greatest in the most distant elements and greater in Br + I than in Br + Cl.

From the researches of MOISSAN and others it follows that Fluorine yields the compound IF_5 which is stable even in the vapour-condition. With Bromine, the compound BrF_3 is formed but no compound is formed with Chlorine. This, also, is in harmony with the above result.

As, however, the compounds with Fluorine have not been studied from the standpoint of the phase-doctrine, there does not exist as yet a reasonable certainty as to their number or their stability.

Mathematics. — "*Second communication on the* PLÜCKER *equivalents of a cyclic point of a twisted curve.*" By Dr. W. A. VERSLUYS. (Communicated by Prof. P. H. SCHOUTE).

§ 1. If the origin of coordinates is a cyclic point (n, r, m) of a twisted curve C the coordinates of a point of C lying in the vicinity of the origin on a branch passing through the origin can be represented as follows:

$$x = a\,t^n,$$
$$y = b_0\,t^{n+r} + b_1\,t^{n+r+1} + b_2\,t^{n+r+2} + \text{etc.},$$
$$z = c_0\,t^{n+r+m} + c_1\,t^{n+r+m+1} + c_2\,t^{n+r+m+2} + \text{etc.}$$

Let q_1 be the greatest common divisor of n and r, let q_2 be that of r and m, q_3 that of m and $n+r$ and finally q_4 that of n and $r+m$.

If $q_1 = q_2 = q_3 = q_4 = 1$ the PLÜCKER equivalents depend only on n, r and m. In a preceding communication[1]) I gave the PLÜCKER equivalents for this special case [2]).

§ 2. If the 4 G. C. Divisors q are not all unity, the PLÜCKER equivalents of the cyclic point (n, r, m) depend on the values of the coefficients b and c, just as in general for a cyclic point of a plane curve given by the developments:

$$x = t^n,$$
$$y = t^{n+m} + d_1\,t^{n+m+1} + d_2\,t^{n+m+2} + \text{etc.},$$

the vanishing of coefficients d influences the number of nodal points and double tangents equivalent to the cyclic point (n, m) [3]).

1) Proceedings Royal Acad. Amsterdam, Nov. 1905.

2) The deduction of these equivalents is to be found among others in my treatise: "*Points sing. des courbes gauches données par les équations:* $x = t^n$, $y = t^{n+r}$, $z = t^{n+r+m}$," inserted in "*Archives du Musée Teyler*", série II, t. X, 1906.

3) A. BRILL and M. NOETHER. Die Entwicklung der Theorie der algebraischen Functionen, p. 400. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, III, 1892—93.

If the coefficients c and b are not zero, if no special relations exist between these coefficients and if besides n, r and m are greater than *one*, the cyclic point (n, r, m) is equivalent to

$$n - 1 \text{ stationary points } \beta \text{ and to}$$

$$\{(n-1)(n+r-3)+q_1-1\}:2 \text{ nodes } H.$$

The osculating plane of the curve C in the cyclic point (n, r, m) is equivalent to

$$m - 1 \text{ stationary planes } \alpha \text{ and to}$$

$$\{(m-1)(r+m-3)+q_2-1\}:2 \text{ double planes } G.$$

The tangent of the curve C in the cyclic point (n, r, m) is equivalent to

$$r - 1 \text{ stationary tangents } \theta, \text{ to}$$

$$\{(r-1)(n+r-3)+q_1-1\}:2 \text{ double tangents } \omega \text{ and to}$$

$\{(r-1)(r+m-3)+q_2-1\}:2$ double generatrices ω' of the developable O formed by the tangents of the curve C.

§ 3. The cyclic point (n, r, m) of the curve C is an $n+r$-fold point of the developable O of which C is the cuspidal curve.

The cyclic point (n, r, m) counts for

$$(n+r-2)(n+r+m)$$

points of intersection of the cuspidal curve C with the second polar surface of O for an arbitrary point.

Through the cyclic point (n, r, m) of the cuspidal curve C pass

$$\{n(n+2r+m-4)+q_3-q_2\}:2$$

branches of the nodal curve of the developable O.

All these nodal branches touch in the cyclic point (n, r, m) the tangent of the cuspidal curve C (the x-axis).

They have with this common tangent in the point of contact

$$\{(n+r)(n+2r+m-4)+q_4-q_2\}:2$$

points in common.

The nodal branches passing through the cyclic point (n, r, m) all have in this point as osculating plane the osculating plane $z=0$ of the cuspidal curve C.

These nodal branches have with their osculating plane $z=0$ in the cyclic point (n, r, m)

$$\{(n+r+m)(n+2r+m-4)+q_1-q_2\}:2$$

points in common.

§ 4. The case of an ordinary stationary plane α, the point of contact of which is a cyclic point $(1, 1, 2)$, shows that through a

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cyclic point branches of the nodal curve can pass not touching in this point the cuspidal curve.

These intersecting nodal branches exist only when $q_3 \geq 1$. If $r > 1$ the coefficients b and c must satisfy special conditions. If $r = 1$ then through the cyclic point (n, r, m) of the cuspidal curve pass either $q_3 : 2$, or $(q_3 - 1) : 2$ of these nodal intersecting branches. All intersecting nodal branches have a common tangent in the plane $z = 0$ if $r = 1$.

§ 5. The case of an ordinary stationary point β (2, 1, 1) shows that through a cyclic point of the cuspidal curve nodal branches can pass which have the same tangent, but not the same osculating plane as the cuspidal curve. These particular nodal branches exist only when $q_4 > 1$. If $q_4 > 1$ and $m = 1$ these particular nodal branches are always present. If $q_4 > 1$ and also $m > 1$ the coefficients b and c must satisfy special conditions. These particular nodal branches have in the cyclic point (n, r, m) a common osculating plane (differing from the plane $z = 0$) if $m = 1$.

§ 6. The tangent to C in the cyclic point (n, r, m) is an r-fold generatrix g on the developable O. The r sheets of the surface O passing through the generatrix g all touch the osculating plane $z = 0$ of C in the point (n, r, m).

The generatrix g moreover meets in $\varrho - (n + 2r + m)$ points R a sheet of the surface O, when O is of order ϱ.

In every point R the generatrix g meets r branches of the nodal curve. These r branches form, when $m > r$ a singularity $(r, r, m-r)$ and the osculating plane of these nodal branches is the tangent plane of O along g.

If $m < r$ these r nodal branches form a singularity $(r, m, r - m)$ and the osculating plane of these r nodal branches is the tangent plane of O along the generatrix intersecting g in R.

If $r = m$ these r nodal branches form a singularity $(r, r, 1)$.

§ 7. In general the singular generatrix g will meet only nodal branches in the cyclic point (n, r, m) and in the points R. If $q_2 > 1$ the generatrix g may meet moreover nodal branches arising from the fact that some of the r sheets, which touch each other along g penetrate each other. These nodal branches meet g in the same point Q. If $q_2 > 1$ and $n = 1$ there is always such a point of intersection Q. If $q_2 > 1$ and $n > 1$ the coefficients b and c must satisfy some special conditions if the sheets passing through g are to penetrate each other.