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the value of $\frac{dT_z}{dx} = 0$, and for $T_1 = T_z$ the value of $\frac{dT_z}{T_z dx} = -\frac{1}{6} \frac{db}{b dx}$

so the well-known limits for mixtures for which $\frac{dp_c}{p_c dx}$ can be equal to 0.

For the initial direction of the section normal to the p-axis, the following equation holds:

$$-\frac{1}{T}\left(\frac{dT}{dx_1}\right)_p = \frac{RT}{r}\left\{\frac{x_2 - x_1}{x_1}\right\} = \frac{RT}{r}\left\{e^{\frac{u}{RT}\frac{d\rho e}{dx}} - 1\right\}$$

and

$$-\frac{1}{T} \left(\frac{dT}{dx_2} \right)_p = \frac{RT}{r} \left\{ \frac{x_2 - x_1}{x_2} \right\} = \frac{RT}{r} \left\{ 1 - e^{-\frac{u}{RT} \frac{dpe}{dx}} \right\}$$

Both yield at the critical temperature of the components:

$$-\left(\frac{dT}{Tdx}\right)_{p} = \frac{RT}{r} \frac{u}{RT} \frac{dp_{c}}{dx} = \frac{u}{r} \frac{dp_{c}}{dx} = \frac{\frac{1}{p_{c}} \frac{dp_{c}}{dx}}{\frac{T}{p_{c}} \frac{dp}{dT}} = \frac{1}{7} \frac{1}{p_{c}} \left(\frac{dp_{c}}{dx}\right)_{x}$$

According to results obtained before, we may also write:

$$-\frac{1}{T}\left(\frac{dT}{dx_{\lambda}}\right)_{p}=-\frac{6}{7}\left\{\frac{dT_{k}}{T_{\lambda}dx_{\lambda}}+\frac{1}{6}\frac{1}{b}\frac{db}{dx_{\lambda}}\right\}.$$

Physics. — "The exact numerical values for the properties of the plaitpoint line on the side of the components." By Prof. VAN DER WAALS.

In my two previous communications, inserted in the proceedings of this meeting, viz. I on the properties of the plaitpoint line on the side of the components and II on the properties of the sections of the surface of saturation on the side of the components, it has again appeared, that the thermodynamic treatment of such problems enables us to find a complete general solution — but also that if we want to compute numerical values in special cases, the knowledge of the equation of state is indispensable. In some cases it will be sufficient, if we make use of an approximate equation of state; but as soon as the density of the substance is comparable to that in the critical state, the numerical values calculated by means of the approximate equation of state can deviate strongly from reality. This is specially the case with quantities which either refer to the volume, or are in close connection with it. Thus it is known, that already the critical volume of a simple substance is not

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equal to 3b, the value furnished by the equation of state, in which b is put constant, but that this equation is found rather nearer to 2b. This may be accounted for by taking into account that b is variable and decreases with the volume. In a mixture balso depends on the composition. Accordingly the quantity $\frac{db}{dx}$ is an intricate expression for mixtures, and must in general be distinguished from $\left(\frac{db}{dx}\right)$. If the way in which b depends on volume and composition, was accurately known, then there would not be left any difficulties but those of toilsome and intricate calculations. But It is sufficiently known, that the way in which b, even for a simple substance, depends on v, has not yet been fixed with perfect certainty, and that in any case the knowledge of the numerical values, which occur in given forms of b, is wanting. These considerations led me to believe that this would be an objection to deriving theoretically the properties of the beginning of the plaitpoint line with perfect certainty — and also to determining the numerical values exactly. It has however, appeared to me that the knowledge of how b depends on x and v is not required for this exact determination: but that for this purpose it suffices to know two quantities which have been experimentally determined for the critical state of a simple substance.

Let us call f the value which $\frac{T}{p} \left(\frac{\partial p}{\partial T} \right)_v = \frac{T}{p} \frac{dp}{dT}$ has in the critical conditions of the component, and \varkappa , the critical coefficient, so that $MRT_r = \varkappa(pv)_r$.

From
$$p = \frac{MRT}{v-b} - \frac{a}{v^2}$$
 follows $\frac{MRT}{p(v-b)} = f$ and $\frac{a}{pv^2} = f - 1$.

The equality of $MRT = \varkappa pv = f(v-b)p$, gives the value

$$v = \frac{f}{f - \varkappa} b$$

for the critical volume, in which we have to keep in view, that now that b is put variable with the volume, b represents the value which this quantity has in the critical state. With f=7 and $\varkappa=\frac{15}{4}$ we find $\frac{v}{b}=\frac{28}{13}$, whereas with f=4 and $\varkappa=\frac{8}{3}$ we should find the value $\frac{v}{b}=3$. For carbonic acid Keesom has found f=6,7 and $\varkappa=3,56$, from which would follow $\frac{v}{b}=\frac{6,7}{3,14}=2,134$.

If in $MRT = \varkappa pv$ we put the value of v, we find:

$$MRT \stackrel{\perp}{=} pb \frac{\kappa f}{f - \kappa}$$

With f = 4 and $\varkappa = \frac{8}{3}$ the factor of pb = 8, and with f = 7 and $\varkappa = \frac{15}{4}$ this factor is found to be only slightly different viz. $8 \frac{1}{13}$. For the calculation of the value of b in the critical condition we get therefore:

$$b = \frac{MRT}{p} \frac{f - \varkappa}{\varkappa f} \quad \text{or} \quad \frac{T_r}{p_r} \frac{1}{273} \frac{f - \varkappa}{\varkappa f}.$$

If we put the value of v in the equation $\frac{a}{pv^2} = f - 1$, we find:

$$p = \frac{a}{b^2 \frac{f^2 (f-1)}{(f-x)^2}}$$

The factor of b^2 , which with f = 4 and $\varkappa = \frac{8}{3}$ has the well known value of 27, is found slightly above 27,8 with f = 7 and $\varkappa = \frac{15}{4}$.

If in $MRT = \varkappa pv$ we substitute the values found for p and v, we find:

$$MRT = \frac{a}{b} \frac{\varkappa (f - \varkappa)}{f(f - 1)}.$$

If we again put f = 4 and $\varkappa = \frac{8}{3}$, we find $MRT = \frac{8}{27} \frac{a}{b}$; with f = 7 and $\varkappa = \frac{15}{4}$ the factor $\frac{a}{b}$ is equal to $\frac{1}{3.446}$; also this value differs but little from $\frac{8}{27} = \frac{1}{3.375}$.

For the calculation of α with the critical values of T and p, the formula:

$$a = \frac{(MRT)^2}{p} \frac{f-1}{\kappa^2}$$

holds

The factor $\frac{f-1}{\varkappa^2}$ is equal to $\frac{27}{64} = \frac{1}{2,37}$ with f = 4 and $\varkappa = \frac{3}{3}$. With f = 7 and $\varkappa = \frac{15}{4}$ it is again only slightly different, viz. $\frac{96}{225} = \frac{1}{2,34}$.

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For the critical condition $\left(\frac{\partial p}{\partial v}\right)_T$ must be 0. From this follows:

$$\frac{MRT}{(v-b)^2}\left(1-\frac{\partial b}{\partial v}\right)=2\,\frac{a}{v^3},$$

and after substitution of the values found for MRT and v

$$1 - \frac{\partial b}{\partial v} = 2 \frac{\kappa (f-1)}{f^2}.$$

With f = 4 and $\varkappa = \frac{8}{3}$ it follows naturally that $\frac{\partial b}{\partial v} = 0$, whereas with f = 7 and $\varkappa = \frac{15}{4}$ it follows that:

$$1 - \frac{\partial b}{\partial v} = \frac{45}{49}.$$

In the same way $\left(\frac{\partial^2 p}{\partial v^2}\right)_T$ must be 0 in the critical state. From this follows:

$$-b\frac{\partial^2 b}{\partial v^2}=2\frac{\varkappa(f-1)(f-\varkappa)(f-4)}{f^4}.$$

With f = 4 and $\varkappa = \frac{8}{3}$ this value is of course equal to 0. With f = 7 and $\varkappa = \frac{15}{4}$ we find:

$$-b\frac{\partial^2 b}{\partial v^2}=0,1827^{-1}).$$

Let us now proceed to calculate the value of $\frac{dT}{Tdx}$ at the beginning of the plaitpoint line. We have the formula:

$$\frac{dT}{Tdx_{o}} = \frac{\left(\frac{\partial^{2} p}{\partial v \partial x}\right)_{T} + \frac{1}{MRT} \left(\frac{\partial p}{\partial x}\right)_{vT}^{2}}{-\frac{\partial^{2} \varepsilon}{\partial v^{2}}},$$

and have therefore to determine $\left(\frac{\partial p}{\partial x}\right)_{vT}$ and $\left(\frac{\partial^2 p}{\partial x \partial v}\right)_{T}$ for the critical condition, but on the supposition that b varies with the volume and that

¹⁾ This high value of $-b\frac{\partial^2 b}{\partial v^2}$ supports the hypothesis that b, in its dependence on the volume, has a more intricate form than is represented by a series of ascending powers of $\left(\frac{b_{\infty}}{v}\right)$.

 $\frac{db}{dx}$ has different values depending on the variations of the volume.

$$\left(\frac{\partial p}{\partial x}\right)_{vT} = \frac{MRT\left(\frac{\partial b}{\partial x}\right)_{v}}{(v-b)^{2}} - \frac{da}{dx}\frac{1}{v^{2}} = -\frac{a}{v^{2}}\left\{\frac{1}{a}\frac{da}{dx} - \frac{MRT}{a}\left(\frac{\partial b}{\partial x}\right)_{v}\frac{v^{2}}{(v-b)^{2}}\right\}$$

If we call $\frac{db}{dx}$ the value denoting the change of b with change of x, when we make also the volume vary in such a way that the mixture is again in the state which may be called the critical state of the unsplit mixture, then:

$$\frac{db}{dx} = \left(\frac{\partial b}{\partial x}\right)_{v} + \frac{\partial b}{\partial v} \left(\frac{dv}{dx}\right)_{kr}$$

and v, being $=b\frac{f}{f-\varkappa}$, $\left(\frac{dv}{dx}\right)_{,}=\frac{db}{dx}\frac{f}{f-\varkappa}$, when f and \varkappa are constant, which is the case when the law of corresponding states is fulfilled. We find then:

$$\left(\frac{\partial b}{\partial x}\right)_{v} = \left(1 - \frac{\partial b}{\partial v} \frac{f}{f - \varkappa}\right) \frac{db}{dx}.$$

We have to know:

$$\frac{MRT}{a} \left(\frac{\partial b}{\partial x}\right)_{v} \frac{v^{2}}{(v-b)^{2}} = \frac{MRT}{\frac{a}{b}} \frac{v^{2}}{(v-b)^{2}} \left\{ \frac{f\left(1 - \frac{\partial b}{\partial v}\right) - \varkappa}{f - \varkappa} \right\} \frac{1}{b} \frac{d}{dx}.$$

When we substitute the values found above for MRT, v and $1-\frac{\partial b}{\partial v}$

we find $\frac{f-2}{f-1}\frac{1}{b}\frac{db}{dx}$ — and so:

$$\frac{1}{p} \left(\frac{\partial p}{\partial x} \right)_{vT} = -\frac{a}{pv^2} \left\{ \frac{1}{a} \frac{da}{dx} - \frac{f-2}{f-1} \frac{1}{b} \frac{db}{dx} \right\}$$

or

$$\frac{1}{p} \left(\frac{\partial p}{\partial x} \right)_{vT} = -(f-1) \left\{ \frac{dl \frac{a}{b}}{dx} + \frac{1}{f-1} \frac{dlb}{dx} \right\}$$

This value is in a high degree dependent on f.

With
$$f = 4$$
 we find $\frac{1}{p} \left(\frac{\partial p}{\partial w} \right)_{vT} = -3 \left\{ \frac{dT_x}{T_r dx} + \frac{1}{3} \frac{db}{b dx} \right\}$.

With
$$f = 7$$
 on the other hand $\frac{1}{p} \left(\frac{\partial p}{\partial x} \right)_{vT} = -6 \left\{ \frac{dT_v}{T_v dx} + \frac{1}{6} \frac{db}{b dx} \right\}$.

In the preceding communication I have concluded to the same value from the equality of $\binom{v_0}{v_n}_v$ and $\frac{dp_c}{dx}$ in the critical circumstances, and by means of the empirical formula $-l\frac{p_c}{p_z} = f\frac{T_z - T}{T}$. For from this formula follows

$$\frac{dv}{dx} = \frac{dv}{dx} - f \frac{dT_{x}}{Tdx}$$

 \mathbf{or}

$$\frac{1}{\frac{d}{dx}}\frac{df}{dx} = \frac{dT_x}{T_x dx} - \frac{1}{b}\frac{db}{dx} - f\frac{dT_x}{T_x dx}$$

OF

$$\frac{1}{t} \begin{pmatrix} a_{f'} \\ a_{s'} \end{pmatrix}_{t,f} = -(f-1) \left\{ \frac{dT_{s}}{T_{s}dx} + \frac{1}{f-1} \frac{db}{bdx} \right\}.$$

But we could arrive at this equation in a much simpler way still. From:

$$d_{T} = \begin{pmatrix} \partial_{f} \\ \partial_{x} \end{pmatrix}_{f} dT \qquad \begin{pmatrix} \frac{\partial p}{\partial v} \\ \frac{\partial p}{\partial v} \end{pmatrix}_{v} dT \qquad \begin{pmatrix} \frac{\partial p}{\partial v} \\ \frac{\partial p}{\partial v} \\ \frac{\partial p}{\partial v} \end{pmatrix}_{T} = 0$$

follows, when dT is possequal to dT_z (taking for dT_z the variation of the critical temperature of the unsplit mixture)

$$\frac{1}{r} \frac{dr}{dr} = \frac{1}{r} \left(\frac{dr}{dr} \right)_{vT} + \left(\frac{T}{p} \frac{dp}{dT} \right) \frac{dT_{x}}{T_{x} dx}$$

and so

$$\frac{1}{P} \begin{pmatrix} \dot{\theta}_P \\ \dot{\theta}_{s'} \end{pmatrix}_{PI} = \frac{1}{P} \frac{dp_s}{dx} - f \frac{dT_{s-}}{T_s dx}.$$

And this equation is not only preferable because it is shorter, but also because it is independent of the circumstance whether the law of corresponding states is applicable or not. The value of f in this derivation is that of the component.

Besides $\begin{pmatrix} a_{ij} \\ a_{ij} \end{pmatrix}_T$ we have to determine the value of $\begin{pmatrix} \frac{\partial^2 p}{\partial x \partial v} \end{pmatrix}_T$

For this quantity we find:

$$\left(\frac{\partial^2 b}{\partial x^{1/2}}\right)_T = -2 \frac{4RT}{(1-b)^2} \left(1-\frac{\partial b}{\partial v}\right) \left(\frac{\partial b}{\partial x}\right)_v + MRT \frac{\frac{\partial^2 b}{\partial x \partial v}}{(v-b)^2} + 2 \frac{\frac{\partial a}{\partial x}}{v^2}$$

OI,

$$\left(\frac{\partial^2 p}{\partial x \partial v} \right)_T = \frac{2v}{v^2} \frac{1}{v} \frac{du}{dx} - \frac{MRT}{a} \frac{v^2}{(v-b)^2} \left(1 - \frac{\partial b}{\partial v} \right) \left(\frac{\partial b}{\partial x} \right)_v + \frac{MRT}{2a} \frac{v^3}{(v-b)^2} \frac{\partial^2 b}{\partial x \partial v} \right\}.$$

In this expression only the quantity $\frac{\partial^2 b}{\partial x \partial v}$ is unknown. We determine it from:

$$\left(\frac{\partial b}{\partial x}\right)_{v} = \left\{1 - \frac{\partial b}{\partial v} \frac{f}{f - \varkappa}\right\} \frac{db}{dx}.$$

From this follows:

$$\frac{\partial^2 b}{\partial x \partial v} = -b \frac{\partial^2 b}{\partial v^2} \frac{f}{f - \varkappa} \left(\frac{1}{b} \frac{db}{dx} \right).$$

If we substitute the values given above for MRT, v, $\left(1 - \frac{\partial b}{\partial v}\right)$ and $-b \frac{\partial^2 b}{\partial v^2}$ in the expression for $\left(\frac{\partial^2 p}{\partial x \partial v}\right)_T$, we find for the value of the second term $-\frac{1}{b} \frac{db}{dx} 2 \frac{(f-2)}{f}$ and for the value of the third term $+\frac{1}{b} \frac{db}{dx} \frac{f-4}{f}$.

The value of $\left(\frac{\partial^2 p}{\partial x \partial v}\right)_T$ is then found equal to:

$$\left(\frac{\partial^{3} p}{\partial x \partial v}\right)_{T} = \frac{2a}{v^{3}} \left\{ \frac{1}{a} \frac{da}{dx} - \frac{1}{b} \frac{db}{dx} \right\}$$

0ľ

$$\left(\frac{\partial^2 p}{\partial x \partial v}\right)_T = \frac{2a}{v^3} \frac{dT_x}{T_x dx},$$

and for $\frac{\left(\frac{\partial^2 p}{\partial x \partial v}\right)_T}{-\frac{\partial^2 \epsilon}{\partial v^2}}$ we find the simple value $\frac{dT_{\lambda}}{T_{\lambda}dx}$; so exactly the same

value as follows from the equation of state, in which b is put constant. This gives rise to the conjecture that this relation might be found merely from thermo-dynamic relations independent of the knowledge of the equation of state, and this is indeed the case.

Let us consider the quantity $\left(\frac{\partial p}{\partial v}\right)_T$. It is equal to 0 in the critical state of the component. Let us pass from this homogeneous critical phase to another in which the volume has changed with dv, the composition with dx, and the temperature with dT.

Let us put dT again equal to dT_c , so let us assume that the mixture with dx molecules of the second kind is again in an homogeneous critical phase, then $\left(\frac{\partial p}{\partial v}\right)_T$ is again equal to 0.

From:

$$d\left(\frac{\partial p}{\partial v}\right)_{T} = \left(\frac{\partial^{3} p}{\partial v^{2}}\right)_{Tx} dv + \left(\frac{\partial^{2} p}{\partial x \partial v}\right)_{T} dx + \left(\frac{\partial^{2} p}{\partial v \partial T}\right)_{x} dT_{x}$$

follows, because $d\left(\frac{\partial p}{\partial v}\right)_T$ and $\left(\frac{\partial^2 p}{\partial v^2}\right)_{Tx}$ are equal to 0:

$$\left(\frac{\partial^{3}p}{\partial x \partial v}\right)_{T} = -\left(\frac{\partial^{2}p}{\partial v \partial T}\right)_{x} \frac{dT_{r}}{dx},$$

and from the relation:

$$T\left(\frac{\partial p}{\partial T}\right)_{v} - p = \frac{\partial \varepsilon}{\partial v}$$

follows:

$$T\left(\frac{\partial^2 p}{\partial v \partial T}\right) = \frac{\partial^2 \varepsilon}{\partial v^2}.$$

And from this we find again now only from thermodynamic relations what we have derived already above.

As we found, also by means of mere thermodynamics:

$$\left(\frac{\partial p}{\partial x}\right)_{vT} = -\int p \left\{ \frac{dT_{\times}}{T_{\nu}dx} - \frac{1}{f} \frac{dp_{\nu}}{p_{\nu}dx} \right\}$$

we may put without making use of the equation of state:

$$rac{dT}{T_{dx_0}} = rac{dT_r}{T_r dx} - rac{f^2 p^2}{MRT rac{\partial^2 \epsilon}{\partial v^2}} \left\{ rac{dT_r}{T_r dx} - rac{1}{f} rac{dp_r}{p_r dx}
ight\}^2.$$

The factor $-\frac{f^2 p^2}{MRT \frac{\partial^2 \varepsilon}{\partial v^2}}$ may be reduced to a simple form, but for

the determination of the value of this factor it is required to know the equation of state. If we write $fp = \frac{MRT}{v-b}$, and $\frac{\partial^2 \varepsilon}{\partial v^2} = -2\frac{a}{v^3}$, this

factor becomes equal to $\frac{MRT}{(v-b)^2}:\frac{2a}{v^3}$ and as $\frac{MRT}{(v-b)^2}\left(1-\frac{\partial b}{\partial v}\right)=\frac{2a}{v^3}$

follows from $\left(\frac{\partial p}{\partial v}\right) = 0$, we get:

$$-\frac{f^2 p^2}{MRT \frac{\partial^2 \varepsilon}{\partial v^2}} = \frac{1}{1 - \frac{\partial b}{\partial v}} = \frac{f^2}{2\varkappa (f-1)},$$

so with f = 7 and $z = \frac{15}{4}$ the value of this factor becomes equal to $\frac{49}{45}$. Hence we have:

$$\frac{dT}{Tdx_0} = \frac{dT_x}{T_x dx} + \frac{49}{45} \left\{ \frac{dT_x}{T_x dx} - \frac{1}{7} \frac{dp_x}{p_x dx} \right\}^2.$$

If we introduce the quantity b instead of $\frac{dp_x}{p_r dx}$, we find:

$$\frac{dT}{Tdx_0} = \frac{dT}{T_cdx} + \frac{f-1}{2\varkappa} \left\{ \frac{dT}{T_cdx} + \frac{1}{f-1} \frac{db}{bdx} \right\}^2.$$

With f = 4 and $\varkappa = \frac{8}{3}$ we find again $\frac{f-1}{2\varkappa} = \frac{9}{16}$, but with f = 7

and $\varkappa = \frac{15}{4}$, $\frac{f-1}{2\varkappa}$ rises to 0,8. With f = 6,7 and $\varkappa = 3,56$ (Keesom's values for carbonic acid) the value is not appreciably different from 0,8. If we calculate with $\frac{dT}{T_c dx} = -0,493$ and $\frac{1}{b} \frac{db}{dx} = -0,271$,

f=6.7 and z=3.56 the value of $\frac{dT}{Tdx_0}$, we find for this value -0.259. Though 0.259 is smaller than the values calculated from Keesom's observations, 0.284 for x=0.1047 and 0.304 for x=0.1994, we must not forget that the calculated value would hold for the limiting case, viz x=0, and the fact that for $\Delta x=0$ a smaller value than 0.284 would have to be expected is at least in harmony with the circumstance that the amount is found higher for a higher value of x.

It is evident from all this that though we cannot do quite without the equation of state for the calculation of $\frac{dT}{Tdx}$ for the plaitpoint line, yet it is not necessary to know the form of the quantity b.

For the calculation of the quantity $\frac{T}{p}\frac{dp}{dT}$ for the beginning of the plaitpoint line we have from the formula:

$$dp = \left(\frac{\partial p}{\partial x}\right)_{vT} dx + \left(\frac{\partial p}{\partial T}\right)_{vx} dT$$

the relation:

$$\left(\frac{T}{p}\frac{dp}{dT}\right)_{pl} = \frac{T}{p}\left(\frac{\partial p}{\partial T}\right)_{v} + \frac{1}{p}\left(\frac{\partial p}{\partial x}\right)_{Tv}\frac{Tdx_{0}}{dT}$$

or

$$\left(\frac{T}{p} \frac{dp}{dT} \right)_{pl} = f + \frac{-(f-1) \left\{ \frac{dT_{c}}{T_{c}dx} + \frac{1}{(f-1)} \frac{1}{b} \frac{db}{dx} \right\}}{\frac{dT_{c}}{T_{c}dx} + \frac{f-1}{2\varkappa} \left\{ \frac{dT_{c}}{T_{c}dx} + \frac{1}{(f-1)} \frac{1}{b} \frac{db}{dx} \right\}^{2} }$$

or in numerical value for the mixture of oxygen and carbonic acid:

$$\left(\frac{T}{p}\frac{dp}{dT}\right)_{\mu l} = 6.7 + \frac{-5.7\{-0.493 - 0.047\}}{-0.259} = 6.7 - 11.28 = -4.58$$

With the mixture x = 0.1047 KEESOM has found -6.3 and with x = 0.1995 the amount found was -6.08.

If we take the product of $\frac{T}{p}\frac{dp}{dT}$ and $\frac{dT}{Tdx}$ we find the value of $\frac{1}{p}\frac{dp}{dx}$ for the beginning of the plaitpoint line. As both $\frac{T}{p}\frac{dp}{dT}$ and $\frac{dT}{Tdx}$ are negative for the mixture of carbonic acid and oxygen, the value of $\begin{pmatrix} 1 & 1/p \\ 1 & 1/p \end{pmatrix}$ is positive.

Anatomy. "Bolk's centra in the cerebellum of the mammalia".

By D. J. Hulshoff Pol (from the laboratory for Psychiatry and Neurology at Amsterdam). (Communicated by Prof. C. Wishiel).

In his well-known researches about the cerebella of mammalia 1); Bork concludes: that "the Lobus anterior cerebelli does contain the centre of coordination for the muscle-groups of the head, the Lobus simpley the centrum of coordination for those of the neck; the non-symmetrical centre of coordination for both left and right extremity is simulated in the Lobus medianus posterior, whilst each of the Lobuli austicianes is the seat of one of the symmetrical central respectively for both right, and for both left extremities." 2)

Within the same line of research, VAN RIJNBERK 3) at LUCIANI'S laboratory in Roma, experimenting on two dogs, extirpated a portion of the condiction. with the aim of taking away the right part of the Loine simplex.

The secondary symptoms, which were observed during the first caps after the operation, having passed away, the animal experimented mon continued shaking its head, as if it-meant to say "no".

This symptom resembled very much a trouble in the coordination, and such being indeed the case, it would have confirmed the hypothesis of Bolls. Therefore it was important to determine with as much executes as possible, which portion of the cerebellum had been removed.

To the purpose the preparation, fixed in formol, was offered

Prof. Dr. L. Bolk, Das Gerebellum der Säugetiere, Petrus Camper, Vol III, Fert. L. Angeleichen.

P. of. L. B. x. Over de physiologische beteekenis van het cerebellum. De Erven Bouv. Hagnan. 1903.

 $[\]beta$) G. A. v.: N R. ме: ак. Tentative di localisazione funzionali nel cerveletto. Archivio di fistologia, No. I. Fasc. V.