through which passes a surface out of each of the pencils. Through the points P, P', P'' and P''' of such a quadruple pass three sheets of the surface L and three branches of the double curve. The 12 branches of the double curve through those four points we can call $P1, P2, P3, P'1, P'2, P'4, P''1, P''3, P''4, P'''2, P'''3, P'''4$, in such a way that the triplet of points is movable along the branches $P1, P'1, P''1$, along $P2, P'2, P'''2$, along $P3, P''3, P'''3$ and along $P'4, P''4, P'''4$. If the sheet of L passing through $P1$ and $P2$ is called $P12$, then the corresponding sheets (i. e. sheets along which the pair of points not lying on the double curve is movable) are $P12$ and $P'12, P13$ and $P''13$, etc.

Geophysics. — "*Current-measurements at various depths in the North Sea.*" (First communication). By Prof. C. H. WIND, Ltt. A. F. H. DALHUISEN and Dr. W. E. RINGER.

In the year 1904 accurate measurements of the currents in the North Sea [1]) were started by the naval lieutenant A. M. VAN ROOSENDAAL, at the time detached to the "Rijksinstituut voor het Onderzoek der Zee", having been proposed and guided by the Dutch delegates to the International Council for the Study of the Sea.

By him four apparatus were put to the test, viz. 2 specimens of the current-meter of PETTERSSON [2]), one of that of NANSEN [3]) and one of that of EKMAN [4]), all destined to determine the direction and the velocity of the current at every depth.

The experiments were partly made on the light-ship "Haaks", where Dr. J. P. VAN DER STOK, the Marine Superintendent of the Kon. Nederl. Meteorologisch Instituut, also took part in them. Other experiments were made in the harbour of Nieuwediep and further, from the research-steamer "Wodan", in the open North Sea at a station (H2) of the Dutch seasonal cruises [5]), situated at Lat. 53°44′ N. and Long. 4°28′ E.

[1]) Cons. Perm. Intern. p. l'expl. de la mer, Publications de circonstance No. 26 : A. M. VAN ROOSENDAAL und C. H. WIND, Prüfung von Strommessern und Strommessungsversuche in der Nordsee. Copenhague, 1905.

[2]) Publ. de circ. No. 25.

[3]) „ „ „ No. 34.

[4]) „ „ „ No. 24.

[5]) Quarterly cruises of the countries taking part in the international study of the sea, along fixed routes, observations being made at definite points or "stations".

The apparatus of NANSEN appeared to be unfit for the measurements on the North Sea; it was not calculated for the strong tidal currents occurring there (e.g 60—100 cm/sec.), and also the putting out of the apparatus in unfavourable weather was hardly possible without doing harm to the instrument. In more quiet water, however, it seems to be very useful.

The apparatuses of PETTERSSON and EKMAN appeared to be better fit for the observations in the North Sea. Some improvements in the construction were proposed, partly also put into practice, by VAN ROOSENDAAL and WIND, by which the instruments have gained in fitness. For a description of the construction of the current-meters used, and the experience made in using them, we may refer to the publications mentioned. The following few words may be sufficient here.

It appeared that pretty large oscillations, e. g. 15° to both sides round the longitudinal axis, did not yet render observation impossible. In 32 out of nearly 200 observations by VAN ROOSENDAAL as much as the figure 4 was noted for the motion of the sea, in 40 to 50 cases the oscillations amounted to 10 à 20° to either side, and yet the accuracy and certainty of these measurements were only exceptionally insufficient.

In the parallel-observations with the apparatus of PETTERSSON and EKMAN the agreement in indicating the *velocity* appeared satisfactory. In one series of 23 measurements e. g. the average difference amounted to 4.8 cm/sec, whilst the smallest was 3.1, the greatest 6.3.

Nor did the indications of *direction*, as given by the two instruments, show great differences. The observations with EKMAN's apparatus bear to some extent a check in themselves, as, by the construction of the instrument, every observation includes a series of consecutive readings at small intervals. In by far the greater part of the readings-observations these separate did not considerably vary. In 128 cases the direction of the current could be estimated from them:

To less than 10°	in 105 cases,
10—20	15
20—30	2
30—40	0
40—50	2
more than 50°	4.

Compared with the probable direction, as derived from the instrument of EKMAN, that which was determined by means of PETTERSON's instrument deviated :

in 65	cases less than	10
37		10—20
15		20—30
5		30—40
1		40—50
8		more than 50°.

VAN ROOSENDAAL and WIND took from the whole of observations made at station H_2 the most probable values direction and velocity of current at the various depths and represented them graphically. They constructed for the different series of observations, each lasting 12 or 24 hours, in the first place *central vector-diagrams,* by drawing from a fixed point the successively determined currents as radii-vectores and connecting the terminal points by means of straight lines or of a curve, and in the second place *progressive vector-diagrams,* by drawing the current-vectors, this time interpolated for the successive full hours, one after and attached to the other. In the first kind of diagrams the periodical currents, and in the second the residual currents make themselves most apparent.

The measurements were continued at the station H2 during all the following seasonal cruises of the "Rijksinstituut", first by VAN ROOSENDAAL and afterwards by the naval lieutenant DALHUISEN, who succeeded the former in his detachment. At the more recent measurements the current-meter of EKMAN was always made use of.

The following table gives the dates of the series of observations and the number of measurements [1]).

No.	Time.	Number of Measurements.	Depth (M.)	Apparatus	Observer.
1.	from 16 Aug. '05 4.12 p m. till 17 „ „ 4.54 »	56	5,20,35	EKMAN.	VAN ROOSENDAAL.
2.	from 7 Nov. '05 7.48 a.m. till 8 „ „ 12.34 p m	53	» » »	»	VAN ROOSENDAAL and DALHUISEN
3.	from 7 Febr. '06 7.20 p.m. till 8 „ „ 5.53 a m.	18	» » »	»	DALHUISEN.
4.	from 2 May '06 6.35 a m. till 3 „ „ 6.41 »	54	» » »	»	»

1) A more detailed description of these observations forms the contents of the last issue of the "Publications de circonstance" No. 36.

At these researches wind and weather were on the whole favourable; the wind was in a few cases noted 7 at most, at which force, however, the observations had to be put a stop to in February 1906 [1]).

On the plate added, the new measurements are again represented graphically in central and progressive vector-diagrams. Also the central diagrams, have been constructed this time with the aid of values interpolated for full hours, the directly measured values however, having still been indicated by dots.

It is principally to give a full idea of the variability in direction and velocity of the currents, that these diagrams of the new series of observations have been reproduced fully here.

Comparing the values of the velocity near the surface and in the depth, we see that in 3 out of the 4 cases they show a rather distinct decrease at an increase of depth. Also at the former series of observations at H2 (3—4 Aug., 8—9 Aug. and 2—3 Nov. 1905 [2]), also 8—9 Febr. 1905 [3])) the same result was arrived at.

Also differences of phase in the periodical currents are noticed in most cases between the surface and the depth, though a distinct law may not immediately be obvious here.

The striking difference in amplitude of the tidal currents during the observations in August 1905 and February 1906 on the one side and that of November 1905 and May 1906 on the other, is certainly connected with the age of the tide, as it was with the first nearly spring-tide ($15^1/_2$ and 14 days after N. M.), with the last nearer to dead neap (10 and 0 days after N.M.).

The small number of series of observations that can be disposed of, does of course not allow at all to already think of a calculation of tidal constants, nor to give a correct description of the average variation of the currents. The unmistakable general agreement, however, between the different current-diagrams justifies sufficiently an attempt to compose them. As no doubt moon-tide will have played

1) The reliability of the new observations is no doubt greater than that of the former, if we take into consideration, that in August and November 1905 and in February and May 1906 the Wodan lay moored, so that her motion was considerably smaller than on the former occasions, when she had cast only one anchor.

It may still be mentioned that an experimental and theoretical investigation was started about the influence of the movements of the ship upon the indications of the current-meter, which, however, has not yet led to a satisfactory result.

2) Publ. de Circ. N⁰. 26.

3) „ „ „ N⁰. 36.

the principal part, we have thought best for this purpose to compose for the successive full moon-hours the current-values as they follow by interpolation from the different diagrams. The averages thus obtained have been combined in new diagrams, which are represented on the plate, in the last column of figures, and that by black curved lines.

In order to complete the matter and to allow comparisons, in the same way average diagrams have been derived from the observations made in the past year at H2 (see above) and represented in the same figures on the plate by black-and-white curves.

The arrows drawn in these figures indicate: in the central diagrams the direction of the current at the moon's transit, in the progressive diagrams the total residual current during a half moon-day.

A comparison of the average current-diagrams for various depths or also of the newer with the older ones might give rise to all kinds of remarks. With a view to the small number of data, however, on which the diagrams are based, it would perhaps be inconsiderate to mention all of them here. We therefore confine ourselves to what follows.

Difference in Phase of the tide at different depths.

	August 1905 – May 1906		August—November 1904	
	20 M.—5 M.	35 M.—20 M.	20 M.—70 M.	30 M.—20 M.
☾ Transit	18°	5°	0	3°
one hour after »	24	— 6	—13	14
2	23	— 3	— 8	5
3	20	5	— 3	6
4	25	— 2	5	14
5	26	0	18	— 5
6	25	8	3	22
5 » before »	19	17	0	17
4	25	15	— 9	17
3	8	— 6	— 3	25
2	4	10	— 6	24
1	6	11	— 6	11
Average	13°85′	4°30′	—1°30′	12°45

The tidal curve shows not only at different depths, but also in the older and newer observations, generally the same *shape*. Its *size*, on the other hand, both in the older and more recent observations, appears to be smaller near the bottom than near the surface. Also its *orientation* and the situation of the point in it, which relates to the moment of the moon's transit, or, more generally, the *phase* of the tidal current, seems to change in a definite sense as the depth increases. This last relation may be specially illustrated by the following table.

It appears from the table, that the tide is on the whole accelerated in the depth, compared with higher layers; but the table also proves that the phenomenon underlies varying influences, besides constant causes, among which perhaps may be reckonned the shape of the bottom of the sea and the rotation of the earth.

The residual current is by no means constant; at the new observations it has been much stronger than at the old; it shows considerable fluctuations also, when the progressive diagrams of the different days of observation are compared. At the new observations this residual current was on an average stronger near the surface than in deeper layers. This particular may perhaps be principally attributed to the action of persisting winds, which at least on the observations of August 1905 and May 1906 had a very marked influence, rendered quite obvious by the special diagrams for these dates.

The figures for the residual current as deduced from the newer observations are the following:

Depth.	Direction.	Velocity.
5 M	N 304° E	1/4 mile p. hour
20	317	1/8
35	309°	1/9 ,

as deduced from the older:

Depth.	Direction.	Velocity.
7[1]) M.	N 319° E	1/19 mile p. hour
20	295°	1/26
30	323°	1/18 .

These results are worth comparing with the following table of values for the year-average of the residual current at the Noord-Hinder (Lat. 51°35'.5 N., Long. 2°37,E.), calculated by VAN DER STOK [2]) from current-estimations near the surface during five consecutive years.

1) Average of depths of 1, 4, 5, 6, 10 M.; at a depth of 35 M. measurements were made by VAN ROOSENDAAL only in February 1905.

2) J. P. VAN DER STOK, Etudes des Phénomènes de Marée sur les côtes néerlandaises; Kon. Ned. Met. Inst. No. 90, II. p. 67, 1905.

Year.	Direction.	Velocity.
1890	N 16° E	0.024 miles p. hour.
91	15	62
92	16	35
93	29	47
94	27	47
Average	N 21° E	0.044 miles p. hour.

Here it appears that the average residual current, which — as we mention in passing — has at this point quite another direction than at H2, even from year to year does not at all remain constant in strength which may perhaps be an indication for differences in the quantity of Atlantic water, entering through the English Channel from year to year.

The question may be put, whether and how far the results attained by the current-measurements described, deviate from what is known from the charts, in general use, about the currents near the station H2, The subjoined table allows of a comparison with statements, borrowed from a chart, published by the British Admiralty [1]), and shows

Hour.	From the Charts.		Observed.	
	Direction.	Velocity (m. p. h.).	Direction.	Velocity (m. p. h.).
5 before H.W.Dover	N 90° E	0,3—0,2	N 73° E	0,3
4	110	0,5—0,3	115	0,4
3	135	0,9—0,6	147	0,4
2	160	0,6—0 4	189	0,3
1	180	0,3—0.2	227	0 4
H. W Dover	—	0	266	0,5
1 after H.W. Dover	260	0,[illegible]—0,2	280	0 5
2	300	0,6—0,4	296	0,6
3	300	1,0—0,7	331	0 5
4	315	0,6—0,4	342	0,4
5	0	0 3—0,2	9	0,4
6	50	—	40	0,4

1) Tidal Streams North Sea 1899.

A. F. H. DALHUISEN, W. E. RINGER and C. H. WIND. „Current-Measurements at various depths in the North Sea."

CURRENTS OBSERVED at H_2 (Lat. 53°44', Long. 4°28').

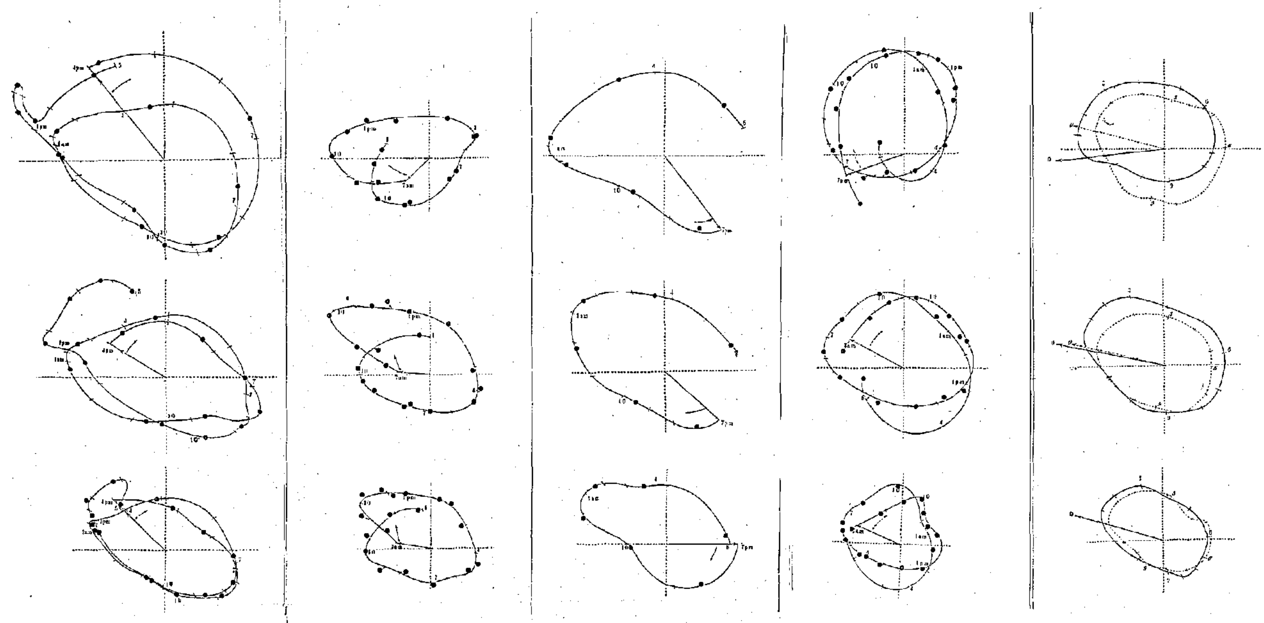
(True direction.)

1905, August 16—17. | 1905, November 7—8. | 1906, Februar 7—8. | 1906, May 2—3. | Averages (at figures indicate moon-hours).

CENTRAL DIAGRAMS.

cm. p. sec. / miles p. hour.

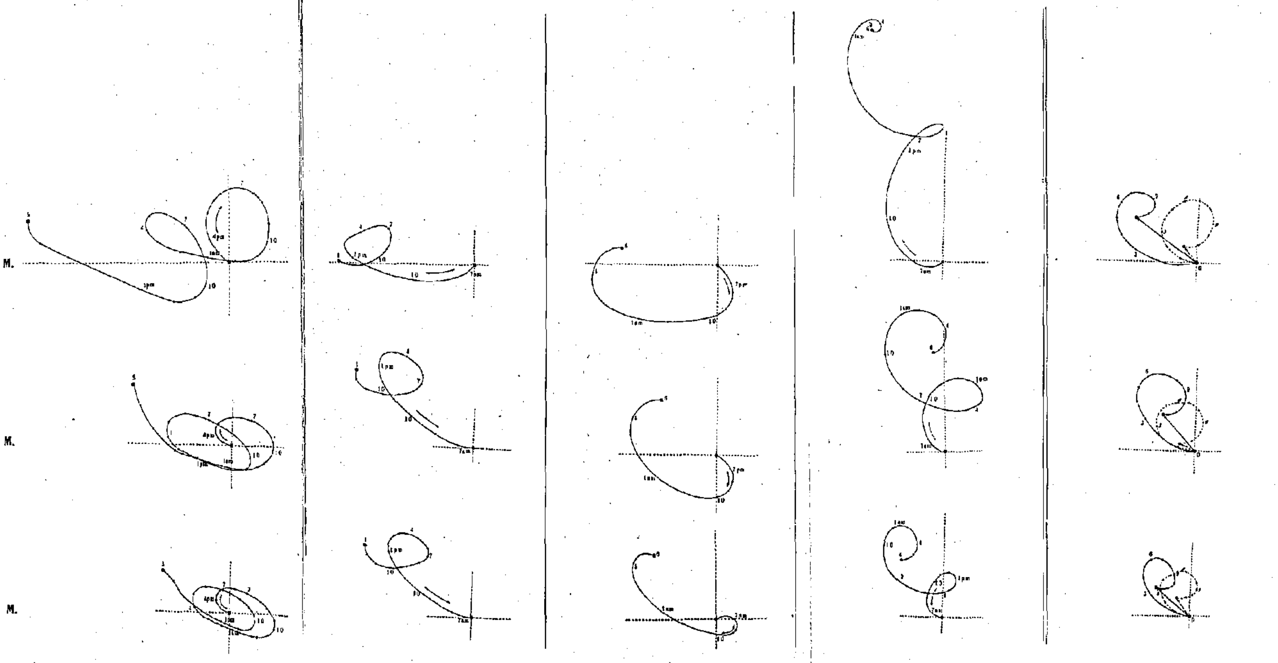
5 M. / 20 M. / 35 M.



PROGRESSIVE DIAGRAMS.

miles.

5 M. / 20 M. / 35 M.



that the deviations for a part considerably exceed the limits of accurateness of the statements.

It should be observed that the charts refer to currents near the surface, whereas the values of the table derived from our observations refer to a depth of 5 M.

Finally we may mention that the observations at station H2 up till now have been continued in the same way, that is to say, they are still made every quarter of a year, as far as possible, during 24 hours. Moreover, owing to the kind co-operation of His Excellency the Minister of Marine, a current-meter of PETTERSSON has been placed on the lightship "Noord-Hinder", with which since November 1906 daily, in so far as the state of the weather permits, with intervals of three hours, measurements at various depths are made by the ordinary staff of the lightship. The lists of observation are forwarded to the "Rijksinstituut" and promise to yield important material, especially for the inquiry into the way in which the tidal and residual currents differ in layers of different depth.

Mathematics. — *"The locus of the pairs of common points of $n+1$ pencils of $(n-1)$-dimensional varieties in a space of n dimensions."* By Dr. F. SCHUH.

(Communicated by Prof. P. H. SCHOUTE).

1. Let (V_i) $(i = 1, 2, \ldots, n+1)$ be $n+1$ pencils of $(n-1)$-dimensional varieties in the space of operation Sp^n of n dimensions and let r_i be the order of the varieties V_i of the pencil (V_i). Let moreover a_i be the number of points of intersection of the n varieties $V_1, V_2, \ldots, V_{i-1}, V_{i+1}, V_{i+2}, \ldots, V_{n+1}$ not of necessity lying in the base-varieties.

When considering the locus of pairs of points P, P' through which a variety of each of the pencils passes we have exclusively such pairs in view of which neither of the two points lies of necessity on a base-variety of one of the pencils and we call the locus thus arrived at the *locus proper* L.

We determine the order of L out of its points of intersection with an arbitrary right line l. To this end we take on l an arbitrary point $Q_{12\ldots n}$ and we bring though it varieties $V_1, V_2, V_3, \ldots, V_n$, having $a_{n+1}-1$ points of intersection not lying on $Q_{12\ldots n}$ and the base-varieties. Through each of those points we bring a V_{n+1} and arrive in this way at $a_{n+1}-1$ varieties V_{n+1} intersecting together line l in $(a_{n+1}-1)\, r_{n+1}$ points Q_{n+1}. So to $Q_{12\ldots n}$ correspond $(a_{n+1}-1) r_{n+1}$ points Q_{n+1}.