Two further questions connected with those views and perhaps liable also to be answered by way of experiment, are these:

1. whether the air molecules on the outside of the aluminium window of LENARD emit R.-rays in appreciable quantity;

2. whether the γ-rays of a radio-active substance, except by the substance itself, are to a considerable extent emitted also by the atoms of air in its neighbourhood on their being pierced by the electrons constituting the β-rays.

Physics. — *"On the motion of a metal wire through a piece of ice."* By Dr. J. H. MEERBURG. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of January 26, 1907).

During the last and the preceding winter I made some measurements with a purpose of testing the formulae, expressing the velocity of descent of a metal wire through a block of ice, which Mr. L. S. ORNSTEIN had derived from the theory of regelation [1]).
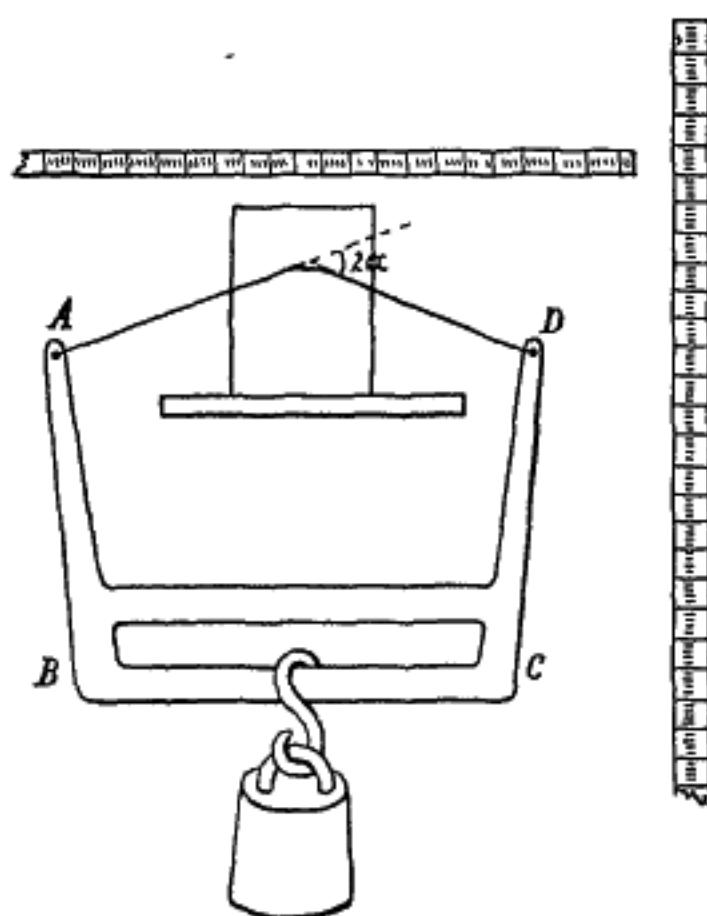
In my experiments the metal wire was fastened at both ends to

---

[1]) L. S. ORNSTEIN. These Proc. VIII, p. 653.

the legs A and D of an iron frame, which, in order to secure greater rigidity, had been cut from an iron plate. In the first measurements the downward displacements of the wire were observed by means of a small reading telescope, turning round a horizontal and a vertical axis, and were determined on a measuring rod, mounted at the side of the frame. The breadth of the ice-block was also read on a horizontal measuring rod. In the later experiments a cathetometer was used, placed at my disposal by the professors of the Technical University at Delft. I wish to express here my sincere thanks to these gentlemen, especially to Prof. DE HAAS. The fall of the wire was always derived from the change in the difference of level between the top of the wire and the upper edge of a small bubble, existing somewhere in the interior of the ice. Every ten minutes or, when the descent was quicker, every five minutes, the difference of level was measured in order to ascertain whether the fall was regular. Each experiment lasted 20 to 40 minutes.

The ice used was artificial commercial ice. From a larger block a clear smaller one was sawn out, in which some bubbles should be present to serve as marks. The faces were melted flat by pressing them against a metal plate, so that errors, caused by irregular refraction, were avoided.

Heat conduction along the wire was prevented by hanging small pieces of ice on the wire on both sides of the block. Still small grooves were occasionally formed when the descent was slow.

The experiments were made with wires of steel, german silver and silver. The thickness of the wires was measured by means of caliber compasses, giving results accurate to 0.01 mm. The thickness was 0.5, 0.4 and 0.3 mm. Deviations from these numbers, amounting to some hundredths of a millimetre, were occasionally found.

For the case, realised in my experiments, in which the two straight ends of the wire make a certain angle 2α with each other, formula (IIIa) of Mr. ORNSTEIN's paper [1]) holds:

$$v = \frac{2aCP}{d_1 \sin \alpha}. \qquad \ldots \qquad \ldots \qquad (1)$$

in which v represents the velocity of descent of the wire, P the total weight and d the breadth of the ice-block. C is a constant. The value of this constant I calculated by formula (1) from the values of v, found in my experiments.

The results are summarised in the following table:

---

1) l. c.

Steel wire Diameter 0.5 mM.

P (in grammes).	C	average of C.
455	0.0162	
755	0.0151	
1255	0.0172	0.017
2160	0.0185	
2205	0.0169	
5160	0.019	
	diameter 0.4 mM.	
455	0.030	
755	0.029	0.029
1255	0.029	
	diameter 0.3 mM.	
755	0 043	0.0425
1255	0.042	

German silver. diameter 0.5 mM.

P	C	average of C.
755	0.0134	
1255	0.0119	0.014
2150	0.0143	
5160	0.0172	
	diameter 0.4 mM.	
755	0.0196	
1255	0.0204	0.022
2150	0.0208	
5160	0.0255	
	diameter 0.3 mM.	
555	0.0306	
755	0.0348	0.035
855	0.0393	

Silver wire. diameter 0.5 mM

P	C	average of C.
755	0.0207	0.023
1255	0.0255	
	diameter 0.4 mM.	
536	0.0367	
755	0.0384	0.039
1036	0.0392	
1255	0.0404	
	diameter 0.3 mM	
555	0.0347	0 041
755	0.0467	

The quantity C is not expressed here in C.G.S. units, since the dimensions have been taken in millimetres, the velocities in millimetres per minute and the forces in grammes. In order to reduce them to C.G.S. units, the value of C has to be multiplied by 170×10^{-9}.

The values given in the table are averages of several measurements. In order to show the deviations of different measurements, made with the same weight, I give here an arbitrarily chosen set of separate measurements.

German silver wire, diameter 0.4 mm.

Number of the experiment	P	v	d_1	2α	C	averages.
8	1036	1.017	39.0	42°	0.0368	
10	1036	1.11	37.4	42°	0 0393	0.0392
14	1036	1.27	35.4	37°	0.0411	
17	1036	1.73	29.3	30°	0.0393	
87	1255	0.99	52 3	50°	0.0401	
112	1255	1.09	51.27	53°	0.043	0.0404
115	1255	0.756	66.46	53°	0.0387	

The value of C is calculated by Mr. ORNSTEIN in formula (I) of his paper He finds

$$C = - \frac{\left(\dfrac{dt}{dp}\right)_0 \left[k_2 \dfrac{k_1 - {}^d/R\,(k_1 - k_2)}{k_2 + {}^d/R\,(k_1 - k_2)} + k_3 \right]}{\pi\, R^2\, WS_i} \quad \cdots \quad (2)$$

Here k_1, k_2 and k_3 are the coefficients of heat conductivity respectively of the wire, of water and the ice, $\left(\dfrac{dt}{dp}\right)_0$ is the rise of the melting temperature by pressure, measured at the melting temperature, W is the latent heat of melting ice, S_i the specific gravity of ice, R the radius of the wire and d the thickness of the layer of water. Now the value of C cannot be calculated by this formula, since the quantity d is unknown. But besides the equation (I) Mr. ORNSTEIN gives in his formula (II*) an expression, found by a hydrodynamical reasoning, in which the quantity d likewise occurs. This relation is [1]:

$$v = \frac{S_w}{S_i} \frac{2\alpha P}{12\,\pi\,\mu\,d_1\,\sin\alpha} \left(\frac{d}{R}\right)^2 \quad \cdots \cdots \cdots \quad (3)$$

---

[1] In Mr. ORNSTEIN's paper this formula is given without the factor $\dfrac{S_w}{S_i}$ since this latter has no perceptible influence.

Here S_w is the specific gravity of water at $0°$, μ the viscosity coefficient. By equalising (1) and (3) we find.

$$C = \frac{S_w}{S_i} \cdot \frac{1}{12\,\pi\,\mu} \left(\frac{d}{R}\right)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and we should now have to eliminate $\frac{d}{R}$ between (2) and (4). In order to perform this elimination we simplify (2). We consider the form in (2) between the brackets [ ] and keep in mind that $\frac{d}{R}$ is very small, that k_1 is very much greater than k_2 and that k_2 may be neglected with respect to the first term (which amounts to neglecting the conduction of heat through the ice).

We may then write:

$$k_2\,\frac{k_1 - d/R\,(k_1 - k_2)}{k_2 + d/R\,(k_1 - k_2)} = k_2\,\frac{k_1}{k_2 + d/R\,k_1} = \frac{k_1}{1 + d/R\,\dfrac{k_1}{k_2}}.$$

Then we have

$$C = -\,\frac{\left(\dfrac{dt}{dp}\right)_0 \dfrac{k_1}{1 + d/R\,\dfrac{k_1}{k_2}}}{\pi\,R^2\;WS_i}$$

If we put

$$\left(\frac{dt}{dp}\right)_0 = -\,7.4 \times 10^{-9}, \quad S_i = 0.9167, \quad W = 79.2,$$

C becomes

$$C = 3.3 \times 10^{-11} \times \frac{1}{R^2}\,\frac{k_1}{1 + d/R\,\dfrac{k_1}{k_2}}.$$

In (4) we substitute

$$S_w = 1, \quad S_i = 0.9167, \quad \mu = 0.0181,$$

then

$$C = 1.600 \left(\frac{d}{R}\right)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equalising the two values of C we have:

$$\left(\frac{d}{R}\right)^3 = 2.0 \times 10^{-11}\,\frac{1}{R^2}\,\frac{k_1}{1 + \dfrac{d}{R}\dfrac{k_1}{k_2}}$$

or

$$\left(\frac{d}{R}\right)^3 + \left(\frac{d}{R}\right)^4 \frac{k_1}{k_2} = 2.0 \times 10^{-11}\,\frac{k_1}{R^2}.$$

From this equation $\frac{d}{R}$ can easily be found by a tentative method, when k_1, k_2 and R are given. In the different cases we find (in CGS-units),

Steel wire
$k_1 = 0.166$ $k_2 = 0.0015$ $R = 0.025$ $\frac{d}{R} = 0.00166$

$k_1 = 0.166$ $k_2 = 0.0015$ $R = 0.020$ $\frac{d}{R} = 0.00190$

$k_1 = 0.166$ $k_2 = 0.0015$ $R = 0.015$ $\frac{d}{R} = 0.00229$

German silver wire $k_1 = 0.070$ $k_2 = 0.0015$ $R = 0.025$ $\frac{d}{R} = 0.00128$

$k_1 = 0.070$ $k_2 = 0.0015$ $R = 0.020$ $\frac{d}{R} = 0.00149$

$k_1 = 0.070$ $k_2 = 0.0015$ $R = 0.015$ $\frac{d}{R} = 0.00179$

Silver wire $k_1 = 1.50$ $k_2 = 0.0015$ $R = 0.025$ $\frac{d}{R} = 0.00239$

$k_1 = 1.50$ $k_2 = 0.0015$ $R = 0.020$ $\frac{d}{R} = 0.00279$

$k_1 = 1.50$ $k_2 = 0.0015$ $R = 0.015$ $\frac{d}{R} = 0.00318$

C is then found by substitution in (5). These values are given below, together with the values found by experiment, but now expressed in CGS-units.

		Calculated	Found
Steel wire	$R = 0.025$	73×10^{-10}	29×10^{-10}
	$R = 0.020$	110×10^{-10}	49×10^{-10}
	$R = 0.015$	192×10^{-10}	72×10^{-19}
German silver wire	$R = 0.025$	34×10^{-10}	24×10^{-10}
	$R = 0,020$	53×10^{-10}	37×10^{-10}
	$R = 0.015$	91×10^{-10}	59×10^{-10}
Silver wire	$R = 0.025$	218×10^{-10}	46×10^{-10}
	$R = 0.020$	347×10^{-10}	66×10^{-10}
	$R = 0.015$	519×10^{-10}	70×10^{-10}

The agreement must be called bad for the silver wire, satisfactory for the german silver wire. It may be called satisfactory, since different circumstances may be mentioned which make us expect a too small value. Leaving aside the great uncertainty in the values of the heat conductivities of metals, to which we cannot here ascribe the bad agreement, since we do not know in which direction this

will influence the result [1]), the following causes may be mentioned.

1. The roughness of the wires. Already Mr. ORNSTEIN pointed this out. If the wire is not entirely smooth, the hydrodynamical deductions are uncertain and hence also formula (3). In order to ascertain the influence of this roughness I made some experiments with a steel wire that had for a moment been scoured with fine sand-paper in the direction of its length. Macroscopically no result of this manipulation could be discovered on the wire. Yet the effect proved considerable, for the following results were found:

	P	C	
	455	0,009	
steel wire, diameter 0,5 m.M.	1255	0,011	average 0.011 m.M.
	2205	0,014	
steel wire, diameter 0,3 m.M.	1255	0,028	

So we find a diminution of about 40 °/₀ in the value of C. After having observed this influence I tried to obtain smooth wires, but unsuccessfully, all the wires that were used in the experiments showed under the microscope numberless grooves in the direction of their length and of a breadth that might be estimated at somewhat less than 0.01 mm.

Since it is easily deduced from the calculated values of $\dfrac{d}{R}$ that the thickness of the layer of water increases with the size of the radius of the wire and since the influence of the roughness of the wire will be smaller with a greater thickness of this layer of water, I have still made some measurements with a thicker steel wire of 0,87 mm. diameter and heavier weights. The result was:

P	C		C (in C.G.S. units)
25200	0,00803		13.8×10^{-10}
25200	0,00822	0,0081	
5200	0,00667		11.3×10^{-10}

while calculation gives

$$k_1 = 0,166, \quad k_2 = 0,0015, \quad R = 0,0435 \quad \frac{d}{R} = 0,00120 \quad C = 27.7 \times 10^{-10}$$

The agreement is now better indeed; the value found is half the calculated one, while with the thinner steel wires it was slightly more than a third.

---

[1] The values given by F. KOHLRAUSCH (Lehrbuch der praktischen Physik 10 Auflage 1905), steel $k = 0.06$ to 0.12 and silver $k = 1.01$, would give a much better agreement.

2. In the deduction of formula (1) it was assumed that within the layer of water the relation

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0.$$

holds. This relation, however, holds for a body at rest. Here, on the other hand, we have to deal with a streaming liquid, in which case the following formula holds:

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = \frac{\varrho S_w}{k_2} \left(u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} \right).$$

Here ϱ is the specific heat of the liquid, u and v the velocity components in the X- and Y-directions. If we use this formula we take into account that the heat, conducted through the wire, does not entirely serve for melting the ice, but that it is partly conveyed upwards again by the streaming liquid. This also must result in a diminution of the velocity of descent. Prof. LORENTZ informed me, however, that it can be shown that this influence must be regarded as a quantity of the second order, so that the differences cannot be explained in this way.

3. If the temperature in the interior of the block of ice is not exactly 0°, but lower, the velocity of descent will also become smaller. But I observed no phenomena which point to a lower temperature in the interior. Blocks of ice that had been kept for 24 hours in a space above 0° gave the same results as blocks that had just been received. Moreover the wires as a rule went down at a distance of only a few millimetres from one of the faces of the block, and in some experiments they even came out of the block by melting of that face. Yet in the last moment, before the wire came out, no acceleration of the descent was observed.

Nor does theory support such an explanation. Prof. LORENTZ informed me that when the surface of a ball of ice of 3 centimeters diameter and at a temperature of $-2°$, is raised to 0° and kept at this temperature, it may be shown that in less than an hour the temperature at the centre has risen to $-0.01°$.

4. Another important influence on the velocity of descent is found in the fact that it is possible that not all the ice, melting at the lower side of the wire, freezes again exactly at the upper side, but that this water perhaps flows off laterally. It is clear that this must have a great influence since then the heat, necessary for the melting, is furnished by conduction through the ice. Already J. THOMSON

BOTTOMLEY [1]) showed that the lateral flow of water causes a great retardation.

The experiments now showed that this lateral flow really exists. For even when the ice was perfectly clear, in the places where the wire had passed through it various small bubbles were observed. Consequently not all the ice had been re-formed which had been there.

In this respect I also mention a curious change, found in the values of C: these values rise with the weight. This is very conspicuous with the silver and german silver wires, but also with the steel wires it exists, especially with the thick one of 0.87 mm.

Accordingly it was often seen that the bubbles on the path of the wire were more numerous with small than with heavy weights. This became particularly clear in experiments in which, during one descent, first a heavy and then a small weight was used. With the smaller weight more water flows off laterally.

I still made several experiments in which the wire was pulled upwards through the ice, hoping to prevent this lateral flow. The result was not the expected one, for bubbles also appeared and the values, found for C, were even somewhat smaller than in the former case. In regard to this question it would be desirable to investigate the descent of a whole body, e.g. of an iron ball, through perfectly clear ice.

In my opinion this lateral flow is the chief reason why theory and observation disagree. It also explains why with the silver wires larger differences were found than with the german silver and the steel wires. For if the heat is only partly furnished by the freezing process above the wire and if the rest has to be furnished by conduction through the ice, it seems to be of little consequence whether the wire be a good conductor of heat.

5. Ice is a crystalline substance. This also may have its effect. Perhaps the melting point is not the same at the different faces of the crystals which the wire touches. Though this influence may exist, we cannot say in which direction it would modify the result.

In order to find out whether such an influence makes itself felt, I made the wire pass several times through the same block of ice in three mutually perpendicular directions. But no perceptible difference was found.

As the general result of the experiments I think we may state, that they indicate that the regelation theory will be found capable of explaining the phenomena not only qualitatively but also quantitatively.

---

[1]) Pogg. Ann. 148, p. 492, 1871.