mentum, the instrument (PROBST's covered hook) passing through the middle crus cerebelli. In this cat were found a certain number of fibres degenerated, which passed through the regio reticularis of the side of the lesion and then, crossing the raphe and running upward in the predorsal region of the other side, took their way towards the red nucleus. This experiment tends to show, that there are direct fibres, coming from the basal cerebellar nuclei, which do not join the superior crus, but follow the ventral course to arrive at the red nucleus. LEWANDOWSKY,s fibres O. P. (in fig. 66 and 37) are not to be identified with these fibres on account of their entirely different course.

In cat LXII the anterior crus cerebelli was partially cut, and at the same time an incision made into the middle crus. Also in this animal there was found no degeneration on the distal side of the lesion except the bundle of Monakow. Were the ventral bundle to be regarded as being formed by descending collaterals of the anterior crus this result could hardly be explained. A simular result was obtained in cat LXVIII, where hemisection of the pons was effected. Also here there was no degeneration on the distal side of the lesion.

Mathematics. — *"Equilibrium of systems of forces and rotations in Sp_4."* By Dr. S. L. VAN OSS. (Communicated by Prof. P. H. SCHOUTE).

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Referring to the following well-known properties:

a. The coordinates p_{ij} and π_{ij} of a line p and a plane π satisfy the five relations:

$$P_i \equiv p_{kl}\, p_{jm} + p_{lj}\, p_{km} + p_{jk}\, p_{lm} = 0 \quad , \quad \pi_i = \sum_i \pi_{kl}\, \pi_{jm} = 0, \quad . \quad (1)$$

of which relations three are mutually independent.

b. The condition that a line p and a plane π intersect each other is expressed by

$$\Sigma\, p_{ij}\, \pi_{ij} = 0 \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

c. The coordinates of the point of intersection X of two planes π, π' and that of $Sp_2 \Xi$ through two lines p, p' are:

$$x_i = \pi_{kl}\, \pi'_{jm} + \pi_{lj}\, \pi'_{km} + \pi_{jk}\, \pi'_{lm} + \pi_{jm}\, \pi'_{kl} + \pi_{km}\, \pi'_{lj} + \pi_{lm}\, \pi'_{jk} \, ;$$

$$\xi_i = \sum_i p_{kl}\, p'_{jm} \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

we wish to draw the attention to the following properties:

If (ij) are ten *arbitrary* quantities: i.e. not satisfying the relations

$\sum_{i} (jk)(lm) = 0$, we shall continually be able to break up each of these quantities into two parts $(ij)'$ and $(ij)''$, so that $\sum_{i}(kl)'(jm)' = \sum_{i}(kl)''(jm)'' = 0$. It is easy to see that this decomposition can be done in ∞^4 ways.

For each decomposition holds good:

$$\sum (kl)'(jm)'' = \sum (kl)(jm) \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

for:

$$\sum_{i} (kl)'(jm)'' = \sum_{i} (kl)'\{(jm) - (jm)'\} = \sum_{i} (kl)'\,(jm),$$

likewise:

$$\sum_{i} (kl)'(jm)'' \qquad\qquad = \sum_{i} (kl)''(jm),$$

from which by addition:

$$\sum_{i} (kl)'(jm)'' = \sum_{i} (kl)(jm).$$

Giving a geometrical interpretation we regard a homogeneous system of 10 arbitrary quantities a_{ij} and α_{ij} as the coordinates of

a system a of ∞^4 lines, in pairs a', a'' conjugated by the relations:

$$a'_{ij} + a''_{ij} = a_{ij} \quad . \quad . \quad . \quad (5)$$

All these lines lie in one Sp_3 Ξ having as coordinates:

$$\xi_i = \sum_{i} a_{kl}\, a_{jm} \quad . \quad . \quad . \quad . \quad (6)$$

a system α of ∞^4 planes, in pairs α', α'' conjugated by the relations:

$$\alpha'_{ij} + \alpha''_{ij} = \alpha_{ij} \quad . \quad . \quad . \quad . \quad (5')$$

All these planes pass through one point X, having as coordinates:

$$x_i = \sum_{i} \alpha_{kl}\, \alpha_{jm} \quad . \quad . \quad . \quad (6')$$

We now annul the homogeneousness of the p-, π-, a- and α-coordinates.

This causes those elements to assume vector-nature and makes them interpretable respectively as force, as rotation, as dynam and as double-rotation. The equations (5), (5′) determine the reduction of the *vectors* a and α on the conjugate pairs of lines and of planes of the systems a and α under consideration and not yet partaking vector-nature, whose structure now becomes revealed.

II. In connection with the meaning given in b of the equation $\sum p_{ij}\,\pi_{ij} = 0$ we interpret

$$\sum \alpha_{ij}\, p_{ij} = 0 \quad . \quad . \quad . \quad . \quad (7)$$

as the condition that a line p cuts a pair of conjugated planes of system α.

$$\sum a_{ij}\, \pi_{ij} = 0 \quad . \quad . \quad . \quad . \quad (7')$$

as the condition that a plane π cuts a pair of conjugated lines of system a.

This gives us a very fair survey of the structure of the linear complex of lines and planes. The reduction of the equation of the complex of planes to its diametral space is now easy to do; likewise the further reduction to the simplest form $(kl) = \sigma\,(jm)$, assumed by the equation when the edges kl and jm, the planes ijm and ikl of the simplex of coordinates are conjugated elements of the systems a or α.

III. If we assign to the elements p, π, a, α vector-nature, expressions $\Sigma\, \alpha_{ij} p_{ij}$, $\Sigma\, a_{ij} \pi_{ij}$ become of importance as *virtual coefficients* (in BALL's theory of screws) and the disappearing of these coefficients then gives the *condition that the force p performs no work at a displacement in consequence of a double rotation α, resp. that the dynam a performs no work at a rotation π.*

So in BALL's notation the equations (7), (7)′ give the *condition of reciprocity between force and double rotation, resp. between dynam and rotation.*

In like manner the equation

$$\Sigma\, a\ \alpha_{ij} = 0\,, \qquad \qquad (8)$$

which includes (7) and (7)′ and likewise (2), gives the condition of reciprocity between the dynam a and the double rotation α.

IV. We shall now pass to the general equilibrium of forces and rotations. It will be convenient to understand by p, π, a, α vectors *unity* and to indicate the intensity of these vectors by a factor.

It will be sufficient to limit ourselves to the equilibrium of forces, leaving the treatment of the dual case to the reader.

In the first place we regard the case of n forces, $n > 10$ working along lines given arbitrarily.

It goes without saying that for the equilibrium it is necessary and sufficient that the intensities $k^{(\nu)}$ satisfy the ten conditions:

$$\Sigma\, k^{(\nu)}\, p^{(\nu)} = 0 \qquad \qquad (9)$$

We can therefore in general bring arbitrary intensities along $n - 10$ vectors, those on the other ten then being determined by the above equation (9).

In particular for $n = 11$ the theorem holds:

To vectors along eleven lines given arbitrarily belongs in general only one distribution of ratios of intensity, so that the system on those lines is in equilibrium.

The generality of the case is circumscribed by the requirement that no ten lines can satisfy one and the same linear condition in the form $\Sigma\, a_{ij}\, p^{\nu}_{ij} = 0$, where the coefficients a_{ij} do not depend on ν, in consequence of a well-known property of determinants tending to zero.

So if there are among n lines at most 10 belonging to a linear complex we can satisfy the equations (9) by choosing all intensities except those belonging to these 10 equal to 0 and then (if not all subdeterminants of order 9 tend to 0) we shall be able to bring along these last only *one* distribution of intensity differing from 0 in such a way that the system of forces obtained in this manner is in equilibrium.

We have thus at the same time arrived at the following theorems:

For the equilibrium of ten forces it is necessary that these belong to one and at most to one linear complex. In this case always one and not more than one distribution of intensity is possible.

If we continue the investigation of the equations (9) we then obtain successively the conditions of equilibrium of 9, 8, 7, 6, 5 forces. We can express the result as follows:

In order to let n forces, $11 > n > 4$, admit only of one distribution of intensity in equilibrium, it is necessary and sufficient for them to be the common elements of exactly $21-n$ linear complexes.

In particular for $n = 5$ we find the condition that the forces must belong to a system of associated lines of SEGRE.

This has given us a connection with a former paper in which we treated this case synthetically.

V. The condition that ten forces in equilibrium belong to one complex follows almost immediately out of the interpretation of the equation $\Sigma a_{ij} p_{ij} = 0$ as condition of reciprocity of force and double rotation.

Let e.g. ten forces be given in equilibrium; nine of these forces chosen arbitrarily determine a complex, so also the double rotation α for which none of them can perform labour. The united system of ten forces, as being in equilibrium doing no labour for no motion whatever, it is necessary for the tenth force to be likewise reciprocal with respect to the double rotation α, i.e. this force belongs with the former nine to the selfsame complex.

Equally simple is the deduction of the conditions of equilibrium for nine forces.

For eight forces determine a simply-infinite pencil of complexes whose conjugate double rotations $\alpha + \lambda\alpha'$ are all reciprocal with respect to these eight forces. So they must also be reciprocal with respect to the ninth force in equilibrium with these, i. o. w. the latter must belong to all linear complexes to which the eight others belong.

And so on.

VI. We shall now denote still, by means of a few words, in which way we can arrive at an extension of the screw-theory of BALL by the application of the principle of exchange of space-element to the equations $\sum_{1}^{10} a_i \xi_i = 0$.

By interpreting this equation either

1st. as condition of united position of a point X and an Sp_8 Ξ in Sp_9,

2nd. as condition of reciprocity (BALL) of a dynam X and a double rotation Ξ,

we make a connection between the point- and Sp_8-geometry in Sp_9 on one hand and the geometry of dynams and double rotations on the other hand.

To each theorem of the former corresponds a theorem of the latter geometry. Nov the remarkable fact makes its appearance that *the fundamental theorems of the geometry of Sp_6 correspond to the fundamental theorems of the theory of screws of* BALL *in* Sp_3.

With this as basis we shall show, though it be but by means of some few examples of a fundamental nature, that the principles of a generalisation of the theory of screws are very easy to be arrived at by transcription of the simplest properties of the point- and Sp_8-geometry in Sp_9 which examples can at the same time be of service to explain the above observations on the theory of BALL in Sp_3.

To avoid prolixity we introduce the following notation. We call:

dynamoid the system of lines whose conjugate pairs can serve as bearers of a dynam.

rotoid the system of planes whose conjugate pairs can serve as bearers of a double rotation: So dynamoid and rotoid correspond to dynam and double rotation as in the notation of BALL "screw" to dynam and helicoidal movement.

Let the following transcriptions be sufficient to explain the application of the above principle.

σX:	Point X bearing a mass σ.	Dynamoid X bearing a dynam of intensity X.
$\sigma\Xi$:	Sp_8 Ξ with a density of mass σ;	Rotoid Ξ bearing a double rotation of intensity σ.
$(X'X'')$:	Right line, locus of the centres of gravity of variable masses in the points X' and X''.	Pencil of dynamoids, locus of the bearers of the resultants of two variable dynams on the dynamoids X' and X''.
$(\Xi'\Xi'')$:	Sp_8-pencil.	Pencil of Rotoids.
	A right line has always	A pencil of dynamoids always

a point in common with an Sp_8.

An Sp_8 is determined by nine points.

p spaces Sp_8 cut each other according to Sp_{9-p}.

contains a dynamoid reciprocal to a given rotoid.

A rotoid can always be determined lying reciprocal with respect to nine dynamoids.

The dynamoids reciprocal to the movements of a body with p degrees of freedom form a $(9-p)$-fold infinite pencil.

Etc. etc.

We shall now apply the above to the problem: "To decompose a dynam according to *ten* given dynamoids", this problem being a transcription of the following:

"To apply to ten given points a distribution of mass so that the centre of gravity finds its place in a given point."

We again put side by side the results.

To be defined successively:

a. An Sp_8 through nine of the given points.

b. The right line through the remaining point and the centre of gravity.

c. The point of intersection of this right line with the Sp_8 found in *a.*

d. The decomposition of the mass in the centre of gravity according to this point of intersection and the 10^{th} point named in *b*, which is possible, these three points being collinear; gives at once the mass to be applied in the last named point.

The other must necessarily become the centre of gravity of the remaining nine points.

e. These treatments to be repeated for the determination of mass in the other points.

The rotoid reciprocal to nine of the given dynamoids.

The pencil of dynamoids through the remaining dynamoid and the bearer of the given dynam.

The dynamoid on this pencil reciprocal with respect to the rotoid found in *a.*

The decomposition of the given dynam according to the dynamoid found in *c* and the 10^{th} dynamoid named in *b*, which is possible, these three dynamoids belonging to one pencil, gives at once the intensity of the dynam on the last mentioned dynamoid.

The other must necessarily bear the resultant of the dynam to be applied to the remaining nine dynamoids.

These treatments to be repeated for the determination of intensity on the other dynamoids.

Zalt-Bommel, March 28, 1907.