Physics. — *"Supplement to Communication N°. 95a from the Physical Laboratory of Leiden, on the comparison of the thermo-element constantin-steel with the hydrogen thermometer".* By Prof. H. KAMERLINGH ONNES and C. A. CROMMELIN.

§ 14. *Corrected representation of the observations by a five term formula.*

As appears from note 1 the calculations in § 12 were made with not perfectly accurate values of the temperature at — 182° and in the same way the mean errors were derived from the assumption of those less accurate values. [1])

If the correct values of those temperatures for the calculations of the deviations $W-R_1$, $W-R_2$, $W-R_3$, $W-R_4$ in Table VIII, are used, the mean errors in microvolts become:

for formula (*B*I) ± 3.0
(*B*II) ± 3.4
(*B*III) ± 2.8 (2.5 without — 217°)
(*B*IV) ± 2.1

instead of

(*B*I) ± 2.8
(*B*II) ± 3.2
(*B*III) ± 2.6 (2.1 without — 217°).
(*B*IV) ± 1.8

which would also have been obtained if the observations at — 182° were excluded.

Now it was necessary to examine whether a repetition of the adjustment would diminish these mean errors. It appeared convincingly that this was not possible to an appreciable degree for (*B*I), (*B*II), (*B*III). It appeared possible for (*B*IV) to distribute the errors more equally. However, this only reduced the sum of squares from 26,57 to 26,14.

Instead of the coefficients a_4, b_4, c_1, e_4 and f_4 (see § 12) we get then

$$a'_4 = +4.32513 \qquad e'_4 = +0.023276$$
$$b'_4 = +0.409153 \qquad f'_4 = -0.0025269$$
$$c'_4 = +0.0015563$$

The deviations are given in Table IX under $W-R'_4$.

---

1) The correction amounted to 0°,081 in temperature or to 1.7 microvolt. in electromotive force.

§ 15. *Representation of the observations by means of a four term formula.*

We have now quite carried out the calculation of a formula of the form

$$E = a\left(\frac{t}{100}\right) + b\left(\frac{t}{100}\right)^2 + c\left(\frac{t}{100}\right)^3 + e\left(\frac{t}{100}\right)^4 \quad . \quad . \quad . \quad (C)$$

announced in note 2 of § 11, by the method of E. F. v. D. SANDE BAKHUYZEN, which proved to facilitate matters greatly again.

Four solutions (C) were found, viz. (CI), (CII), (CIII) representing the observations down to — 253°, whereas in (CIV) only agreement down to — 217° has been sought for.

The coefficients in millivolts are the following:

	1	2	3	4
a	+ 4.30192	+ 4 30571	+ 4 30398	+ 4 33031
b	+ 0.357902	+ 0 366351	+ 0.363681	+ 0.421271
c	— 0.0250934	— 0 0192565	— 0 020071	+ 0 018683
e	+ 0 0257462	+ 0 0270158	+ 0.0270044	+ 0 035268

The residuals have been given in tenth parts of microvolts in Table IX under $W—R_{CI}$, $W—R_{CII}$, $W—R_{CIII}$, $W—R_{CIV}$.

Just as with the five term formula, the residual at — 182° appeared also now greater than the others.

In calculation 3 it was tried to distribute the errors more equally, but the sum of squares appeared now to have increased.

The mean errors are if we include the observations down to — 253° for (CI), (CII), (CIII), and only those down to — 217° for (CIV), for

(CI)	± 3.0
(CII)	± 2.9
(CIII)	± 3.0
(CIV)	± 2.3

If — 182° is excluded, they become:

(CI)	± 2.7
(CII)	± 2.6
(CIV)	± 1.8

The mean errors of (CI), (CII), (CIII) must be compared with those of (BI) and (BIII), those of (CIV) with those of (BIV).

This comparison teaches that the four term formula for the representation of the observations may be considered to be almost equivalent to the five term formula, and that therefore (this remark is in harmony with note 2 of §13) for the calibration to — 217° the lowest number of temperatures for which observations are required, amounts to *four*. That three are not sufficient was already proved in § 11. This appears also clearly, when the mean error is determined, which rises to ± 7.6 microvolts for the three term formula.

TABLE IX.

DEVIATIONS OF THE CALIBRATION-FORMULAE FOR THE THERMO-ELEMENT CONSTANTIN-STEEL.

I	II	III	IV	V	VI	VII
N°.	t	$W-R_4'$	$W-R_{CI}$	$W-R_{CII}$	$W-R_{CIII}$	$W-R_{CIV}$
22	— 29°82	— 12	+ 20	+ 15	+ 18	— 19
24 and 20	— 58 75	+ 16	+ 30	+ 26	+ 29	+ 4
21 and 23	— 88 15	+ 14	+ 1	+ 1	+ 1	+ 1
1 and 17	— 103 70	— 6	— 29	— 28	— 30	— 20
16 and 18	— 139 86	+ 1	— 26	— 24	— 31	— 17
19	— 158 83	— 10	— 10	— 10	— 18	— 10
3, 11 and 5	[— 182 73]	+ 26	+ 46	+ 44	+ 35	+ 34
4, 28 and 6	— 195 19	+ 2	+ 23	+ 21	+ 12	+ 11
12, 27 and 7	— 204 70	— 20	— 9	— 11	— 19	— 18
26, 14, 13 and 8	— 212 85	+ 24	+ 21	+ 21	+ 13	+ 21
29, 15 and 25	— 217.55	— 15	— 30	— 29	— 37	— 23
30	— 252 93	+ 280	0	+ 20	+ 20	+ 150
31	— 259 24	+ 485	+ 115	+ 141	+ 143	+ 313