

Citation:

J.P. van der Stok, On frequency curves of meteorological elements, in:
KNAW, Proceedings, 8 I, 1905, Amsterdam, 1905, pp. 314-327

less pigmented. (A transversal serially ranged segmental excedent contrast).

b. animals with narrow dark transversal stripes, more numerous than the segments of the body (mammalia, e.g. zebra's). These stripes correspond with zones of intersegmental summation. (A transversal serially ranged intersegmental excedent contrast).

6. EIMER's type of the animals with longitudinal stripes includes:

a. fishes, in which the dark longitudinal stripes, or else the dark dots and spots ranged in long rows, correspond apparently with the points of entrance into the hypodermis of the skin-branches of the peripheral nerves. (An excedent contrast *ex introitu*).

b. amphibians and reptiles. Probably the precedent hypothesis holds likewise for these.

c. mammalia. In the viverridae the longitudinal stripes apparently have been produced by the confluence of rows of spots, which were originally distributed intersegmentally. (Pseudo-longitudinal stripes).

7. EIMER's spotted type in the mammalia includes:

a. Irregular spotting. This is caused by segmental excedent and defect-variability.

b. Uniform dotting. We may imagine this to have been produced by the fragmenting of stripes, that occur un-interrupted in kindred species of animals (leopards).

Meteorology. — "*On frequency curves of meteorological elements.*"

Bij Dr. J. P. VAN DER STOK.

1. The application of the theory of probability to the results of meteorological investigations has hitherto been more limited than the nature of the data would lead us to expect.

It is not difficult to indicate the reason for this fact. Nearly all applications of the theory of errors to physical and astronomical problems are induced by the desire to determine a quantity with the greatest attainable precision; the remaining uncertainty affords a criterion for the value of the different methods employed and leads to experimental improvements, by means of which the errors, or departures from the average value, may be minimized.

These reasons for the application of the theory of errors fail in meteorology: for the greater part of meteorological quantities and climatological regions it is impossible to calculate average values within a reasonable time and with a moderate degree of precision and, if this were at all possible (e.g. for tropical stations), an increase

of precision would scarcely afford any advantage as we are unable to reduce the deviations by improving the observations. Moreover the knowledge of the most probable value is of minor importance as the frequency curves in general are very flat and we cannot attach the common idea of errors to the deviations which, after all, are more characteristic of meteorological conditions than absolute values.

Meteorological constants in the true sense of the word and to which the methods and terminology of the theory of errors are applicable, are nearly exclusively FOURIER constants, obtained by the analysis of periodical phenomena such as daily and annual variations, and to these it is certainly desirable to apply the criterion of the theory of errors more extensively than has hitherto been the case: the theory of errors in a plane can be immediately and advantageously used to get a clear understanding of the value of the results obtained.

If however we abandon this basis of the theory of errors and proceed upon the lines which have of late been followed by the sociological and biological sciences, the matter appears in a different light; in these sciences the principal object to be obtained is not so much the mean value as the occurrence of deviations, or rather the nature of the frequency curves.

Monthly means e.g. of barometric heights may be identical for January and July as far as the absolute values are concerned, but we may confidently expect the frequency curves for these months to bear a totally different character. It is also extremely probable that the frequency curves will show a considerable difference for places in different latitudes or differently situated in relation to the main tracks of depressions.

The constants which occur in the analytical expressions for these curves may then be considered as characteristics of the climate and, as in meteorology we possess more data than in most other branches of science, a more thorough study of details is possible.

The principal questions are:

a. In how far are monthly means in accordance with the common law of probability.

b. What is the form of the frequency curves constructed from daily means or from observations made at fixed hours in as far as these curves may be considered symmetrical.

c. An investigation of the skewness of these curves.

In this communication only the first of these problems will be considered.

2. The material chosen for this inquiry consists of:

1st. monthly means of barometric pressure at Helder, calculated for the 60 years period Aug. 1843 to July-1902, the total number being 720.

2nd. monthly means of barometric pressure at Batavia for 37 years, 1866- 1902, altogether 444 data.

3rd. monthly means of atmospheric temperature for the whole of France during the 50 years period 1851-1900, altogether 600 data.

Up to 1873 the data for Helder have been taken from a meteorological journal kept by Mr. VAN DER STERR and, after his death, from the annals issued by the K. Met. Instituut.

A NEWMAN standard-barometer at Helder, which is known to have been in use as early as 1851, has recently been tested and does not show any appreciable errors, so that it may safely be assumed that also the records of the station-barometer are sufficiently accurate for our purpose.

The monthly means for Batavia have been taken from the returns published by the K. Magn. en Met. Observ., and those for France from ANGOT's "*Études sur le climat de la France, Température,*" published in the Ann. du Bureau Central Météor. de France, Année, 1900, I. Mémoires, Paris, 1902, p. 34-118.

Table I gives the results of the calculations for Helder.

Let ε be the deviations of the individual data from the corresponding general average value and n the number of data available, then:

$$M = \sqrt{\frac{|\varepsilon^2|}{n-1}}, \text{ mean deviation,}$$

$$\vartheta = \frac{|\varepsilon|}{n}, \text{ average deviation,}$$

$$h = \frac{1}{M\sqrt{2}}, \text{ factor of steadiness,}$$

$$h' = \frac{1}{\vartheta\sqrt{\pi}}, \text{ idem.}$$

A = number of years required to obtain a general mean value with a probable error of ± 0.1 mm. for the barometric height and of ± 0.1 C. for the atmospheric temperature.

This number, calculated from the formula:

$$\sqrt{A} = \frac{0.6745 M}{0.1},$$

is given instead of the probable error of the result with a view of showing how difficult, if not how impossible, it is to fix normal

values of meteorological elements, at least in high latitudes. The application of this formula is justified by the consideration that monthly means for a given month may, as far as our actual knowledge goes, be regarded as independent of each other, whereas e.g. daily means are certainly not so.

If the deviations are distributed according to the normal, exponential law :

$$\frac{h}{\sqrt{\pi}} e^{-h^2 x^2} dx \dots \dots \dots (1)$$

the quantity h' must be equal to h . Another criterion to ascertain whether the distribution of deviations is regulated by the normal law, as advocated by CORNU¹⁾, is obtained by calculating π by means of the formula :

$$\pi = \frac{2 M^2}{g^2} \dots \dots \dots (2)$$

it is equivalent to the criterion previously mentioned as it holds only when $h = h'$.

The quantities M and h may be regarded as a measure of the

TABLE I. Monthly means of barometric height, Helder.

	M	g	h	h'	A	π
January.....	5 01 mm.	4.04 mm	0.141	0.140	1149	3.083
February.. . . .	4.85	3.82	0.146	0.148	1071	3.222
March.....	4.21	3.45	0.168	0.164	805	2.971
April.....	3.36	2.74	0.211	0.206	514	3.007
May.....	2 34	1.91	0.302	0.296	250	3.022
June.	2.22	1.76	0.318	0.321	225	3.190
July.....	2.09	1.65	0.339	0.343	198	3.212
August	2.12	1.69	0.334	0.334	206	3.158
September.....	3.01	2.45	0.235	0.230	411	3.079
October	3.35	2.62	0.211	0.215	510	3.262
November.. . . .	3.74	3.02	0.189	0.187	637	3.063
December.....	4 99	4.00	0.142	0.141	1132	3.106
Mean..						3.198

¹⁾ Ann. de l'Observ. de Paris, XIII, 1876.

variability and steadiness of the climate from year to year in so far as this is determined by the oscillation of the atmospheric pressure. By analogy to the secular variation of the elements of terrestrial magnetism this instability might also be called secular variability.

Assuming this criterion to be correct, it appears from Table I that there is every reason to suppose that at Helder the deviations follow the normal law, the average value of π not differing more than 1.8 % from the real value.

On comparing the climate at Helder, which is highly variable from year to year, with the climate at Batavia (in so far as in this case also the variability of atmospheric pressure may be taken as a measure), we find totally different conditions.

A period of about ten years for the Eastmonsoon, and of twenty years for the Westmonsoon months is already sufficient to obtain total monthly means of the barometric height with a probable error of ± 0.1 mm. and for the dry months the available series of 37 years is quite sufficient to obtain a degree of certitude twice as great.

TABLE II. Monthly means of barometric height, Batavia.

	M	σ	h	h'	A	π
January.....	0.84 mm.	0.71 mm.	0 845	0.792	32	2.759
February.....	0.75	0.62	0 938	0.917	26	3.004
March.....	0.63	0.52	1.115	1 085	18	2.974
April	0.42	0.36	1.701	1.581	8	2.715
May.....	0.44	0.32	1.603	1.751	9	3.752
June	0 40	0.28	1.779	1.990	7	3 931
July.....	0.44	0 34	1.604	1.610	9	3.282
August.....	0.47	0.33	1.492	1.689	10	4.028
September.....	0.44	0.35	1.598	1.606	9	3.173
October.....	0.51	0 41	1 375	1.370	12	3.118
November.....	0.65	0.53	1.088	1.063	19	2.999
December.....	0.61	0.49	1.166	1.155	17	3.086
Mean.....						3.235

The application of the criterion as to whether the deviations follow

the normal law leads to a far less satisfactory result for this place than for Helder. The two values h and h' of the factor of steadiness show considerable and systematic discrepancies, the calculated values of π for May to August being collectively too great, and those for the other months too small. Although the total mean, 3.235, does not differ more than 3 % from the real value, these differences amount to + 15.7 % in the five dry months and to - 6.5 % in the seven months of the wet season.

Here, therefore, the secular variability cannot be regarded as a purely accidental quantity unless another law, more complicated than the normal one, applies and which is in some degree dependent upon the monsoons. This might be the case if the atmospheric pressure were dependent (and in a different manner in different seasons) upon another factor, for instance the temperature, the variability of which might still be according to the law of accidental quantities.

Similar systematic differences, varying with the season, between the calculated and the real value of π are not apparent in the results of the calculations for the atmospheric temperature in France, and the general average value of π does not differ from the real value more than 0.13 %.

TABLE III. Monthly means of atmospheric temperature, France.

	M	s	h	h'	A	π
January.....	2.07 C.	1.73 C.	0.341	0.326	195	2.869
February.....	2.03	1.70	0.356	0.332	188	2.855
March.....	1.59	1.25	0.446	0.452	115	3.230
April.....	1.20	0.92	0.588	0.616	66	3.444
May.....	1.32	1.07	0.536	0.529	79	3.067
June.....	1.14	0.91	0.629	0.623	59	3.150
July.....	1.29	1.00	0.548	0.565	76	3.347
August.....	1.08	0.88	0.653	0.641	53	3.029
September.....	1.19	0.94	0.594	0.600	64	3.205
October.....	1.25	1.02	0.565	0.551	71	2.991
November.....	1.50	1.22	0.472	0.464	102	3.043
December.....	2.41	1.84	0.294	0.306	264	3.418
Mean.....						3.137

In the paper already quoted, Mr. ANGOT assumed that the deviations do not show systematic differences in different months, and he subjects the deviations taken conjointly to the criterion of the law of errors.

This assumption is not justified by the results given in Table III, from which it is evident that the values of h are subject to considerable and systematic variations and, if a satisfactory agreement is still found between theory and observation, this can only be accounted for by the fact that the probability of the occurrence of deviations between fixed limits is expressed in a number of decimals too restricted to indicate the differences which, as for Helder and Batavia, must here exist between theory and practice.

No more can it be affirmed that, if a satisfactory accordance exists between the calculated and the observed number of deviations between given limits, the average value will also be the most probable one. In applying this criterion, as well as in calculating h' and π , a possible (and probable) skewness of the frequency curve is not taken into account because, by treating the deviations without regard to their sign, symmetry with respect to the ordinate of the centre of gravity of the figure is tacitly assumed.

As the number of years over which the observations extend is still far too small to allow frequency curves to be drawn for each month separately, it is still worth while to consider the deviations collectively, provided that at the same time the question be put, what form the law of deviations will assume when they are composed of groups which individually follow the normal law, the factor of steadiness being different for different groups. Even then the available data are insufficient to indicate with certainty a small degree of skewness in the frequency curve, so that only the symmetrical form can be sought for.

3. If, as in our case, the different groups occur with equal (sub) frequency, it is not difficult to indicate in what respects such a curve, the resultant of many elements, must differ from the normal curve. The groups characterised by large factors of steadiness will raise the number of small deviations above the number corresponding with an average factor and contribute only in a small degree to the number of large deviations, whereas, on the contrary, flat curves with small factors will give rise to a greater number of large deviations than is consistent with the normal law. Deviations of average magnitude will then occur to a less degree than is required by the common law; consequently in drawing the two curves, they will be seen to intersect at four points, as a minimum.

In a paper ¹⁾ published some years ago, SCHOLS has drawn the attention to the fact that differences of this description are almost always found when sufficiently extensive series of errors are put to the test of the normal law; in this paper he shows that these differences cannot be explained by the omission of terms in BESSEL's development of the exponential law and suggests that their origin must be sought for in the superposition of observations of different degrees of precision.

In the observations alluded to by SCHOLS, it will in general not be possible to estimate these degrees of precision any more than the relative subfrequencies with which the different groups are represented in the result; in the case of monthly means such as are being discussed here, the factors of steadiness are approximately known and the subfrequencies of the different groups are all identical.

If we arrange the 12 groups according to increasing values of h , it appears that we may take its change to be uniform; consequently it is possible to find an approximate solution of the problem in finite form.

We have then to consider h as a variable quantity z and to ask what form the expression will assume for a sum of elementary surfaces:

$$C \int_{-\infty}^{\infty} e^{-z^2 x^2} dx \dots \dots \dots (3)$$

if z varies in a continuous manner from h to H . If the subfrequency of these elementary groups be also regarded as a function of z (which occurs e.g. in the case of wind-frequencies), (3) must be equated to $\varphi(z) dz$, $\varphi(z)$ being subject to the condition:

$$\int_h^H \varphi(z) dz = 1 \dots \dots \dots (4)$$

The constant C is determined by the expression

$$C = \frac{z\varphi(z)dz}{\sqrt{\pi}} \dots \dots \dots (5)$$

and if, as in our case,

$$\varphi(z) = c$$

$$c = \frac{1}{H-h}, \quad C = \frac{zdz}{(H-h)\sqrt{\pi}}$$

¹⁾ Versl. Wis. Nat. Afd. K. Akad. Wet. I. 1893 (p. 194—202).

the resulting probability of a deviation being situated between x and $x + dx$ is then:

$$\frac{dx}{(H-h)\sqrt{\pi}} \int_h^x z e^{-z^2} dz$$

and the equation of the frequency curve:

$$y = \frac{1}{2(H-h)\sqrt{\pi}} \left[\frac{e^{-h^2x^2} - e^{-H^2x^2}}{x^2} \right] \dots \dots \dots (6)$$

Developing this expression we may put:

$$y = \frac{H+h}{2\sqrt{\pi}} e^{-\frac{H^2+h^2}{2}x^2} \left[1 + \frac{(H^2-h^2)^2}{2^2 \cdot 3!} x^4 + \frac{(H^2-h^2)^4}{2^4 \cdot 5!} x^8 \dots \right] \dots (7)$$

If we put:

$$\mu_n = 2 \int_0^\infty x^n y dx$$

we find with the help of:

$$2 \int_0^\infty x^n e^{-x^2} dx = \Gamma\left(\frac{n+1}{2}\right)$$

and

$$\int_0^\infty \frac{e^{-pz} - e^{-qz}}{z} dz = \log \frac{q}{p},$$

for the moments of different order with respect to the maximum ordinate:

$$\begin{aligned} \mu_0 &= 1 \quad , \quad \mu_2 = M^2 = \frac{1}{2Hh} \\ \mu_1 &= \mathfrak{D} = \frac{1}{(H-h)\sqrt{\pi}} \log \frac{H}{h} \quad , \quad \mu_3 = \frac{1}{2\sqrt{\pi}} \frac{H+h}{H^2h^2} \dots \dots (8) \end{aligned}$$

From a series of deviations following the law (6) the two characteristic constants H and h can be derived by computing the moments of the second and third order. They are found to be equal to the roots of the quadratic:

$$\begin{aligned} X^2 - pX + q &= 0 \\ p &= \frac{\mu_3\sqrt{\pi}}{2\mu_2} \quad , \quad q = \frac{1}{2\mu_2} \dots \dots \dots (9) \end{aligned}$$

If we had put a similar series to the test of the normal law (1) we should have found for the equation of the frequency curve:

$$y = \sqrt{\frac{Hh}{\pi}} e^{-Hhx^2}$$

or

$$y = \sqrt{\frac{Hh}{\pi}} e^{-\frac{H^2+h^2}{2} x^2} \left[1 + \frac{(H-h)^2}{2 \cdot 1!} x^2 + \frac{(H-h)^4}{2^2 \cdot 2!} x^4 \dots \right]. \quad (10)$$

On comparing this expression with (7) it is at once seen that in this manner too great a number of small deviations must be found, as the module of the deviation zero, computed by (10)

$$\sqrt{\frac{Hh}{\pi}},$$

is always smaller than that derived from (7):

$$\frac{H+h}{2\sqrt{\pi}}.$$

The position of the four points where the two curves intersect are found by equating the expressions (7) and (10); if the development can be stopped at the third term they are given by the roots of the biquadratic:

$$p X^4 - q X^2 + s = 0 \dots \dots \dots (11)$$
$$p = \frac{(H+h)^3}{6} - \sqrt{Hh} (H-h)^2, \quad q = 4 \sqrt{Hh}$$
$$s = \frac{4 (\sqrt{H} - \sqrt{h})^2}{(H-h)^2}.$$

With the help of the form. (8) for \mathfrak{D} , it can be shown that, if a series of figures follows the law (6) the computation of π according to (2) must necessarily lead to values which are somewhat too high:

$$\frac{2\mu_2}{\mathfrak{D}^2} = \pi \frac{(H-h)^2}{Hh} \left(\log \frac{H}{h} \right)^{-2}$$

Putting:

$$H + h = p, \quad H - h = q,$$

we find:

$$\log \frac{H}{h} = 2 \frac{q}{p} \left(1 + \frac{1}{3} \frac{q^2}{p^2} + \frac{1}{5} \frac{q^4}{p^4} + \dots \right)$$
$$\frac{2\mu_2}{\mathfrak{D}^2} = \pi \frac{1 + \frac{q^2}{p^2} + \frac{q^4}{p^4} + \dots}{\left(1 + \frac{1}{3} \frac{q^2}{p^2} + \frac{1}{5} \frac{q^4}{p^4} + \dots \right)^2} > \pi \dots \dots (12)$$

4. In the following applications of these reasonings to deviations taken collectively for all months, the frequencies are reduced to a total number of 1000: by exponential law is understood the simple, normal law of errors (1).

TABLE IV. Barometer, Helder.

Dev. mm.	Observ.	Exp L	Diff.	Dev. mm.	Observ.	Exp. L.	Diff.
0.0—0.45	104	100	+ 4	5.95—6.45	21	25	- 4
0.45—0.95	129	108	+21	6.45—6.95	17	19	- 2
0.95—1.45	121	106	+15	6.95—7.45	14	15	- 1
1.45—1.95	101	100	+ 1	7.45—7.95	7	11	- 4
1.95—2.45	97	92	+ 5	7.95—8.45	18	8	+10
2.45—2.95	86	84	+ 2	8.45—8.95	8	6	+ 2
2.95—3.45	68	75	- 7	8.95—9.45	10	4	+ 6
3.45—3.95	50	65	-15	9.45—9.95	7	3	+ 4
3.95—4.45	43	56	-13	9.95—10.45	2	2	0
4.45—4.95	38	47	- 9	10.45—10.95	1	1	0
4.95—5.45	31	39	- 8	10.95—11.45	0	0	0
5.45—5.95	25	32	- 7	11.45—11.95	2	2	0

$$\mu_1 = 2.769, \mu_2 = 12.867, \mu_3 = 77.427,$$

$$h(\text{Exp. L}) = \frac{1}{\sqrt{2\mu_2}} = \sqrt{Hh} = 0.1971,$$

$$\pi(\text{form. 2}) = 1.069 \times 3.142.$$

Points of intersection observ. near dev. 2.95 and 7.95,

$$H = 0.2712, h = 0.1433(\text{form. 9}).$$

$$\pi(\text{form 12}) = 1.044 \times 3.142.$$

Points of intersection (form. 11) at dev. 2.60 and 9.19.

The sums of the differences between the limits of the observed points of intersection are, as given in Table IV, +48, -70, +22.

If we also wish to compare these quantities with the result of the theory, we have to integrate (6) between the limits deduced from (11). For the limits α and zero we find the frequency:

$$\frac{1}{\alpha(H-h)\sqrt{\pi}} [e^{-\alpha^2 H^2} - e^{-\alpha^2 h^2}] + \frac{2}{(H-h)\sqrt{\pi}} \left[H \int_0^{\alpha H} e^{-\tau^2} d\tau - h \int_0^{\alpha h} e^{-\tau^2} d\tau \right] \quad (13)$$

By means of this formula we find between the limits calculated by means of (11):

α	Form. (6)	Form. (10)	Diff.	Obs.
0—2.60	553	509	+44	+48
2.60—9.19	440	476	-36	-70
9.19—etc.	7	15	- 8	+22

As the situation of the second point of intersection according to the observations (7.95) shows a rather large discrepancy with that given by theory (9.19), it is natural that only the sums of the positive differences between the limits zero and the first point of intersection agree closely.

Taken as a whole it may be stated that the secular variability of barometric pressure at Helder is regulated by the law of accidental events as completely as might have been expected considering the scantiness of the material available.

A possible skewness of the curve is left out of consideration as has been already remarked; it can, however, be but unimportant as in 720 deviations 364 are positive and 356 negative.

The same cannot be ascertained of the secular variability of barometric pressure at Batavia; the differences between the observed frequencies and those calculated according to the exponential law are not of such a well marked description as for Helder, so that a determination of the points of intersection is out of the question; their situation can only be calculated as a result of theory.

TABLE V. Barometer, Batavia.

Dev. mm.	Observ.	Exp. L.	Diff.	Dev. mm.	Observ.	Exp. L.	Diff.
0.000—0.095	146	135	+11	0.995—1.095	34	25	+ 9
0.095—0.195	149	136	+13	1.095—1.195	7	18	—11
0.195—0.295	126	129	— 3	1.195—1.295	7	12	— 5
0.295—0.395	117	118	— 1	1.295—1.395	10	8	+ 2
0.395—0.495	101	104	— 3	1.395—1.495	5	5	0
0.495—0.595	95	89	+ 6	1.495—1.595	7	3	+ 4
0.595—0.695	50	74	—24	1.595—1.695	2	2	0
0.695—0.795	52	59	— 7	1.695—1.795	2	1	+ 1
0.795—0.895	59	46	+13	1.795—1.895	0	1	— 1
0.895—0.995	29	35	— 6	1.895—etc.	2	0	— 2

$$\mu_1 = 0.4395, \mu_2 = 0.3156, \mu_3 = 0.2915.$$

$$h (\text{Exp. L.}) = 1.2586, \pi (\text{form. (2)}) = 1.040 \times 3.142,$$

$$H = 1.989, h = 0.796.$$

$$\pi (\text{form. (12)}) = 1.0115 \times 3.142.$$

Points of intersection (form 11) at dev. 0.399 and 1.620. For the sums of deviations between these limits we find (form 13):

α	Form. (6)	Form. (10)	Diff.	Obs.
0 — 0.399	559	522	+ 37	+ 17
0.399 — 1.620	431	474	— 43	— 19
1.620 — etc.	10	4	+ 6	+ 2

It appears from these results that the calculation of π cannot always be regarded as a good criterion of the variability being regulated by the law of accidental events. From a series of numbers, composed, as the barometric departures for Batavia are, of groups which follow neither the simple normal law nor the more complicated law (6), still the calculation of π leads to a value which is correct within 1%.

TABLE VI. Temperature, France.

Dev. C°.	Observ.	Exp L.	Diff.	Dev. C°.	Oserv.	Exp. L.	Diff.
0.00—0.15	73	78	— 5	2.15—2.35	27	36	— 9
0.15—0.35	113	101	+12	2.35—2.55	18	29	—11
0.35—0.55	108	98	+10	2.55—2.75	8	24	—16
0.55—0.75	87	95	— 8	2.75—2.95	25	19	+ 6
0.75—0.95	100	88	+12	2.95—3.15	15	15	0
0.95—1.15	83	82	+ 1	3.15—3.35	12	11	+ 1
1.15—1.35	77	74	+ 3	3.35—3.55	8	9	— 1
1.35—1.55	60	66	— 6	3.55—3.75	12	6	+ 6
1.55—1.75	70	59	+11	3.75—3.95	5	5	0
1.75—1.95	58	50	+ 8	3.95—4.15	2	3	— 1
1.95—2.15	30	43	—13	4.15—etc.	9	9	0

$$\mu_1 = 1.207, \mu_2 = 2.394, \mu_3 = 6.703.$$

$$h(\text{Exp. L.}) = 0.4570, \pi(\text{form. (2)}) = 1.046 \times 3.142,$$

$$H = 0.7627, h = 0.2739,$$

$$\pi(\text{form. (12)}) = 0.140 \times 3.142.$$

Position of points of intersection (form. (11) at dev. 1.09 and 4.87. Sums of deviations between these limits (form. 13):

α	Form. (6)	Form. (10)	Diff.	Obs.
0 — 1.09	565	519	+ 46	+ 22
1.09 — 4.87	429	480	— 51	— 22

As a general result of this investigation it can be stated that, according to theory, in all three series the number of small deviations is greater than the simple exponential law would require, but to a somewhat less degree than would follow from the law formulated in (6).

The deviations of barometric pressure at Helder are in almost perfect accordance with this frequency law and, therefore, for each month separately with the normal law; the curve of deviations of atmospheric temperature in France still shows many irregularities, but, in general, it accords well with the law of form. (6); the secular variability of atmospheric pressure at Batavia is not regulated by the law of accidental events and its frequency curve shows characteristic peculiarities in different seasons.

Microbiology. — "*Methan as carbon-food and source of energy for bacteria*". By N. L. SÖNINGEN. (Communicated by Prof. M. W. BEIJERINCK).

Methan, which is incessantly produced from cellulose in the waters and the soil, through the agency of microbes, and which, since vegetable life became possible on our planet must have been formed in prodigious quantities, yet occurs only in traces in our atmosphere.

As this gas is very resistant against chemical influences its disappearance in this way is highly improbable. But the conversion of methan into carbon dioxid and water produces a considerable quantity of heat, and so it seemed worth investigating whether there should exist any organic beings capable of feeding and living on it.

In the first place green plants were examined as to their power of decomposing methan in the light. To this end some waterplants were chosen, which seemed to offer most chance of success, considering that the formation of methan, as an anaerobic process, takes especially place in stagnant waters.

In this way positive results were obtained with several species of plants as *Callitriche stagnalis*, *Potamogeton*, *Elodea canadensis*, *Batrachium*, *Hottonia palustris*, *Spirogyra*. So, for example, in one of the experiments in the light of a window to the North, with *Hottonia palustris*, put in a flask containing 500 cc. of methan and 500 cc. of oxygen, and inversely placed in a vessel filled with water, all the methan disappeared from 7—21 May, so within a fortnight.