Citation:

J.A.C. Oudemans, Supplement to the account of the determination of the longitude of St. Denis (Island of Reunion) executed in 1874, containing also a general account of the observation of the transit of Venus, in:
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$$z^{IV} = -z^3\left(6\frac{z^2}{z^3} + \frac{\ddot z}{z^2} - 5\frac{\ddot z^2}{z^4} - \frac{20}{3}\frac{\ddot z}{z^2}\frac{\dot z^2}{z^3} + \frac{40}{3}\frac{\dot z^4}{z^6}\right)$$

As $\frac{1}{a}$, p and the 3 heliocentric distances r are known, $\dot z^2$, $\ddot z$ and z^{IV} can be computed for each of the 3 points P_1, P_2 and P_3.

For a circular orbit all the derivatives of z are equal to zero and the function U becomes $\frac{\sin \tau \sqrt{z}}{\sqrt{z}}$. According to the preceding development we obtain for an elliptic orbit the following approximate formula for U, which still contains the 6th power of the interval:

$$U = \frac{\sin \tau \sqrt{z}}{\sqrt{z}} + \frac{\tau^5}{20} z^2 \left(9 - 4\frac{r}{a} - 5\frac{p}{r}\right). \quad . \quad . \quad (VIII)$$

By means of the values which take U_1, U_2 and U_3 for the values τ_1, τ_2 and τ_3 of the argument τ, we obtain for n_1 and n_3 values containing the terms of the 5th order with respect to the intervals; while the approximation may be extended to the 6th order, if we add to the above mentioned expression for U:

$$-\frac{\tau^7}{5040} z^3 \left(-\lambda - 120\mu + 170\lambda^2 + 340\lambda\mu - 1020\mu^2\right).$$

where λ and μ denote:

$$\lambda = \frac{3}{2}\left(9 - 4\frac{r}{a} - 5\frac{p}{r}\right)$$

$$\mu = 3\left(2 - \frac{r}{a} - \frac{p}{r}\right).$$

Astronomy. — "*Supplement to the account of the determination of the longitude of St. Denis (Island of Réunion), executed in* 1874, *containing also a general account of the observation of the transit of Venus*". By Prof. J. A. C. OUDEMANS.

When I set about to correct the imperfections left in my first communication, I began by calculating for the times of observation of the occultations the correction of NEWCOMB's parallactic correction, mentioned on p. 603 of my previous paper; as said there this correction amounts to

$$+ 0''67 \sin D + 0''05 \sin (D - g) - 0''09 \sin (D + g'),$$

where D stands for the mean elongation of the moon from the sun, g for the moon's mean anomaly, and g' for that of the sun.

D and g could be derived from Tables I and II at the end of NEWCOMB's paper [1]; there, however, in agreement with the method introduced by HANSEN in his tables of the moon, the unit is not a degree, but a mean day, so that the numbers derived from those tables must be multiplied by 12°,19 and 13°,065 respectively in order to be reduced to degrees. g' could be derived with a sufficient accuracy for our purpose from the tables of LARGETEAU in the Conn. des Temps for 1846.

The corrections found are, however, not to be applied to the true, but to the mean longitude, called by HANSEN $n\,\delta z$, and therefore must be reduced to corrections of the true longitude by multiplication by $(1 + 2e \cos g \ldots.)$, and these must again be reduced to corrections of right ascension and declination; for the latter reduction I used the moon's hourly motions in R. A. and declination from the Nautical Almanac, whence the direction of the moon's motion with regard to the parallel could be directly derived.

The corrections of the moon's ephemeris in the Nautical Almanac, given by NEWCOMB on p. 41 of his *Investigation* for each day from 1 Sept. 1874 to 31 January 1875, were corrected for the two first months by means of the values found; and in the calculation of the longitude from the occultations we have now applied these corrected corrections, instead of the corrections furnished by the meridian observations. It then would become evident which corrections were to be preferred, and it soon appeared that it was the former. The large corrections in declination found for 19 Sept. and 16 Oct. 1874, (— 4″.3 and — 4″.1) for instance, in consequence of which the second occultation observed on 19 Sept. was formerly rejected, were apparently due to the inaccuracy of the meridian observations.

I now shall give the details of my calculation. (see table p. 112).

These corrections were added to those given on p. 609 of my first account as being interpolated from NEWCOMB, and then the required alterations were made in all the calculations of the occultations.

Before I passed on to this second communication, I have once more thoroughly revised all the computations and thus was able to apply some corrections; some occultations which had been rejected, could now be retained after the error had been corrected.

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1) S. NEWCOMB. Investigation of corrections to HANSEN's Tables of the Moon; with tables for their application, forming Part III of papers published by the Commission on the Transit of Venus, Washington, Government Printing Office, 1876.

1874	M.T.Gr.	D	g	g'	0″67 $\sin D$	0″05 $\sin(D-g)$	— 0″09 $\sin(D+g')$	Sum $= \Delta n \delta z$	$\Delta \lambda$	$\Delta \alpha$	$\Delta \delta$
Sept. 19	5^h8	105°5	256°7	257°	+0″64	—0″02	0″00	+0″62	+0″60	+0″68 = +0^s04	+0″01
21	7.5	130.5	283.5	259	+0.51	—0.02	—0.05	+0.44	+0.45	+0.48 = +0.03	+0.11
22	3.6	140.9	294.6	260	+0.42	—0.02	—0.06	+0.34	+0.35^5	+0.34 = +0.02	+0.11
22	10.3	144.1	298.1	260	+0.39	—0.02	—0.06	+0.31	+0 32^5		
26	4.8	190.3	347.5	264	—0.12	—0.02	—0.09	—0.23	—0.26	—0 22 = —0.01^5	—0.12
Oct. 2	11.5	267.1	69.8	270	—0.67	—0.01^5	—0.00^5	—0.69	—0.71	—0.79 = —0 05	+0 07
4	11.6	291.5	95.9	272	—0.62	—0.01	+0.04	—0.59	—0.58	—0.60 = —0.04	+0.18
15	3.8	61.7	235.3	283	+0.59	—0.01	+0.02	+0.60	+0.56	+0.63 = +0.04	—0.09
16	5.0	74.3	248.8	284	+0.64	—0.00^5	0.00	+0.64	+0.61	+0.70 = +0.05	—0.02
17	7.2	87.7	263.2	285	+0.67	0.00	—0.02	+0.65	+0.64	+0.72 = +0.05	+0.06
18	8.3	100.3	276.7	286	+0.66	0.00	—0.04	+0.62	+0.62^5	+0.68 = +0.04^5	+0.13
19	7.4	112.5	289.7	287	+0.62	0.00	—0.06	+0.56	+0.58	+0.60 = +0.04	+0.17

I shall briefly record the modifications of my previous account [1]).

The following are the numbers originally belonging to the observations mentioned there: 1, 3, 4, 7, 9, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37.

Hence were rejected the numbers: 2, 5, 6, 8, 13, 15, 17, 19, 21, 26, 33, 38, 39.

N°. 2, disappearance of Arg. Z. 233, N°. 77, observed by me on the ground of the harbour office on 19 Sept., yielded $-21^s.31$ for the correction of the eastern longitude. It appeared that this large value was for the greater part due to the large correction ($-4''3$) applied to the moon's declination as derived from the meridian observations. The corrected corrections of Newcomb were $-0^s.45$ and $+0''.3$ (that of the declination even with another sign), and the correction of the eastern longitude became $-6^s.85$, not larger than several others.

N^os^. 5 and 6, disappearances of Arg. Z. 311, N^os^. 72 and 75 observed by me on the ground of the harbour office on 21 Sept. It appeared that in reducing these two observations the correction of the chronometer had been taken from the journal with a wrong sign. After rectification of this error the results were satisfactory.

N°. 8, disappearance of a 9th magnitude star, observed by me on the ground of the harbour office on 22 Sept. at $7^h38^m25^s,07$, hence 34^m9^s after that of 33 Capricorni. I have not succeeded in rectifying this observation. Judging from the map which by means of Argelander's Zonae had been constructed preliminary to the observations, it seemed that the star could be no other than N°. 18 of A. Z. 255, but then the correction of the eastern longitude would have been $+58^s,24$. Supposing that an error might have occurred in noting down the minute of the time of observation, I repeated the calculation adopting the time to be 1 minute later, but now I got: Corr. of the E. longitude $+20^s,07$. The time of observation ought therefore to be taken another half minute later, but I did not hold myself justified to do so.

There still followed two occultations, which were missed through clouds, probably one of these two has been A. Z. 255 N°. 18, and the star observed by me does not occur in A. Z. Neither Schönfeld's southern atlas, nor Gill's catalogue could help me to arrive at a conclusion.

N°. 13, disappearance of 73 Piscium on 26 September, N°. 15, disappearance of 53 Geminorum and N°. 17, disappearance of a star

[1]) In the 3rd column on p. 607 a few clerical or printing errors have crept in: for Cordoba III 1589 read Cordoba XVIII 1589 and for Cordoba XVIII 124 read Cordoba XVIII 1612.

of the $6^{1}/_{2}$th magnitude, both on 2 October, were recorded as uncertain and besides the results were too discordant. Full moon had occurred on 25 September; so these disappearances took place at the bright limb of the moon, and it is well-known how uncertain their observation is then. In this case a sudden disappearance can only be observed with stars of the 1st or 2nd magnitude.

N°. 19, disappearance of B. A C. 5800 on 15 October, also yielded a large negative correction of the eastern longitude (—13s,14), but there was no reason for rejecting it; the disappearance took place at the dark limb, the star was of the $6^{1}/_{2}$th magnitude, hence very bright in the telescope, and in the journal of observation uncertainty is *not* mentioned.

N°. 21, disappearance of A. Z. 223 N°. 48, observed by Mr. E. F. van de Sande Bakhuyzen on 16 October, yielded + 33s,70; N°. 26 disappearance of a star of which the place was $20^{h}5^{m}16^{s}$ — 25°10′46″, gave — 120s,6. Both had therefore to be rejected.

Nor was I more fortunate with N°. 33. The star as determined at Leyden gave an unsatisfactory result (+ $21^{m}38^{s}$) and I could not find in the catalogues another star which fulfills the requirements.

I succeeded better with N°. 38. N°. 38 had been noted by Mr. Bakhuyzen as disappearance of ζ Piscium on 28 October; it appears however that this star was not occulted and that the occulted star could be no other than 24 Piscium; assuming this, I arrived at a satisfactory conclusion.

In the case of N°. 39, disappearance of 59 Geminorum at the dark limb on 26 November, we could only obtain a result, that was not wholly inadmissible, by assuming a combination of errors. Although each of these was in itself not quite improbable, it was thought necessary to reject also this observation.

About N°. 27 I remark that on p. 607 I noted as observers S.B. i. e. that both Mr. Soeters and Mr. Bakhuyzen observed the occultation (disappearance at the dark limb); as the time recorded by Mr. Bakhuyzen was 4 seconds *later* than that noted by Mr. Soeters, I accepted the result of the former as the more probable one, the more so as it agreed better with the other results.

Generally, the endeavours to rectify the occultations, which at first seemed to have failed, have cost more work than those where nothing was wrong.

Finally I must remark that Mr. Soeters himself had corrected a small error of computation in his reduction of the observations, made to determine the relative position of our different observing places, but had neglected to enter the corrected value in the final table of his results. In consequence of this error we must read for the longitude east of Greenwich of the observing place on the ground of our dwelling house, as given on p. 604 $3^h 41^m 48^s.06$ instead of $48^s.11$.

Taking into account the corrections mentioned above, the list of results communicated in the Proceedings of March (p. 607 of the preceding volume) must be modified as given at the end of this paper; see tables I*a* and I*b*.

We then find as correction of Germain's longitude:

Using disapp. and reapp. indifferently . . $-2^s.90 \pm 0^s.64$ (m.error)
Treating them separately . . . . $-0\ .81 \pm 1\ .22$ („ „)

It is much to be regretted we did not succeed in observing more reappearances. There is always a greater chance to observe the disappearances than the reappearances at the dark limb of the moon. A short time after new moon until a few days after first quarter we can easily see with a good telescope on the east side of the moon stars of the 8^{th}, 9^{th} or perhaps even the $9^1/_2{}^{th}$ magnitude, of which the disappearance may be easily observed; no preparation is required for this.

For the observation of reappearances at the dark limb, a preparation by means of star maps is necessary, which takes up much time. We must calculate from hour to hour the parallax of the moon in R. A. and declination and hence derive its apparent places, draw them on the map, and then derive geometrically the instants at which the stars considered must reappear. For the most southern declinations the star maps themselves had to be constructed first by means of ARGELANDER's southern Zonae. Moreover it is always desirable finally to derive more accurate results by a calculation according to the known formulae.

The operations described here have been executed as well for the days preceding full moon as for those following it, and it was our bad luck that in the latter part of the lunation the weather was always unfavourable.

After this revision of the calculations a small negative correction of the longitude of St. Denis according to Germain seems probable, although its exact amount is uncertain. We have however, still to consider what follows:

When in 1884 AUWERS wanted to determine a fundamental meridian for Australia [1]), for which purpose he chose that of SYDNEY, he used 78 occultations observed from 1873 to 1876 in Windsor (N.

[1]) Astron. Nachr. Vol. 110 p. 289—346.

S. Wales) by TEBBUTT and 18 occultations observed by ELLERY in 1874 and '75 at Melbourne. He applied to the ephemerides of the moon of the Nautical Almanac the corrections of NEWCOMB's *Investigation* and took for the relation between the moon's radius and the horizontal parallax the value $k = 0.27264$ found by me. (*Versl. en Meded. Akad. Amsterdam Afd. Nat.* 1st *Reeks, Vol. X* p. 25).

But as nevertheless a constant error might occur in the obtained results, he calculated, as a test, a large number of occultations, which had been observed either at Greenwich or at places of which the longitude had been determined by telegraph, viz: 31 observed at Greenwich, 25 at Washington, 40 at Nikolajef, 44 at Oxford, 30 at Luxor, 46 at Strassburg, 13 at Leipzig, 7 at Vienna, 2 at Königsberg, 2 at Moscow, 2 at Pulkowa and 1 at Kiel.

Thus he was able to derive the correction to be applied to a longitude determined by a disappearance at the dark limb, and found for this after a graphical compensation :

1873,0	+ $1\overset{s}{,}08$,
1873,5	+ 1,63,
1874,0	+ 2,12,
1874,5	+ 2,52,
1875,0	+ 2,84,
1875,5	+ 3,11,
1876,0	+ 3,27,
1876,5	+ 3,38,
1877,0	+ 3,52.

I have on purpose given this table in full to show how constant is the positive sign of the correction. For the reappearances AUWERS found a correction which in the mean was larger by + $0^s,23$. (Although this value has been found having regard to weights, it yet seems to me rather uncertain; I find for its mean error ± $0^s,64$). It appears from this that this correction is due not to an erroneous value of the moon's radius, but to a slowly varying error still left in the tables of the moon.

Now if we want to apply this correction — and I consider this as quite justified — we must also take for the relation k between the apparent radius of the moon and the horizontal parallax the same value as AUWERS has used and hence apply the necessary corrections to our longitude.

The formulae required for this could be easily derived. Let the difference in right ascension between the moon's centre and the place of the star, after the moon's parallax, calculated for the point where the occultation took place, has been added to it with a contrary sign, be denoted by I; let the geocentric radius of the moon (as it was used for this calculation) be $= R$, the horizontal parallax $= \Pi$ and the difference in declination between the reduced place of the star and the moon's centre $= v$, then we have

$$\mathrm{I} = \frac{1}{15}\, sec.\, \frac{d' + D}{2} \sqrt{R^2 - v^2},$$

and hence

$$\partial \mathrm{I} = \frac{\mathrm{I}\, R}{R^2 - v^2}\, \partial R,$$

but as

$$R = \Pi k$$

we have

$$\partial R = \Pi \partial k$$

hence

$$\partial \mathrm{I} = \frac{\mathrm{I} R \Pi}{R^2 - v^2}\, \partial k = \frac{\mathrm{I}\, R^2}{R^2 - v^2} \cdot \frac{\partial k}{k}.$$

The reduction to be added to the star's R. A. to get that of the apparent moon's centre is $\mp \mathrm{I} + \mathrm{II}$, where the 2^d term is independent of k and the upper sign of the first is to be used for disappearances, the lower for reappearances.

If the hourly motion of the moon in R. A. is $\Delta\alpha$, the correction of the Greenwich mean time of an occultation is $\frac{\mp \mathrm{I} + \mathrm{II}}{\Delta\alpha} \times 3600^s$ and the correction of the eastern longitude derived from it: $\frac{\pm \mathrm{I} - \Pi}{\Delta\alpha} \times 3600^s$. Now as the assumed value of k was 0,272525 we have $\partial k = + 0{,}000115$:

and

$$\partial \, \mathrm{E.\,L.} = \pm\, 3600 \times \frac{0.000115}{0.272525} \cdot \frac{\mathrm{I}\, R^2}{(R^2 - v^2)\, \Delta\alpha}.$$

$$= \pm\, [0{,}1814]\, \frac{\mathrm{I}\, R^2}{(R^2 - v^2)\, \Delta\, \alpha},$$

where the value in square brackets is a logarithm, and the logarithms of the other factors may be derived from the former calculation. The $+$ sign is to be used for disappearances, the $-$ sign for reappearances.

In this way I have found the corrections given in table II and thus obtained corrected values for the longitude. I think it best to use indistinctly the results from disappearances and reappearances.

We then find as mean correction of
GERMAIN's longitude: — $2^s,15 \pm 0^s,79$ (mean error)
the supplementary correction according
to AUWERS is for 1874,80: $+ 2^s,71 \pm 0^s,50$ [1])
and the final correction is $+ 0^s,56 \pm 0^s,93$.

Although our calculations were somewhat modified and a systematic correction was applied, which seems to be required, we arrive at the same conclusion as in our first paper, viz. that the correction of the longitude of St. Denis found by GERMAIN, in so far as we may judge from the occultations observed by us, is very small. If we pay attention to the mean error of our result, it is not even certain whether it is negative or positive, though there is a greater probability in favour of a small positive correction.

In my previous paper I have not mentioned that the reduction to 1874 of the places of the stars from all the available catalogues has been very carefully executed by Mr. H. KRESS, "amanuensis" at the Observatory at Utrecht. The derivation of the most probable places from the whole material I have made myself.

It will be interesting to record that the meridian observations of the moon, made at Leyden in Sept. and Oct. 1874 by Mr. H. HAGA, then assistant at the observatory (now professor of physics at the university of Groningen), has yielded the following corrections of the places in the Nautical Almanac, previously corrected according to NEWCOMB's *Investigation*: (see table p. 119)

1) This mean error has been estimated, and is based on the argument that the value of the correction, which was found by graphic compensation, rests on about 25 occultations, while AUWERS has arrived at the result (*A.N. Bd.* 110, *column* 336) that one disappearance at the dark limb yields a longitude, of which the mean error may be considered to be $\pm 2^s,5$.

1874	Limb.	Obs — Comp. $\Delta\alpha$	Obs — Comp. $\Delta\delta$	Remarks.
September 21	I upper	-0^s14	$-4''7$	Very unsteady.
24	I upper	+0.07	+1.3	Clouds.
» 26	II lower	-0 28	+0 1	
» 27	II lower	-0 08	+1.9	
» 30	II lower	-0 04	-5.1	Clouds.
October 1	II	-0.07		
» 15	I	+0 35		Very faint, uncertain.
» 20	I upper	+0.22	-1.4	Very unsteady.
» 22	I upper	+0.13	+0.8	
» 24	I upper	+0.15	+1.8	
» 26	II lower	-0.08	+1.1	
» 27	II lower	-0 12	+1.3	
» 28	II lower	-0.14	+2.3	Clouds.
» 30	II upper	-0.14	-0.5	
	Mean value :	-0^s01	$-0''1$	

These results have not yet been published, but have been lately communicated to me by Dr. E. F. VAN DE SANDE BAKHUYZEN.

It will be desirable also to consider the other determinations of the longitude of St. Denis de la Réunion. Dr. E. F. VAN DE SANDE BAKHUYZEN kindly communicated them to me. These determinations, whose results only we shall mention for brevity, were made by Lord LINDSAY and Dr. COPELAND on the one side and by Messrs. Löw and PECHÜLE on the other side, on their respective observing-stations Belmont and Solitude, both on the isle of Mauritius, the differences of longitude of those stations and St. Denis being determined by transportation of chronometers.

Lord LINDSAY and Dr. COPELAND [1]) found for Belmont:

[1]) Dun-Echt Observations. Vol. III. p. 171.

by means of 52 chronometers on the home voyage [1])	$3^h50^m40^s.03$
from observations of the moon:	
from 11 occultations (7 disappearances and 4 reappearances at the dark limb)	$3^h50^m40^s.60 \pm 0^s.33$
from 12 culminations of the moon	42 .6

Assigning to these two results weight 2 and 1, we get as mean result $3^h50^m41^s.27$

Reduction on St. Denis flag-staff determined by transportation of chronometers $-8^m53^s.41$

Hence longitude of St. Denis flag-staff E. of Gr.

from the chronometers	$3^h41^m46^s.62$
from observations of the moon	47 .86

The German observers found for the longitude of Solitude [2])

from 6 culminations of the moon	$3^h50^m39^s.52 \pm 3^s.29$
from 3 occultations	40 .33 ± 1 .91
whence in the mean	3 50 40 .13 ± 1 .65

Now Solitude is situated	$0^s,89$	west of Belmont,
Belmont	8^m53 ,41	east of St. Denis,
hence Solitude	8 52 ,52	„ „ „ „ .

Hence longitude of St. Denis (flag-staff) E. of Gr. $3^h41^m47^s.61$

Combining all these results, omitting only that from the chronometers (comp. footnote) we have:

			Weight
by means of the longitude of BELMONT, (observations of the moon)	$3^h41^m47^s,86$	$\pm 0^s,90$	1,25
by means of the longitude of SOLITUDE, (observations of the moon)	47 ,61	± 2 ,00	0,25
determination by GERMAIN (culminations of the moon):	47 ,40	± 0 ,76	1,72
determination by OUDEMANS and BAKHUYZEN, (occultations with corrections according to AUWERS):	47 ,96	± 0 ,93	1,16
Adopted longit. of St. Denis flag-staff	$3^h41^m47^s,69$	$\pm 0^s,44$	4,38

[1]) Unfortunately the outward voyage has not yielded any result, because the rates of the chronometers after the landing could not be determined, as it had been neglected to wind them up. And this accident also takes off much of the value of the home voyage, because through this the difference between the rate at sea and that on land could not be eliminated. AUWERS has already made this remark in: Die Venus-Durchgänge 1874 und 1882. Bericht über die Deutschen Beobachtungen, Vol. VI p. 265, and therefore we shall also leave this result out of account.

[2]) A. AUWERS. Die Venus-Durchgänge 1874 und 1882. Bericht über die Deutschen Beobachtungen Vol VI.

The longitude of our station now being determined as well as possible, I shall proceed to communicate our contact observations of Venus and the Sun during the Transit on December 9th.

Our place of observation was on the battery, in the immediate neighbourhood of the pavilion of the heliometer; its longitude must therefore be accepted to be $3^h41^m47^s,81 + 0^s,26 = 3^h41^m48^s,07$.

The ingress took place very early in the morning, the sun being only five degrees above the horizon. Unfortunately at sunrise the sky was not quite clear. In the east, a few degrees above the horizon there was a dark stratus, and it was to be feared that at the instant of the second contact the sun would just be behind it. So it happened, and this was the more unfortunate as the station Réunion had expressly been chosen for the observation of that contact.

At the first contact the sun's limb was very unsteady. At $5^h38^m20^s$ mean time St. Denis, I thought that I saw an impression on the sun's limb, which I held to be made by Venus. A passing cloud, however, prevented me from seeing whether I had been right. When, after a minute the sun reappeared, I could not distinguish the impression on the limb any more. At $5^h41^m20^s$ it could however be seen plainly. The place where I then saw it was exactly the same as that where I had thought to see it 3 minutes earlier. However, as Venus had moved on 6″ during those 3 minutes, the observation of the first contact must be considered as having failed. The mean between the two instants mentioned is no more than a very rough approximation.

As said already, the second contact was missed.

But both Mr. Soeters and I observed the two last contacts.

The formulae, given in the Nautical Almanac of 1874 on p. 434 for the calculation of the contacts, are:

For the first external contact:

$$t = 13^h45^m58^s - [2,5773]\,\varrho \sin l - [2,7049]\,\varrho \cos l \cos(\lambda + 136°39'.9),$$

for the first internal contact:

$$t = 14^h15^m24^s - [2,6992]\,\varrho \sin l - [2,7462]\,\varrho \cos l \cos(\lambda + 147°55'.7),$$

for the second internal contact:

$$t = 17^h57^m26^s + [2,8253]\,\varrho \sin l + [2.5265]\,\varrho \cos l \cos(\lambda - 55°37'.8),$$

for the second external contact:

$$t = 18^h26^m54^s + [2,7374]\,\varrho \sin l + [2,5014]\,\varrho \cos l \cos(\lambda - 37°50'9);$$

The times are given in Greenwich mean time; and ϱ stands for the radius, l for the geocentric latitude and λ for the longitude east of Greenwich of the place of observation.

If in these formulae we substitute $log\ \varrho\ sin\ l = 9.5488\ (-)$, $log\ \varrho\ cos\ l = 9.9707^5\ (+)$; $\lambda = 55°26'95$ and add to the obtained times the adopted longitude of the battery $3^h41^m48^s.1$, the error of which does probably not exceed one second, as appears from the preceding investigation, we obtain the following results.

Contact	M. T. Greenw. comp.	M. T. St. Denis comp.	Obs. O.	S,	Obs-Comp. O.	Obs-Comp. S.
I	$13^h55^m55^s$,0	$17^h37^m43^s$,1	$17^h39^m50^s$:		$+1^m17^s$	
II	14 26 19, 3	18 8 7 ,4	missed			
III	17 58 43, 5	21 40 31 ,6	21 39 16 ,2	17^s,6	—1 15 ,4	-1^m14^s,0
IV	18 28 23, 4	22 10 11 ,5	22 9 9 ,7	12 ,5	—1 1 ,8	—0 59 ,0

Neither Mr. SOETERS who observed with the telescope of the heliometer, nor myself who used the Fraunhofer telescope of Mr. DE BEAUFORT [1]) have seen anything of the so-called black drop. The former telescope was provided with the strongest eyepiece, magnifying 86 times, the latter with one magnifying 121,5 times.

I shall say only a few words here on the observations with the heliometer and the photoheliograph.

The heliometer made by MERZ at Munich has caused me through its numerous imperfections much trouble and numerous investigations relative to the instrument proved later to be valueless. Only a few days before the transit took place we detected a defect in the construction of the instrument, which rendered the adjustment of the parallactic stand illusory, so that all the measured position angles were unreliable. Nevertheless I have made complete sets of observations with the heliometer, viz. distances of the "Perseus-stars", for the determination of the scale value and during the transit two sets of eight distances between the limbs of Venus and of the sun. This was as much as the cloudy state of the atmosphere prevailing during the whole transit would allow to do. The first set was made in the ordinary manner, the other along the most advantageous chord. (*Versl. en Meded. Kon. Akad. Amsterdam, Nat. Afd.* 2[e] *Reeks, Vol. IX*, p. 127).

The division errors of the scales must still be determined; I hope to do this soon and then to revert to these measurements.

As to the observations with the photoheliograph, unfortunately the atmosphere, even in the moments that it allowed measurements with the heliometer, had a very bad effect on the clichés made. The

[1]) In my previous paper I have erroneously mentioned Mr. STOOP as the owner of this telescope; he possessed it in 1835, when KAISER used it for his observations of the comet Halley.

limbs of Venus are generally so ill defined that there is no question of making microscopic measurements. Dr. P. J. KAISER, assisted by Mr. M. B. ROST VAN TONNINGEN, has done everything in his power to succeed, but specially in making photographic observations we are powerless against atmospheric conditions.

I cannot finish this paper without expressing our thanks to the Dutch and Dutch East Indian Government, who have assisted the expedition to Réunion as much as they could, also to the Teyler Society, the "Hollandsche Maatschappij der Wetenschappen", both at Haarlem, to the "Bataafsch Genootschap" at Rotterdam and to Mr. DE BEAUFORT, who contributed efficiently (the TEYLER Society and Mr. DE BEAUFORT by lending the photo-heliograph and a telescope) to procure the necessary means to the expedition. We have also cordially to thank the Governor of Réunion, (Mr. DE LORMEL) [1]), the Maire of St. Denis, (Med. Dr. Le SINÈR), the director of the "Banque de la Réunion", (Mr. BRIDET), who often assisted and advised us, and further several other inhabitants of St. Denis, who opened their houses to us. Among these I mention Messrs. BERTHO, HUGOT, DE TOURIS and PEZZANI. In Mr. CHAILLIEY, watchmaker, we fortunately found a clever instrumentmaker, who several times made the necessary reparations to our instruments.

I must also mention that Mr. SOETERS (engineer of the Geographical service at Java) during the passage from Batavia via Aden to Réunion suffered already from the first attacks of a liver-complaint, which a few years later, April 10th 1879 carried him to his grave. At St. Denis he was sometimes for several days unable to assist me at the heliometer, then Mr. BAKHUYZEN obligingly took his place. On these occasions he was looked after with the greatest care by the military surgeon Mr. MUIRCEC.

Lastly I mention the valuable assistance in several respects given by the "amanuensis" of the expedition Mr. T. F. BLANKEN.

1) Scarcely had we cast anchor in the harbour of St. Denis, when the harbour-master arrived in a boat to offer us in the name of the Government his assistance to carry ashore the passengers, the luggage and the instruments.

TABLE I*a*.

Results for the correction of Germain's value for the eastern longitude of St. Denis-Réunion, obtained from occultations, using disappearances and reappearances indistinctly.

N⁰	1874	Observer	Name or apparent place of the star.	Magnitude	Dis- or reappearance	ΔL	G	$G \Delta L$	ε	$G \varepsilon^2$
1	Sept. 19	O.	Arg. Z. 223 No. 75	8½	*D*	$+1^{s}64$	0.70	$+1^{s}15$	$+4^{s}54$	14.43
2	» »	O.	Cordoba XVIII. N⁰. 77	8	*D*	−6.85	0.60	−4.11	−3.95	9.36
3	» »	O.	» „ N⁰.1589	9	*D*	+8.24	0.74	+6.10	+11.14	91.83
4	» »	O.	» „ N⁰.1612	8	*D*	+8.56	0.60	+5.14	+11.46	78.80
5	» 21	O.	Arg. Z. 311 No. 72	9	*D*	−6.02	0 90	−5.42	−3.12	8.76
6	» »	O.	» » » » 75	8½	*D*	−6.75	0.99^{5}	−6 72	−3.85	14.75
7	» 22	O.	33 Capricorni	5½	*D*	+3 12	0.29	+0.90	+6.02	10.51
9	» »	O.	Arg. Z. 255 No. 27	7	*D*	−6.75*	0 50	−3.38	−3.85	7.41
10	» »	O.	» » » » 32	8	*D*	−0.42	0.63	−0.26	+2.48	3.88
11	» »	O.	» » » » 34	8	*D*	−1.37*	0.89	−1.22	+1.53	2.08
12	» »	O.	» » » » 35	7	*D*	−4.99	0.97	−4.84	−2.09	4.06
14	» 26	O.	73 Piscium	6½	*R*	+3.13*	0.91	+2.85	+6.03	33.09
16	Oct. 2	B.	53 Geminorum	6	*R*	−2.07*	0.28	−0.58	+0.83	0.19
18	» 4	O.	{ $9^{u}\,0^{m}\,39^{s}.60$ / $+22°\,57'\,38''.7$ }	7½	*R*	+1 70	1.00	+1.70	+4.60	21.16
19	» 15	O.	B. A. C. 5800	6½	*D*	−13.14	0.27	−3.55	−10.24	28.31
20	» 16	B.	Arg. Z 223 No. 47	8	*D*	−4.99	1.00	−4.99	−2.09	4.37
22	» »	B	{ $18^{u}\,3^{m}\,3^{s}.135$ / $-28°\,20'\,58''.7$ }	9	*D*	+0.13	0.40	+0.05	+3.03	3.67
23	» »	B.O.	Arg. Z. 223 No. 49	8	*D*	−4.09	0 95	−3.89	−1.19	1.35
24	» »	B.O.	» » » » 52	9	*D*	−4.03	0.51^{5}	−2.08	−1.13	0.66
25	» »	B.O.	» » » » 51	8	*D*	−3.59	0.49	−1.76	−0.69	0.24
27	» »	B.	{ $18^{u}\,6^{m}\,41^{s}.75$ / $-28°\,0'\,56''.8$ }	8½	*D*	−5.53	0.99	−5.47	−2.63	6.85
28	» »	B.	Gould 24851	8½	*D*	−1.00	0.87	−0.87	+1.90	3.14
29	» 17	O.	{ $19^{u}2^{m}35^{s}.76$ / $-27°54'17''.75$ }	9½	*D*	+0.02	0.19	0.00	+2.92	1.62
30	» »	O.	Arg. Z. 241 No. 9	8½	*D*	−6.95	0.58	−4.03	−4.05	9.51
31	» »	O.	» » 231 » 12	8½	*D*	−2.40	0.35	−0.84	+0.50	0.09
32	» »	O.	» » » » 11	8½	*D*	−6.11	0.62	−3.79	−3.21	6.39
34	» 18	B.	» » 239 » 103	8	*D*	−6.47	0.95	−6.15	−3.57	12.10
35	» 19	B.	» » 247 » 99	9	*D*	−2.79	0.98	−2.73	+0.11	0.01
36	» »	B.	χ Capricorni	6	*D*	−4.46	0.97	−4.33	−1.56	2.36
37	» »	B.	{ $21^{u}\,2^{m}\,24^{s}.71$ / $-21°\,33'\,15''.1$ }	8½	*D*	−9.09	0.94	−8.54	−6.19	36.02
38	» 22	B.	24 Piscium	6½	*D*	−2.18	0.73	−1.59	+0.72	0.38
							21.80	+17.89		417.38
								−81.14	30	
									$m^2 =$	13.91
								−63.25	$m =$	$\pm 3^{s}72$
							21.80		$\frac{m}{\sqrt{21.80}} = \frac{3.72}{4.67}$	$= \pm 0^{s}80$
								$-2^{s}90$		

* These occultations have been calculated after Bessel's method. The modification of this method, to which I referred in the footnote of p. 605 of my previous paper, may be found, as I perceived afterwards, in Chauvenet, I p. 556, in § 344. It consists in this that the coordinates x, y, z, ξ, η and ζ are not computed from hour to hour but only for the instant of the occultation, the Greenwich time of which is found by means of an assumed value of the longitude.
The differences between these results and those after my method were:

No.	*R*	*v*	*B*—*O*.
9	16′ 6″6	15′ 47″8	$-0^{s}65$
11	16 7.7	12 5.5	+ 0.43
14	16 41.1	13 16.8	− 0.46
16	15 36.8	13 56.6	− 0.80
21	15 5.1	5 50.3	+ 0.17

It appears that all the differences remain below one second of time.

TABLE I*b*.

Results for the longitude of St. Denis de la Réunion, from disappearances and reappearances separately.

The 3 reappearances give	$\Sigma G = 2.19$	$\Sigma G \Delta L = + 3.97$	hence $\Delta L_{Re} = +1^s.81$
The total sum was	21.80	-63.25	
Hence the disappearances alone	19.51	-67.22	$\Delta L_{Dis} = -3^s.43$
			Mean $-0^s.81$

N°.	ε	$G\varepsilon^2$
Disappearances.		
1	$+5^s07$	17.99
2	-3.42	7.02
3	$+11.67$	100.78
4	$+11.99$	86.26
5	-2.59	6.04
6	-3.32	10.96
7	$+6.55$	12.44
9	-3.32	5.51
10	$+3.01$	5.71
11	$+2.06$	3.77
12	-1.56	2.36
19	-9.71	25.46
20	-1.56	2.43
22	$+3.56$	5.07
23	-0.66	0.42
24	-0.60	0.19
25	-0.16	0.01
27	-2.10	4.37
28	$+2.43$	5.13
29	$+3.45$	2.26
30	-3.52	7.19
31	$+1.03$	0.37
32	-2.68	4.45
34	-3.04	9.19
35	$+0.64$	0.40
36	-1.03	1.03
37	-6.66	30.12
38	$+1.25$	1.14
Reappearances.		
14	$+1.32$	1.58
16	-3.88	4.21
18	-0.11	0.01

Disappearances: $27\,m^2 = 358.07$, $m^2 = 13.26$, $m = \pm\,3^s64$ (not used)

Reappearances: $2\,m^2 = 5.80$, $m^2 = 2.90$, $m = \pm\,1^s70$ (not used)

Together:
$29\,m^2 = 363.87$
$m^2 = 12.55$
$m = \pm\,3^s54$

$\frac{m^2}{19.61} = 0.64 \qquad \sqrt{\ } = \pm\,0^s.80$

$\frac{m^2}{2.19} = 5.27 \qquad \sqrt{\ } = \pm\,2^s.30$

$4\overline{)5.91} \; 1.48 \qquad \sqrt{\ } = \pm\,1^s.22$

9*

TABLE II.

Results for ΔL after the application of the correction for the radius, disappearances and reappearances together.

No.	ΔL	Corr.	ΔL corr.	G	$G.\Delta L$	ε	$G\varepsilon^2$
1	$+1^s64$	$+0^s85$	$+2^s49$	0.70	+1 74	$+4^s64$	16.07
2	−6.85	+1.13	−5 72	0 60	−3.43	−3 75	7 64
3	+8.24	+0 77	+9 01	0 74	+6.67	+11.16	92 17
4	+8.56	+0 81	+9 37	0.60	+5.62	+11 52	79.63
5	−6 02	+0.74	−5.28	0 90	−4 75	−3.13	8.82
6	−6.75	+0.80	−5.95	0.995	−5 92	−3.80	14.37
7	+3.12	+0 90	+4.02	0.29	+1 17	+6.17	11.04
9	−6.75	+3.75	−3.00	0 50	−1.50	−0.85	0.36
10	−0.42	+0.74	+0.32	0.63	+0.20	+2 47	3.84
11	−1.37	+1 12	−0.25	0 89	−0 22	+1.90	3 21
12	−4.99	+0.78	−4.21	0.97	−4 08	−2.06	4.11
14 R	+3.13	−1.24	+1.89	0.91	+1.72	+4.04	14.85
16 R	−2.07	−1 60	−3 67	0.28	−1.03	−1.52	0.65
18 R	+1.70	−0 77	+0.93	1.00	+0.93	+3.08	9.49
19	−13.14	+1.39	−11.75	0 27	−3.17	−9.60	24.88
20	−4.99	+0 75	−4.24	1 00	−4 24	−2.09	4.37
22	+0.13	+0.97	+1.10	0 40	+0.44	+3 25	4.22
23	−4.09	+0.81	−3 28	0.95	−3 12	−1.13	1 22
24	−4 03	+0.88	−3.15	0.515	−1.62	−1.00	0.51
25	−3.59	+0 90	−2 69	0.49	−1.32	−0.54	0.14
27	−5.53	+0.74	−4.79	0.99	−4 74	−2.64	6.90
28	−1.00	+0 84	−0.16	0.87	−0.14	+1.99	3 45
29	+0.02	+1.06	+1.08	0.19	+0.21	+3.23	1.98
30	−6.95	+0.80	−6.15	0.58	−3.57	−4.00	9.28
31	−2.40	+0 92	−1.48	0.35	−0.52	+0.67	0 16
32	−6.11	+0.79	−5.32	0 62	−3.30	−3.17	6.23
34	−6.47	+0.92	−5.55	0.95	−5.27	−3.40	10.98
35	−2.79	+1.06	−1 73	0.98	−1.69	+0.42	0.18
36	−4.46	$+0\ 99^5$	−3.46	0.97	−3.36	−1 31	1.67
37	−9.09	+1.05	−8.04	0.94	−7 56	−5 89	32.61
38	−2.18	+0.77	−1.41	0.73	−1.03	+0.74	0.40
				21.80	+18.70		$30\,m^2 = 374.43$
					−65.58		$m^2 = 12.48$
							$m = \pm\ 3^s53$
					−46.88		
				21.80	-2^s15		$\frac{m^2}{21.80} = 0.572,\ \sqrt{\ } = \pm 0^s76$

Utrecht, June 24, 1905.