude of the ascending node of the fixed plane was  $10^s14^o·37$ , which is intermediate between the two jovicentric longitudes.

As the two satellites moved in the same direction, the hourly change of distance was small, *viz.*  $0·280$ . It would thus require nearly three hours to annul the difference of  $0·81$ .

The remaining conjunctions, however, show a satisfactory accuracy and we may thus expect that the table, as given below, will serve its purpose.

As a second test I have computed, by the aid of the second part of DAMOISEAU, the two superior conjunctions and the intermediate inferior conjunctions of II, and I have compared these to those given

in the *Nautical Almanac* of 1902. The epochs were found a little earlier, to wit:

superior conjunction of	10 July,	10 <sup>h</sup> 46 <sup>m</sup> .9	M.T. Grw.	0 <sup>m</sup> .7	earlier
inferior conjunction (mean					
of ingress and egress)	16 July	5 <sup>h</sup> 40 <sup>m</sup> .0	„ „	0 <sup>m</sup> .3	„
superior conjunction	17 July	23 <sup>h</sup> 54 <sup>m</sup> .4	„ „	0 <sup>m</sup> .7	„

all three less than a minute.

Now, as the conjunctions in the *Nautical Almanac* have been calculated by the aid of DAMOISEAU's *tables écliptiques* (making allowance for some slight corrections indicated by ADAMS) the differences must be solely due to the fact that in DAMOISEAU's second part the *main* terms only of the equations and perturbations have been taken into account.

The same tables represent as accurately the superior conjunction of I on January 1, 1908, 14<sup>h</sup>4<sup>m</sup>.2 M.T. Grw. = January 2, 2<sup>h</sup>13<sup>m</sup>.55 civil time of Paris; the error amounts to 0<sup>o</sup>.07 or 0<sup>o</sup>.01 linear measure only, an arc traversed by the satellite in 0<sup>m</sup>.5.

(On the terms taken into account in the second part of the tables of DAMOISEAU *vide* 3<sup>rd</sup> appendix below).

In his letter Mr. STANLEY WILLIAMS mentions another rare observation, made as well by himself as by the Spanish observer J. COMAS of Valls, (near Taragona), on 14 August 1891, to wit of the coincidence and of the subsequent separation of the shadows of two satellites on the planet. He concludes that an eclipse must have taken place. These phenomena will be treated in the second part of this communication.

(9) Below follows the table which has served for this computation. The unit, the radius of Jupiter, is 18<sup>''</sup>.37. SOUILLART states that he found mentioned in the papers of DAMOISEAU that this number was borrowed from ARAGO. According to HOUZEAU, ARAGO must have made the determination by means of the double image micrometer (an invention made nearly simultaneously by himself and PEARSON; of the latter the observatory at Utrecht possesses a specimen). Particulars about these measures are not known. The number is smaller than that found by other astronomers, *vide* for instance HOUZEAU, p. 647—650; SEE, *Astron. Nachr.* N<sup>o</sup>. 3670 (15 Aug. 1900).

Hourly change of the elongation  $x$  as a function of the amplitude

		+			I		II		III		IV		-			
0 <sup>s</sup>	0 <sup>c</sup>	0 <sup>s</sup> (12)	0 <sup>c</sup>	r	r	r	r	r	r	6 <sup>s</sup>	0 <sup>c</sup>	6 <sup>s</sup>	0 <sup>m</sup>	6 <sup>s</sup>	0 <sup>m</sup>	
			0	0,895	4	0,708	3	0,560	2	0,420	2					
	5	11	25	0,891	10	0,705	8	0,558	7	0,418	4	5	5	25		
	10	11	20	0,881	17	0,697	14	0,551	10	0,414	8	10	5	20		
	15	11	15	0,864	23	0,683	18	0,544	15	0,406	11	15	5	15		
	20	11	10	0,841	30	0,665	23	0,526	18	0,395	14	20	5	10		
	25	11	5	0,811	36	0,642	29	0,508	23	0,381	17	25	5	5		
1	0	11	0	0,775	42	0,613	33	0,485	26	0,364	20	7	0	5	0	
	5	10	25	0,733	48	0,580	37	0,459	30	0,344	22	5	4	25		
	10	10	20	0,685	53	0,543	42	0,429	33	0,322	25	10	4	20		
	15	10	15	0,632	57	0,501	46	0,396	36	0,297	27	15	4	15		
	20	10	10	0,575	62	0,455	49	0,360	39	0,270	29	20	4	10		
	25	10	5	0,513	66	0,406	52	0,321	41	0,241	31	25	4	5		
2	0	10	0	0,447	69	0,354	55	0,280	43	0,210	32	8	0	4	0	
	5	9	25	0,378	72	0,299	57	0,237	45	0,178	34	5	3	25		
	10	9	20	0,306	75	0,242	59	0,192	47	0,144	35	10	3	20		
	15	9	15	0,231	76	0,183	60	0,145	48	0,109	36	15	3	15		
	20	9	10	0,155	77	0,123	61	0,097	48	0,073	36	20	3	10		
	25	9	5	0,078	78	0,062	62	0,049	49	0,037	37	25	3	5		
3	0	9	0	0,000		0,000		0,000		0,000		9	0	3	0	

Finally we will give below, *vide* pp. 334 and 335, two instances of computation; one of a case in which the apparent motion of the two satellites was opposed, the other in which it was in the same direction.

1<sup>st</sup> Appendix. *What is the maximum duration of the several occultations of one satellite by another?*

We have seen above that it took 19<sup>m</sup>.2 to annul the small difference of the elongations of 0'.11 (2".0). This was caused by the minuteness of the relative motion of the satellites. But in the case that the hourly motions, which we will denote by  $u$  and  $u'$ , are *absolutely* equal, the denominator of the fraction  $\frac{u' - u}{u' - u}$  is zero.

The case then corresponds to that of the "Station of Venus" and it is a very ancient problem to compute its epochs.

Let be  $r$  and  $r'$  the radii vectores of two satellites;  $\theta$  and  $\theta'$  the corresponding amplitudes, then for the occultation :

$$r \sin \theta = r' \sin \theta'.$$

The condition of an equal change of longitude leads to :

$$r \cos \theta \frac{d\theta}{dt} = r' \cos \theta' \frac{d\theta'}{dt}.$$

Now, if  $T$  and  $T'$  represent the sidereal periods, we have, neglecting the apparent movement of Jupiter :

$$\frac{d\theta}{dt} : \frac{d\theta'}{dt} = \frac{1}{T} : \frac{1}{T'} = \frac{1}{r^{3/2}} : \frac{1}{r'^{3/2}},$$

consequently :

$$r^{-1/2} \cos \theta = r'^{-1/2} \cos \theta',$$

from which :

$$\cos^2 \theta = \frac{r}{r'} \cos^2 \theta' = \frac{r}{r'} - \frac{r}{r'} \sin^2 \theta'.$$

Adding

$$\sin^2 \theta = \frac{r'^2}{r^2} \sin^2 \theta',$$

we get

$$1 = \frac{r}{r'} + \left\{ \left( \frac{r'}{r} \right)^2 - \frac{r}{r'} \right\} \sin^2 \theta';$$

Therefore, putting  $\frac{r'}{r} = \mu$ ,

$$\sin^2 \theta' = \frac{1 - \frac{1}{\mu}}{\mu^2 - \frac{1}{\mu}} = \frac{\mu - 1}{\mu^3 - 1} = \frac{1}{\mu^2 + \mu + 1}$$

and

$$\sin^2 \theta = \frac{\mu^2}{\mu^2 + \mu + 1}.$$

The equality of the hourly changes of the two elongations of course only lasts for an instant; very soon inequality sets in and the two satellites begin to separate. Meanwhile it may be long ere such becomes perceptible at the telescope, only, in a case like the present, the satellites do not pass each other, but after the conjunction they have the same position the one to the other as before.

As an example take a conjunction of I and II under the circum-

stances in question. Let the amplitudes be between 0 and 3 signs, so that both the satellites, as seen from the Earth, (the head being turned to the North Pole), are to the left of and both receding from the planet. Before the conjunction I is to the right of II, but the motion of I is quicker than that of II. I will overtake II as soon as its amplitude is  $44^{\circ}39'$ , that of II being then  $26^{\circ}14'$ . At the same time, however, the apparent velocities are equal. Now as I approaches its greatest elongation it retards its motion much more considerably than II, the amplitude of which is so much smaller. The consequence is that, after the conjunction, I is left behind, and gets again to the right of II as before conjunction.

This case represents a transition between two other cases. 1. If, under the same circumstances I is somewhat more in advance (has a greater amplitude), it will pass II, but after a while will be overtaken by II, which then, as seen from the Earth, passes behind it. 2. If, however, I is somewhat less ahead, it will continue to be seen to the right of II, the distance I — II going through a minimum but not reaching zero.

Now, in order to answer the question, how long will be the duration of the occultation counted from the first external contact, the apparent radii of the satellites must be known. Owing to the irradiation they are greater at night than in daytime<sup>1)</sup> as several observers have actually found. The observations of the satellites of Jupiter being made nearly exclusively at night time, we will adopt the apparent radii holding for the night. I took the mean of the values found by SEE at the giant telescope at Washington on the one hand and that found by several observers on the other. (I have taken the values as summarised by SEE himself). For the reduction to the unit used throughout for these computations, *viz* the radius of the equator of Jupiter, this radius is taken =  $18''\cdot37$  in accordance with DAMOISEAU.

	Diameter	Radius
I	$1''\cdot07 = 0\cdot058$	$0\cdot029$
II	$0\cdot95 \quad 0\cdot052$	$0\cdot026$
III	$1\cdot56 \quad 0\cdot085$	$0\cdot0425$
IV	$1\cdot41 \quad 0\cdot076$	$0\cdot038$

<sup>1)</sup> Vide e.g. T. J. J. SEE, Observations of the Diameters of the Satellites of Jupiter, and of Titan, the principal Satellite of Saturn, made with the 26 inch Refractor of the U. S. Naval Observatory, Washington; 19 Oct. 1901. *Astr. Nachrichten* No. 3764, (21 Jan. 1902).

Therefore;

	Sum of the diameters	Sum of the radii
I + II . . .	0·110	0·055
I + III . . .	0·143	0·0715
I + IV . . .	0·134	0·067
II + III . . .	0·137	0·0685
II + IV . . .	0·128	0·064
III + IV . . .	0·161	0·0805

For the mean radii vectores we will take two figures more than did DAMOISEAU in his tables, and we will adopt for the purpose the values found by SOUILLART in DAMOISEAU's papers, (SOUILLART, second paper, Mémoires présentés par divers savants a l'Académie des Sciences, Tome XXX, 2<sup>me</sup> Série, 1889; p. 10) <sup>1)</sup>.

I	6·0491,
II	9·6245,
III	15·3524,
IV	27·0027.

The result of our computation is, that the time between the first contact and the central occultation is:

for I and II I and III I and IV II and III II and IV III and IV  
 1<sup>h</sup>·324, 1<sup>h</sup>·245, 1<sup>h</sup>·103, 2<sup>h</sup>·263, 1<sup>h</sup>·774, 3<sup>h</sup>·725;

between the central occultation and the second contact:

1<sup>h</sup>·204, 1<sup>h</sup>·161, 1<sup>h</sup>·059, 2<sup>h</sup>·190, 1<sup>h</sup>·767, 3<sup>h</sup>·725,

therefore in all

2<sup>h</sup>·528, 2<sup>h</sup>·406, 2<sup>h</sup>·162, 4<sup>h</sup>·453, 3<sup>h</sup>·541, 7<sup>h</sup>·450,

or

2<sup>h</sup>32<sup>m</sup> <sup>2)</sup> 2<sup>h</sup>24<sup>m</sup>, 2<sup>h</sup>10<sup>m</sup>, 4<sup>h</sup>27<sup>m</sup>, 3<sup>h</sup>32<sup>m</sup>, 7<sup>h</sup>27<sup>m</sup>.

Still even these numbers do not represent the maximum of the time during which the two satellites may be seen as a single body. For we can imagine the case that the shortest distance becomes equal to  $-(r + r')$ , i. e. that between two central conjunctions there

<sup>1)</sup> According to SOUILLART, DAMOISEAU derived these numbers in the following way: He adopted the mean distance of IV, in accordance with POUND's determination = 496"0, and took 18"·37 for Jupiter's semidiameter, so that, by division  $r_{IV} = 27·00102834$ . The mean distances of the other satellites were then derived from the sidereal periods by the application of KEPLER's third law. But to these mean distances he added the constant terms produced in the radii vectores by the perturbing force.

I beg leave to remark that 496"0 : 18"·37 is not 27·00102834 but 27·000544366. Happily the 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> figure have no appreciable influence on our computations, nor probably on those of SOUILLART. For the rest the 2<sup>nd</sup> appendix, further below, may be consulted on such numbers of many decimals.

<sup>2)</sup> On June 4, 1908, such a conjunction must take place according to our computation. *Vide* the table further below.

occurs a contact on the other side. In this case the duration will, very nearly indeed, have to be multiplied by  $\sqrt{2}$ . It thus becomes for I and II, I and III, I and IV, II and III, II and IV, III and IV

3<sup>h</sup>.574, 3<sup>h</sup>.402, 3<sup>h</sup>.057, 6<sup>h</sup>.296, 5<sup>h</sup>.006, 10<sup>h</sup>.43,

or:

3<sup>h</sup>34<sup>m</sup>, 3<sup>h</sup>24<sup>m</sup>, 3<sup>h</sup> 3<sup>m</sup>, 6<sup>h</sup>18<sup>m</sup>, 5<sup>h</sup> 0<sup>m</sup>, 10<sup>h</sup>26<sup>m</sup>.

These numbers hold only for those very rare occasions in which 1<sup>st</sup>. the occultation is central and 2<sup>nd</sup>. the rate of change of the elongation is equal or nearly so for the two satellites. As soon as there is some difference of latitude the time during which the two satellites are seen as a single body is of course smaller.

2<sup>nd</sup>. Appendix. *Investigation of the uncertainty, existing in the determination of the synodic periods of the satellites.*

In his introduction to the *Tables Ecliptiques*, p. XIX, DELAMBRE says: "Nous n'avons aucune observation d'éclipse antérieure à 1660". Now let us assume that the difference in time between the first eclipse observed in 1660 and the last observed in 1816, two years before the publication of these tables, (taking into account also the next ones in 1660 and the preceding ones in 1816) leaves an uncertainty, in the case of the four satellites, of 20, 30, 40 and 60 seconds, which will be too favourable rather than too unfavourable. If we divide this uncertainty by the number of synodic periods in 156 years, to wit 32193, 16032, 7951 and 3401, we get for the uncertainty of a single period

for I	for II	for III	for IV
0 <sup>s</sup> .00062,	0 <sup>s</sup> .00188,	0 <sup>s</sup> .0050,	0 <sup>s</sup> .0176.

Therefore, if we find that DELAMBRE gives these periods to 9 places of decimals of the second, we cannot attach much importance to the fact.

When DAMOISEAU, 20 years after DELAMBRE, published new eclipse-tables<sup>1)</sup> for the satellites of Jupiter, he adopted the period of I un-

<sup>1)</sup> The tables of DELAMBRE and DAMOISEAU were destined mainly to serve for the prediction, in the astronomical ephemerides, of the eclipses of the satellites caused by the shadow of Jupiter. It is for this reason that both he and DELAMBRE, united all those terms of the perturbations in longitude which have the same argument at the time of the opposition of the satellites, even though these arguments might be different for all other points in the orbit. Therefore it becomes necessary once more to separate these terms as soon as tables have to be computed from which may be derived the longitude and the radii vectores of the four satellites for any point of their orbits, tables such as have been given by BESSEL in his *Astronomische Untersuchungen* and by MARTH in the *Monthly Notices of the Royal Astronomical Society*, Vol. LI, (1891).



changed, but applied the following corrections to the remaining ones :

$$\text{II} + 0^{\circ}.005\ 127\ 374,$$

$$\text{III} + 0^{\circ}.029\ 084\ 25,$$

$$\text{IV} - 0^{\circ}.092\ 654\ 834,$$

the amount of which is even respectively nearly 3, nearly 6 and somewhat over 5 times that of the uncertainties derived just now. But even if we increase the number of intervening years from 156 to 176, our estimated uncertainties are only diminished by about  $\frac{1}{9}$  of their amount. We thus conclude that these periods can only be considered to be determined with certainty :

that of I to 3 decimals                      of the second  
 „ „ II, III and IV to 2 decimals „ „ „ .

The Nautical Almanac, which, where it gives the superior conjunctions of the satellites, gives also the synodic periods, wisely confines itself to three decimals. The use of 9 decimals may therefore provisionally be taken for astronomical humbug. Some other instances of the same kind might be quoted *e.g.* the formerly well known constants,  $20''.4451$  for the aberration and  $8''.57116$  for the parallax of the sun!

3<sup>rd</sup> Appendix. *Meaning of the equations taken into account in the 2<sup>nd</sup> part of the tables of DAMOISEAU.*

On p. 321 we have referred to the 3<sup>rd</sup> appendix for information as to the equations which have been taken into account for each satellite in the second part of the tables of DAMOISEAU. We will now supply this information; we will denote by  $U$ ,  $u_0$ ,  $u_I$ ,  $u_{II}$ ,  $u_{III}$  and  $u_{IV}$  the mean longitudes of the sun, of Jupiter and of the four satellites; by  $\pi_0$  the longitude of the perihelium of Jupiter, by  $\pi'$  that of the Earth, by  $\pi_{III}$  and  $\pi_{IV}$  the perijovia of III and IV; by  $\Pi$  the longitude of the ascending node of Jupiter's equator on its orbit; finally by  $A_{II}$ ,  $A_{III}$  and  $A_{IV}$  the longitudes of the ascending nodes of II, III and IV each on its own fixed plane.

In order to be able to supply the data following below we have taken the daily motion of the argument of each equation from the tables in the second part of DAMOISEAU. This amount was then multiplied by the synodic period expressed in days; the product thus obtained was then compared with the factor by which, in the first

part, p.p. (III), (V), (VII) and (VIII) the letter  $i$  (the number of synodic periods) is multiplied.

These daily motions are so nearly equal for several of the equations of II, III and IV that, in order to make them out, we must take from the tables the motions for a long interval, *e.g.* for 10 years, (duly taking into account the number of periods). These must then be divided by the number of days (10 years = 3652 or 3653 days). Multiplying this quotient by the synodic period in days, we get  $360^\circ +$  a fraction. The  $360^\circ$  are of no account; the fraction is the factor of  $i$ ; we thus recognise which is the equation we have to deal with. In the preface of the second part of DAMOISEAU we look in vain for any information on the subject.

I. For this satellite five terms have been taken into account.

N<sup>o</sup>. 1 with an amplitude of  $1^\circ.16$ , is the equation of the velocity of light; its argument is  $U - u_0$ .

N<sup>o</sup>. 2, (amplitude  $0^\circ.29$ ), is the equation caused by the ellipticity of Jupiter's orbit; the argument is the mean anomaly of Jupiter  $u_0 - \pi_0$ .

N<sup>o</sup>. 3 is  $180^\circ +$  the mean anomaly of the Earth,  $U - \pi'$ ; by its aid and that of N<sup>o</sup>. 1 *i.e.* the difference in longitude between the Sun and Jupiter, we find, in the table of double entry IX, one term of the geocentric latitude of the satellite.

N<sup>o</sup>. 4 with an amplitude of  $0^\circ.45$ , shows the perturbation caused by II in the motion of I. The argument is  $u_I - u_{II}$ .

N<sup>o</sup>. 5, (amplitude  $3^\circ.07$ ) gives the joventric latitude of I, necessary to find the second term of the geocentric latitude. The argument is  $u_I - A_I$ .

II. Seven terms. N<sup>o</sup>. 1, 2 and 3 have the same arguments as the analogous terms for I; the amplitudes of N<sup>o</sup>. 1 and 2 are half those of I. The term of the latitude to be taken from IX, by the aid of 1 and 3, is of course the same for all the satellites.

N<sup>o</sup>. 4, (amplitude  $1^\circ.06$ ), shows the perturbation caused by III in the motion of II. The argument is  $u_{II} - u_{III}$ .

N<sup>o</sup>. 5, 6 and 7 serve for the latitude.

N<sup>o</sup>. 5, (amplitude  $3^\circ.05$ ), has the argument  $u_{II} - \pi_{IV}$ ;

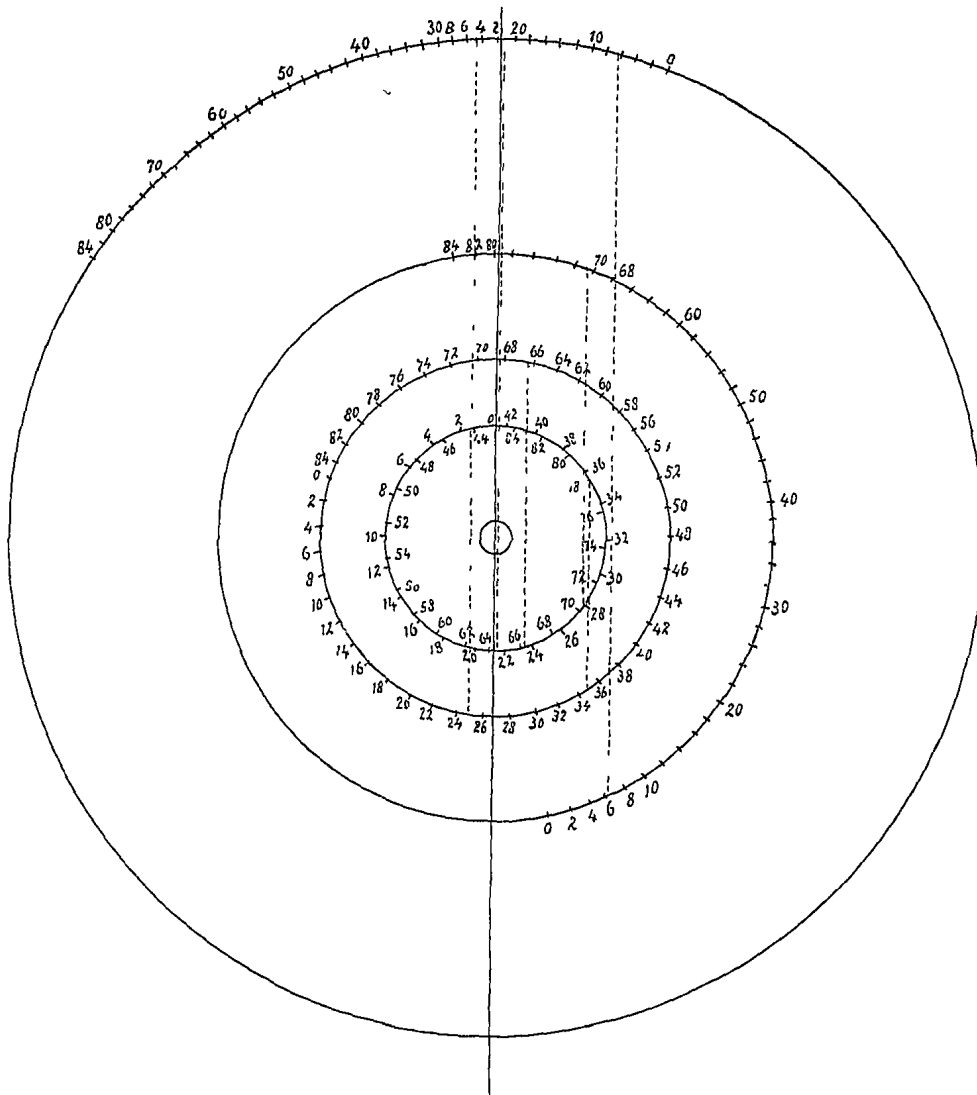
N<sup>o</sup>. 6, ( „ 0.47), „ „ „  $u_{II} - A_{II}$ ;

N<sup>o</sup>. 7. ( „ 0.03), „ „ „  $u_{II} - A_{III}$ .

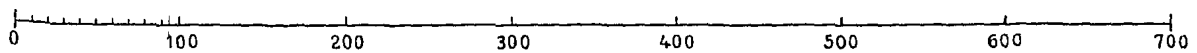
III, Nine terms. Nos. 1, 2 and 3 are the same as for I and II; the amplitudes of N<sup>o</sup>. 1 and N<sup>o</sup>. 2 are  $0^\circ.29$  and  $0^\circ.07$ .

J. A. C. OUDEMANS. „Mutual Occultations and Eclipses of the Satellites of Jupiter in 1908.”

Starting point: the geocentric superior conjunction of I on July 12, 1908,  
 at 11<sup>h</sup> 2<sup>m</sup>.3 M. T. Greenwich = 11<sup>h</sup> 11<sup>m</sup>.65 M. T. Paris = 23<sup>h</sup> 11<sup>m</sup>.65 civil time Paris.



Scale  $\frac{1}{30\ 168\ 000\ 000}$ ; 1mm. = 8'' heliocentric.



Unit: the radius of the earth's equator

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N<sup>o</sup>. 4, (amplitude 0°·07), has the same argument as N<sup>o</sup>. 4 for II, but it now shows the perturbation caused by II.

N<sup>o</sup>. 5, (amplitude 0°·15), is the equation of the centre; argument  $u_{III} - \pi_{III}$ .

N<sup>o</sup>. 6, (amplitude 0°·04), has the argument  $u_{III} - \pi_{IV}$ , it thus must account for a perturbation in III depending on the longitude of the perijovium of IV.

Nos. 7, 8 and 9, with the amplitudes 2°·98, 0°·18 and 0°·03, serve for the latitude. The arguments are respectively  $u_{III} - \pi$ ,  $u_{III} - \Delta_{III}$  and  $u_{III} - \Delta_{IV}$ .

IV. Seven terms.

Nos 1, 2 and 3 are similar to those of the preceding satellites.

N<sup>o</sup> 4, (amplitude 0°·83), is the equation of the centre, argument  $u_{IV} - \pi_{IV}$ .

Nos 5, 6 and 7 serve for the latitude. N<sup>o</sup> 5, (amplitude 2°·64) depends on the mean anomaly of Jupiter; its argument therefore is  $u_0 - \pi_0$ .

N<sup>o</sup> 6, (amplitude 0°·24), depends on the argument of the latitude of the satellite itself; argument  $u_{IV} - \Delta_{IV}$ .

N<sup>o</sup> 7, (amplitude 0°·04), is a minute perturbation, caused by III; its argument is  $u_{IV} - \Delta_{III}$ .

Now in regard to the following table of the computed conjunctions.

The first column contains the ordinal numbers.

The second shows the epoch of the conjunction, accurate to the nearest minute, expressed in civil time of Paris. This time is reckoned from midnight and has been used by DAMOISEAU in his tables; it thus represents the direct result of our computations. In the cases that the computed time was just a certain number of minutes and a half, the half minute has been set down. By subtracting 12<sup>h</sup> 9<sup>m</sup> or, where necessary, 12<sup>h</sup> 9<sup>m</sup>·35, the mean time of Greenwich was found, which is contained in the third column.

The 4<sup>th</sup> and the 5<sup>th</sup> column contain the numbers of the occulted and the occulting satellite. The appended letters *f* and *n* show whether the satellite is *far* or *near* (vide supra p. 304). The satellite is *far* if its amplitude is between 9<sup>s</sup> and 3<sup>s</sup>, *near* if it is between 3<sup>s</sup> and 9<sup>s</sup>. Furthermore *ee* denotes an eastern elongation, for which the amplitude is about 3<sup>s</sup> and *we* a western elongation, for which the amplitude differs little from 9<sup>s</sup>.

At the conjunction the elongations, counted along the orbit of Jupiter, are equal; they are to be found in the next column. If the elongation is  $+$ , the satellite, as seen by a northern observer, using a terrestrial telescope, will be to the left of the planet. Therefore if he uses an inverting telescope, as is the rule for the observation of the heavenly bodies, he will see it to the right.

The three following columns contain the ordinates of the two satellites and their difference; northerly latitudes are positive. The tenth column shows the duration, which the eclipse would have, if the conjunction were central. In a few cases (Nos. 20, 23, 30, 48, 53 and 64), we find  $y' = y$ , consequently  $y' - y = 0$ . If the tables were correct these conjunctions would be central. But in testing the tables by the conjunctions observed by Messrs FAUTH, NIJLAND and STANLEY WILLIAMS the difference of the  $y$ 's did not completely agree with the observations and even a small difference may considerably change the duration of any eventual occultation. Therefore, not to fill a column with figures, which, likely enough, may be contradicted by the observations, I omitted the value found by calculation for the true duration.

We remarked before (p. 308) that, if at all, any conjunction will be visible at a determined place of observation only for a short time, *viz.* between sunset and the setting of Jupiter. As a consequence the list will be of little use, unless observatories distributed over the whole of the earth cooperate in the work. The last column was added as a help to such cooperation. It contains on every line an observatory, at which the conjunction of that line will be visible. It is certainly desirable that other astronomers also, at observatories in the vicinity, examine whether the phenomenon will be visible, and, if so, prepare for its observation.

R E S U L T S.

Geocentric conjunctions of two satellites in June and July 1908.

No.	Civil time at Paris	Mean time at Greenwich	$\begin{matrix} n = \text{near} \\ f = \text{far} \end{matrix}$		$x = x'$	Occulted satellite $y$	Occulting satellite $y'$	$y' - y$	Duration of central occultation	visible at
			Occulted satellite	Occulting satellite						
1	1 June 5h55m	31 May 17h46m	$I_n$	$II_n$	+2r18	-0r16	-0r12 <sup>s</sup>	+0,03 <sup>s</sup>	45m	Mt. Hamilton
2	1 » 13 25	1 June 6 16	$I_{we}$	$II_n$	-6,03	+0,31	+0,29	-0,02	11	Cape
3	2 » 1 55	1 » 13 46	$I_f$	$IV_n$	-3,15	+0,18	-0,12	-0,30	7	Washington
4	2 » 13 16	2 » 1 7	$III_f$	$II_f$	-8,51	+0,51	+0,38	-0,13	60	Madras, H. Kc
5	2 » 14 0	2 » 1 51	$III_f$	$IV_n$	-8,16	+0,51	+0,13	-0,38	11	Madras, H. Kc
6	2 » 14 4,5	2 » 1 55	$II_f$	$IV_n$	-8,20	+0,37	+0,13	-0,24	10	Madras, H. Kc
7	2 » 21 10	2 » 9 1	$III_f$	$II_f$	-4,54	+0,31	+0,20	-0,11	41	Utrecht
8	3 » 3 15	2 » 15 6	$II_f$	$I_n$	-0,33 <sup>s</sup>	0,00	-0,03	-0,03	4	} At the sametime $\sigma'$ :
9	3 » 3 50	2 » 15.41	$III_f$	$I_n$	-0,37	+0,15	0,00	-0,15	6	
10	4 » 14 46	4 » 2 37	$I_n$	$II_n$	$x = +5,01^s$ $x' = +5,07$	-0,29	-0,25 <sup>s</sup>	+0,03 <sup>s</sup>	} Two contacts at the same side. II just reaches central conjunction but then retraces its steps.	
"	4 » 16 0	4 » 3 51	"	"	4,34					
"	4 » 17 15	4 » 5 6	"	"	$x = +3,54$ $x' = +3,48^s$	-0,23	-0,19 <sup>s</sup>	+0,03 <sup>s</sup>		
The satellites will be visible as one body during nearly 2½ hours.										
11	5 » 7 51	4 » 19 42	$I_{we}$	$II_f$	-6,05	+0,33	+0,28	-0,05	12	Sydney
12	6 » 16 24	6 » 4 15	$II_f$	$I_n$	-0,48	0,00	-0,01	-0,01	4	} At the sametime $\sigma'$ and II eclipse Mt. Hamilton
13	6 » 18 11,5	6 » 6 2	$II_f$	$III_n$	+0,82	-0,06	-0,17	-0,11	6	
14	9 » 5 6	8 » 16 57	$IV_f$	$II_f$	-9,19	+0,59	+0,43	-0,16	13	} At the sametime $\sigma'$
15	10 » 4 26	9 » 16 17	$IV_f$	$I_n$	+0,29	+0,14	-0,05 <sup>s</sup>	-0,19 <sup>s</sup>	6	
16	10 » 5 23	9 » 17 14	$II_f$	$I_n$	-0,58	+0,02	-0,01	-0,03	5	} At the sametime $\sigma'$
17	10 » 6 28	9 » 18 18,5	$III_f$	$I_n$	-1,54	+0,15	+0,05	-0,10	6	
18	11 » 3 0	10 » 14 51	$IV_f$	$III_f$	+9,01	-0,32	-0,52	-0,20	133	Washington
19	12 » 10 34	11 » 22 25	$I_{we}$	$II_n$	-6,03	+0,32	+0,28	-0,04	11	Wellington
20	13 » 18 46,5	13 » 6 37	$II_f$	$I_n$	-0,82	+0,01 <sup>s</sup>	+0,01 <sup>s</sup>	0,00	4	$\sigma' \text{ } \text{?}$
21	13 » 21 37	13 » 9 28	$II_f$	$III_n$	+1,31	-0,07	-0,16	-0,09	6,5	II eclipsed (Utrecht)
22	15 » 23 55	15 » 11 46	$I_{we}$	$II_n$	-5,99	+0,32	+0,29	-0,03	10	(Atl. Ocean)
23	17 » 7 53	16 » 19 44	$II_f$	$I_n$	-0,92	+0,02	+0,02	0,00	4	$\sigma' \text{ } \text{?}$

No.	Civil time at Paris	Mean time at Greenwich	$n = \text{near}$ $f = \text{far}$		$x = x'$	Occulted satellite $y$	Occulting satellite $y'$	$y' - y$	Duration of central occultation	Visible at
			Occulted satellite	Occulting satellite						
24	17 June 9h39m	16 June 21h30m	III <sub>f</sub>	I <sub>n</sub>	-2r44	+0r18	+0r10	-0r08	6m	Wellington
25	17 » 20 47	17 » 8 38	II <sub>f</sub>	IV <sub>n</sub>	+7, 33	-0, 37	-0, 52	-0, 15	9	Utrecht
26	18 » 0 33	17 » 12 24	III <sub>f</sub>	IV <sub>n</sub>	+5, 75	-0, 28	-0, 44	-0, 16	10	(Atl. Ocean)
27	18 » 7 30	17 » 19 21	I <sub>f</sub>	IV <sub>n</sub>	+2, 86	-0, 16	-0, 30	-0, 14	6, 5	Sydney
28	18 » 8 28	17 » 20 18	III <sub>f</sub>	II <sub>ee</sub>	+9, 52	-0, 51	-0, 45	+0, 06	15	Sydney
29	19 » 13 13	19 » 1 4	I <sub>we</sub>	II <sub>n</sub>	-5, 94	+0, 32	+0, 29	-0, 03	9, 5	Hong Kong
30	20 » 21 3	20 » 8 54	II <sub>f</sub>	I <sub>n</sub>	-1, 07	+0, 02	+0, 04	+0, 02	4	Utrecht
31	21 » 1 3	20 » 12 54	II <sub>f</sub>	III <sub>n</sub>	+1, 79	-0, 11	-0, 15	-0, 04	6, 5	II eclipsed
32	21 » 11 54	20 » 23 45	I <sub>f</sub>	III <sub>n</sub>	-4, 21	+0, 24	+0, 18	-0, 06	7	Hong Kong
33	23 » 2 32	22 » 14 23	I <sub>we</sub>	II <sub>n</sub>	-5, 90	+0, 32	+0, 29	-0, 03	9	Washington
34	24 » 10 13	23 » 22 4	II <sub>f</sub>	I <sub>n</sub>	-1, 22 <sup>s</sup>	+0, 03	+0, 01	-0, 02	4	Sydney
35	24 » 12 37	24 » 0 27	III <sub>f</sub>	I <sub>n</sub>	-3, 21	+0, 20	+0, 16	-0, 04	6, 5	Hong Kong
36	26 » 11 17	25 » 23 8	IV <sub>f</sub>	I <sub>n</sub>	-5, 60	+0, 35	+0, 30 <sup>s</sup>	-0, 04 <sup>s</sup>	11	Hong Kong
37	26 » 13 43.5	26 » 1 34	IV <sub>f</sub>	II <sub>n</sub>	-4, 61	+0, 31	+0, 25	-0, 06	7	Madras, H. Kong
38	26 » 15 49	26 » 3 40	I <sub>we</sub>	II <sub>n</sub>	-5, 84	+0, 32	+0, 30	-0, 02	9	Madras
39	27 » 0 27	26 » 12 18	IV <sub>f</sub>	I <sub>f</sub>	-0, 18	-0, 01	+0,	+0, 04	20	♄ ♃
40	27 » 13 44	27 » 1 35	IV <sub>f</sub>	I <sub>n</sub>	+5, 66	-0, 17	-0, 3	-0, 14	11	Madras, H. Kong
41	27 » 18 56	27 » 6 47	IV <sub>f</sub>	III <sub>n</sub>	+7, 38	-0, 20	-0, 45	-0, 25	10	Cape
42	27 » 23 24	27 » 11 14	II <sub>f</sub>	I <sub>n</sub>	-1, 38	+0, 03	+0, 07	+0, 04	4	(Atl. Ocean)
43	28 » 14 57	28 » 2 48	I <sub>f</sub>	III <sub>n</sub>	-3, 49	+0, 20	+0, 17	-0, 03	7	Madras
44	30 » 5 5.5	29 » 16 56	I <sub>we</sub>	II <sub>n</sub>	-5, 75	+0, 31	+0, 31	0, 00	4	Mt. Hamilton
45	1 July 12 34	1 July 0 25	II <sub>f</sub>	I <sub>n</sub>	-1, 53	+0, 09	+0, 04	-0, 05	4	Hong Kong
46	1 » 15 40	1 » 3 31	III <sub>f</sub>	I <sub>n</sub>	-3, 93	+0, 23	+0, 21	-0, 02	5	} Central Asia
47	2 » 16 21.5	2 » 4 12	III <sub>f</sub>	II <sub>n</sub>	+9, 14	-0, 48	-0, 42	+0, 06	12	
48	3 » 18 21	3 » 6 12	I <sub>f</sub>	II <sub>n</sub>	-5, 68	+0, 31	+0, 31	0, 00	7	Cape
49	3 » 20 8	3 » 7 58.5	III <sub>ee</sub>	IV <sub>n</sub>	+15, 30	-0, 85	-0, 79	+0, 06	32	Berlin etc.
50	5 » 1 44	4 » 13 35.0	II <sub>f</sub>	I <sub>n</sub>	-1, 67	+0, 03	+0, 10	+0, 07	4	Washington
51	5 » 6 25	4 » 18 16	II <sub>f</sub>	IV <sub>n</sub>	+1, 66	-0, 12	-0, 15	-0, 03	7	II eclipsed

No.	Civil time at Paris		Mean time at Greenwich		$\begin{matrix} z = \text{near} \\ f = \text{far} \end{matrix}$		$x = x'$	Occulted satellite $y$	Occulting satellite $y'$	$y' - y$	Duration of central occultation	visible a
	h	m	h	m	Occulted satellite	Occulting satellite						
52	5 July	8 2.5	4 July	19 53.2	II <sub>f</sub>	III <sub>n</sub>	+2.80	-0.17	-0.29	-0.12	7 <sup>m</sup>	Sydney
53	5 »	17 58	5 »	5 49	I <sub>n</sub>	III <sub>n</sub>	-2.75	+0.15	+0.15	0.00	6	Cape
54	5 »	17 34	5 »	5 25	I <sub>f</sub>	IV <sub>n</sub>	-3.04	+0.14	+0.09	-0.05	7	Cape, Mosc
55	5 »	21 54	5 »	9 45	III <sub>n</sub>	IV <sub>n</sub>	-4.85	+0.28	+0.16 <sup>s</sup>	-0.11 <sup>s</sup>	79	Washington
56	7 »	7 37	6 »	19 28	I <sub>f</sub>	II <sub>n</sub>	-5.60	+0.30	+0.31	+0.01	8	Sydney
57	8 »	14 55	8 »	2 46	II <sub>f</sub>	I <sub>n</sub>	-1.82	+0.04	+0.11	+0.07	4	Madras
58	8 »	18 44.8	8 »	6 35.5	III <sub>f</sub>	I <sub>n</sub>	-4.64	+0.26 <sup>s</sup>	+0.25	-0.01 <sup>s</sup>	8	Moscow
59	9 »	20 13	9 »	8 4	III <sub>f</sub>	II <sub>n</sub>	+8.89	-0.50	-0.40	+0.10	11, 5	Berlin, etc.
60	10 »	20 52	10 »	8 47	I <sub>f</sub>	II <sub>n</sub>	-5.53	+0.31	+0.30	-0.01	7	Utrecht
61	12 »	4 6.5	11 »	15 57	II <sub>f</sub>	I <sub>n</sub>	-1.97 <sup>s</sup>	+0.04	+0.13	+0.09	4	Mt Hamiltc
62	12 »	20 57	12 »	8 47	I <sub>f</sub>	III <sub>n</sub>	-1.95	+0.11	+0.13 <sup>s</sup>	+0.02 <sup>s</sup>	6	Utrecht
63	13 »	5 25	12 »	17 15	IV <sub>f</sub>	III <sub>n</sub>	-6.50	+0.35	+0.40	+0.05	10, 5	Pacific
64	13 »	20 39	13 »	8 30	IV <sub>f</sub>	I <sub>n</sub>	-0.27	+0.05	+0.05	0.00	6	♂ ♀
65	14 »	0 12.5	13 »	12 3	IV <sub>f</sub>	II <sub>n</sub>	+1.20 <sup>s</sup>	-0.01 <sup>s</sup>	+0.01 <sup>s</sup>	+0.03	4	IV eclipsed
66	14 »	10 8.5	13 »	21 59	I <sub>f</sub>	II <sub>n</sub>	-5.42	+0.29	+0.32	+0.03	7	Perth
67	15 »	17 17	15 »	5 7	II <sub>f</sub>	I <sub>n</sub>	-2.12	+0.04	+0.14	+0.10	4	Cape
68	15 »	22 1	15 »	9 51	III <sub>f</sub>	I <sub>n</sub>	-5.27	+0.28	+0.30	+0.02	9	Berlin, etc.
69	17 »	23 11	17 »	11 2	I <sub>f</sub>	II <sub>n</sub>	-5.34	+0.29	+0.31 <sup>s</sup>	+0.02 <sup>s</sup>	6, 5	Charkow
70	19 »	6 0	18 »	18 21	II <sub>f</sub>	I <sub>n</sub>	-2.22	+0.06	+0.14	+0.08	4	Pacific
71	19 »	15 5	19 »	2 56	II <sub>f</sub>	III <sub>n</sub>	+3.77	-0.22	-0.15	+0.07	7	Central Asi
72	19 »	23 56	19 »	11 47	I <sub>f</sub>	III <sub>n</sub>	-1.13 <sup>s</sup>	+0.05	+0.12	+0.07	6	Astrachan



Occultation of II<sub>7</sub> by I<sub>1</sub>, on July 15, 1908 at 17<sup>h</sup>27<sup>m</sup> civil time Paris.

I							II				
	M. Longit <sup>e</sup>	1	2	3	4	5	M. Longit <sup>e</sup>	4	5	6	7
58	9 <sup>s</sup> 5 <sup>m</sup> 74	7 <sup>s</sup> 27 <sup>m</sup> 5	1 <sup>s</sup> 0 <sup>m</sup> 8	0 <sup>s</sup> 1 <sup>m</sup> 4	0 <sup>s</sup> 18 <sup>m</sup> 1	10 <sup>s</sup> 20 <sup>m</sup> 9	8 <sup>s</sup> 17 <sup>m</sup> 66	3 <sup>s</sup> 9 <sup>m</sup> 0	10 <sup>s</sup> 2 <sup>m</sup> 8	0 <sup>s</sup> 18 <sup>m</sup> 6	11 <sup>s</sup> 28 <sup>m</sup> 5
80	6 9,80	9 19,2	11 8,4	0 0,8	2 5,9	7 24,6	4 3,87	10 2,9	5 18,7	5 0,2	9 10,6
30	0 6,20	11 22,6	3 8,9	11 28,6	10 4,2	10 20,7	2 2,03	2 2,1	0 16,5	9 8,9	1 2,3
y	3 21,51	5 13,4	0 15,0	5 28,4	4 2,7	3 21,5	11 18,83	8 1,3	11 18,8	11 24,8	11 20,1
108	7 13,25	10 22,7	4 3 1	5 29,2	5 0,9	8 27,7	2 12,39	11 15,3	3 26,8	2 22,5	10 1,5
a	10 28,85	0 12,6	0 1,2	0 13,8	11 19,6	10 28,8	11 9,25	11 24,8	11 9,2	11 9,7	11 0,3
h	4 24,14	0,6		0,7	2 12,3	4 24,1	1 11,81	1 6,2	2 11,8	2 11,8	2 11,8
m	3,82				1,9	3,8	1,90	0,9	1,9	1,9	1,9
	11 10,06	11 5,9	4 4,3	6 13,7	7 4,7	0 24,4	4 5,35	0 17,2	5 19,7	4 15,9	11 24,5
1	- 1,06						1 - 0,52				
2	- 0,16						2 - 0,08		5 + 0 <sup>m</sup> 53		
4	+ 0,43						4 + 0,60		6 - 0,32		
	11 9,27			5 + 1 <sup>m</sup> 27			4 5,35		7 0,00		
G	4 17,37		- IX + 0,05		S-G		4 17,37		- IX + 0,05		
ude	6 21,90		+ 1,32	Amplitude			11 17,98		+ 0,26		
			$x = -2^s,26$	$y = +0^s,14$					$x' = -2^s,00$	$y' = +0^s,04$	

$$\begin{aligned}
 -2^s,26 - 0,830 \Delta t &= -2^s,00 + 0,691 \Delta t \\
 -1,521 \Delta t &= +0,26 \\
 \Delta t &= -0^h,172 = -10^m,3 \\
 x &= -2^s,12.
 \end{aligned}$$

Result: 15 July at 17<sup>h</sup>16<sup>m</sup>.7 Civil time at Paris,  
 = 5 16 .7 Mean " " "  
 = 5 7 .35 " " " Greenwich.

Due account has been taken of the fact that the number of bissextile days between 1858 and 1908 differs from that between 1830 and 1880. For between (1 January) 1830 and (1 January) 1880 we have 12 bissextile days, against 11 between (1 January) 1858 and (1 January) 1908. The difference has been allowed for by treating 1908 as if it were a *common* year in the line of *July*, which contains the motions for the months January to June (inclusive). For on the pages 201, 209, 217 and 225, which contain the motion of the four satellites for the several months, we find two tables, one headed "Années communes", the other headed "Années bissextiles".

The fifth line of this computation (July 1908) may be used for all the conjunctions of this month.

As a second example we will take the occultation of III<sub>r</sub> by II<sub>r</sub>, for which our drawing gave June 2, 1908, 12<sup>h</sup>9<sup>m</sup>.3 civil time at Paris as a first approximation.

II										III				
1908 June	Mean Longitude 0 <sup>s</sup> 1 <sup>m</sup> 15	1	2	3	4	5	6	7	Mean Longitude 0 <sup>s</sup> 17 <sup>m</sup> 45	5	6	7	8	9
	0 <sup>s</sup> 25 <sup>m</sup> 03	4 <sup>s</sup> 0 <sup>m</sup> 0	4 <sup>s</sup> 23 <sup>m</sup> 06	8 <sup>s</sup> 13 <sup>m</sup> 06	10 <sup>s</sup> 15 <sup>m</sup> 06	9 <sup>s</sup> 40 <sup>m</sup> 03	4 <sup>s</sup> 20 <sup>m</sup> 00		0 <sup>s</sup> 7 <sup>m</sup> 07	2 <sup>s</sup> 24 <sup>m</sup> 03	2 <sup>s</sup> 2 <sup>m</sup> 0	8 <sup>s</sup> 0 <sup>m</sup> 03	6 <sup>s</sup> 26 <sup>m</sup> 04	
2 <sup>d</sup>	3 41,37	0 0,9	0 0,1	0 1,0	1 21,1	3 11,4	3 11,4	3 11,4	1 20,32	1 20,3	1 20,3	1 20,3	1 20,3	1 20,3
12 <sup>h</sup>	1 20,69	0,5		0,5	0 25,5	1 20,7	1 20,7	1 20,7	0 25,16	25,2	25,2	25,2	25,2	25,2
9 <sup>m</sup> .3	0,63				0,3	0,6	0,6	0,6	0,32	0,3	0,3	0,3	0,3	0,3
	2 3,84	9 27,0	4 0,7	5 1,1	11 0,5	3 18,3	2 13,0	9 22,7	3 3,25	2 23,5	5 10,1	4 17,8	10 22,1	9 12,2
1	- 0,28								1 - 0,13					
2	- 0,07								2 - 0,04					
4	- 0,91				5 + 2 <sup>m</sup> 80				4 + 0,04					
					6 - 0,45				5 + 0,15					
2	2,58				7 + 0,03				6 + 0,01			7 + 2 <sup>m</sup> 01		
S-G	4 9,25				- IX - 0,11							8 + 0,11		
Amplitude	9 23,33				+ 2,36				3 3,28			9 + 0,03		
					x = - 8 <sup>s</sup> .84	y = + 0 <sup>s</sup> .40			S-G 4 9,25			- IX - 0,11		
								Amplitude	10 24,03			+ 2,04		
												x' = - 9 <sup>s</sup> .01	y' = + 0 <sup>s</sup> .055	

$$\begin{aligned}
 - 8,84 + 0,280 \Delta t &= - 9,01 + 0,453 \Delta t \\
 + 0,17 &= + 0,173 \Delta t \\
 \Delta t &= + 0^m 983 = + 59^m \\
 \text{First approximation} &: 12 \ 9 \ 3, \\
 \text{Second approximation} &: 13 \ 8 \ 3.
 \end{aligned}$$

If we repeat the computation with this value, we still do not get equality of the elongations. The amplitude of II becomes 9<sup>h</sup>27<sup>m</sup>.56, that for III 10<sup>h</sup>26<sup>m</sup>.08. Furthermore we find :

$$\begin{aligned}
 - 8^s,535 + 0^s,327 \Delta t &= - 8^s,56 + 0^s,4645 \Delta t \\
 + 0,025 &= + 0,1365 \Delta t \\
 \Delta t &= + 0^h,182 = + 10^m,9 \\
 & \quad \quad \quad 13 \ 8 \ 3
 \end{aligned}$$

Third approximation : 13<sup>h</sup> 19<sup>m</sup>.2 Civil time at Paris,  
 = 1 19 .2 Mean " " " "  
 = 1 9 .85 " " " Greenwich.