

if we write  $w_h^A$  for the development of heat in the element  $\mathbf{s}$ , which, in the state A, is due to the current in the principal direction  $h$ .

Now, starting from the expression (50), we shall obtain the total value of  $(\mathcal{E}_{lP})^2$  by an addition, in which all elements  $\mathbf{s}$ , each with its three principal directions, must be taken into account. In a system, completely shut off from surrounding bodies,  $\sum w_h^A$  will be the total amount of energy, emitted by  $P$  in the state A; we can therefore determine it by the formula (48), putting  $a\mathbf{S} = 1$ . This leads to the result

$$(\mathcal{E}_{lP})^2 = \frac{16 \pi k c^2 n dn}{3 \beta v^3}.$$

In the same way, using the theorem of § 7,  $b$  and the expression (49), I find

$$(\mathcal{B}_{lP})^2 = \frac{16 \pi k c^2 n^2 dn}{3 q v^3}.$$

These results being independent of the place of the point  $P$  and the choice of the direction  $l$ , we come to the conclusion that the state of things is the same in all parts of the medium  $L$  and that both the electric and the magnetic vibrations take place with equal intensities in all directions. The amount of the electric and magnetic energy per unit of volume is now easily found. According to § 4 the first is

$$\frac{1}{4} n \beta [(\mathcal{D}_x)^2 + (\mathcal{D}_y)^2 + (\mathcal{D}_z)^2],$$

for the value of which one finds

$$\frac{4 \pi k c^2 dn}{v^3},$$

by remembering that for every direction  $l$ ,

$$(\mathcal{D}_l)^2 = \frac{1}{n^2} (\mathcal{E}_l)^2.$$

The magnetic energy may likewise be determined. Referred to unit volume it has the value

$$\frac{1}{4} q [(\mathcal{B}_x)^2 + (\mathcal{B}_y)^2 + (\mathcal{B}_z)^2],$$

and this is easily calculated, since for every direction  $l$ ,

$$(\mathcal{B}_l)^2 = \frac{1}{n^2} (\mathcal{B}_l)^2.$$

The result is that the two kinds of energy are distributed over the body  $l$  with equal densities. This has been known for a long

time, as has also been the rule implied in our formulae, that these densities are inversely proportional to the cube of the velocity of propagation  $v$ . It must further be noticed that, if the medium  $L$  is aether, the density of the energy of the radiation becomes

$$\frac{8 \pi k dn}{c}.$$

This agrees with the meaning we have originally attached to the coefficient  $k$  (§ 11).

§ 19. There is one point in the foregoing considerations that may at first sight seem strange, viz. that the intensity of the electromotive forces we have imagined should depend on the magnitude of the elements of volume  $\mathbf{s}$ . It must be kept in mind however, that these forces have no real existence, and that we do not pretend to have found something concerning the causes by which the phenomena are produced. That the magnitude of the electromotive forces must be taken inversely proportional to the square root of the volume of  $\mathbf{s}$  is simply a consequence of our assumption that the force has the same phase in all points of such an element. For a given amplitude of the electromotive force, the radiation would therefore be proportional to  $\mathbf{s}^2$ , and we had to make such assumptions concerning that amplitude, that the radiation became proportional to  $\mathbf{s}$  itself.

In connection with these remarks it must be observed that we have no reasons for ascribing to the dimensions of the elements of volume some particular value. These dimensions are indifferent as long as we consider only the radiation at finite distances and the transfer of energy between neighbouring molecules lies outside the theory I have here developed.

**Physiology.** — “*On the ability of distinguishing intensities of tones*”.

By Prof. H. ZWAARDEMAKER. (Report of a research made by A. DEENIK.)

The “*Unterschiedsschwelle*” for impulsive sounds (dropping bullets and hammers) has been studied frequently and many-sidedly, but regarding the “*Unterschiedsschwelle*” for intensities of tone we have had at our disposal till now only some information communicated by M. WIEN in his thesis.

M. WIEN found the value of the “*Unterschiedsschwelle*” for the three tones, to which he limited his investigation to be as follows: for  $a$  average 22.5% (with 18.2 and 27 for extremes) for  $e'$  17.6%

(one determination) for  $a'$  average 14.4% (with 10.8 and 22.5 for extremes). It appeared desirable to perform such an investigation through the whole scale and to establish it in other regards also on larger foundations. At my request Mr. A. DEENIK has executed a very great number of observations of this kind, and I take the liberty to communicate his results here in short, and refer the reader to an ample description in a thesis on this subject by Mr. DEENIK which will soon be published.

*Experiments with the tuning-fork.*

A tuning-fork kept vibrating by electro-magnetism is started in a room at the side of the sound-free cabinet of the physiological laboratory and is kept vibrating at a fixed amplitude. This amplitude may be measured microscopically by means of the triangle of GRADENIGO. Normal to the axis of this tuning-fork a circle divided into grades is placed, to which two hearing-tubes are attached in such a way, that their radial prolongations cut the axis of the tuning-fork in the tuning-centre. These hearing-tubes can be moved along the whole circumference of the scale, and can be brought at pleasure into the interference-planes of KIESSLING, in the planes of maximum-sound or between.

The hearing-tubes are led into the interior of the sound-free cabinet by means of thick-walled caoutchouc tubes which were still further acoustically isolated. There by means of a T—tap alternately the one or the other of the tubes may be listened at or perfect acoustic rest can be obtained by bringing the tap into a closed position.

An assistant now displaces one of the hearing-tubes, while the other hearing-tube is fixed in the plane of maximum sound, every time through some grades at a time into the direction to the interference plane of KIESSLING till a distinct difference has been signalled by the investigator (descending method). After the position of the tube has been read this is pushed on and then brought back in the same way till the investigator observes that the existing difference in intensity becomes indistinct (ascending method). Again the position of the tube is read off and the average is taken.

The observations take place in the above mentioned way "unwisentlich" and at five succeeding times. From the ten figures obtained in this way the average is taken at last, which indicates in grades of the scale a lowest "Unterschiedsschwelle" for the concerned amplitude.

To be able to transpose these angle-values into absolute values,

in the sound free cabinet, which has been internally covered with trichopiese, the greatest distance at which sound is still perceptible is determined for the intensity of sound in the maximumplane and for that in the discovered "Unterschiedsschwelle" plane. If we accept that in case of absence of reverberations, as we may suppose here, the sound intensities decrease proportionate to the quadrates of the distances, the sound intensities stand mutually in the same proportion as the quadrates of those distances. If we call the distance at which the tone sound is perceptible in the plane of maximum sound  $r$  and that for the somewhat weaker sound  $r_u$ , then the quotient  $\frac{r^2 - r_u^2}{r_u^2}$  represents evidently the "Unterschiedsschwelle", which in this case may be indicated as "untere Unterschiedsschwelle" because the stimulation distinguished from the chief is taken weaker than the chief stimulation.

TABLE I. Experiments with the tuning-fork.

Tone level.	Amplitude in microns	$\frac{\Delta r}{r}$	'Unterschiedsschwelle' (average).
$c^1$	640	0.29589	33.2 %
	800	0.34429	
	1040	0.35657	
$c^2$	20	0.22698	29.3 %
	40	0.26932	
	70	0.29825	
	100	0.30835	
	150	0.31003	
	200	0.31540	
$c^3$	300	0.32006	19.5 %
	2	0.23435	
	2	0.20243	
	2	0.14865	

*Experiments with organ-pipes.*

An accurately tuned, wide, covered, wooden organ-pipe is placed in a felt tent in a room at the side of the soundfree cabinet in such

a way that the sound may be listened to through a caoutchouc tube in the cabinet. This organ-pipe is permanently blown by air which was supplied by a pressump driven by water and afterwards dried with chloride of calcium. The supply of this air takes place along a long system of leaden tubes, which shows inside the cabinet a division into two parts and afterwards a reunion. To this two separate branches by micrometer screws removable diaphragm openings of AUBERT are attached, which may be widened or narrowed at pleasure. The reunion takes place in a T-tap, which may also be directed by the investigator, and down the current are placed the necessary measuring apparatus for determining the pressure and volume of the air passing to the organ-pipe. These measuring apparatus are placed within the reach of the investigator, so that he himself can do the reading off.

The investigator arranges in the first place the width of the two diaphragm-openings in such a way that the sound may be called equal in the two positions of the tap. Then he enlarges one of the diaphragmata (the other remains constant) till a distinct difference is perceived (ascending method). This he does five times. After this the difference between the two tone intensities, which were alternately listened to, was enlarged and the diaphragm position was ascertained by descending at which the difference became indistinct (descending method). This again was done five times. The same takes place conformally in narrowing the diaphragm-openings. So the first series leads to a "obere" the second to an "untere Unterschiedsschwelle". The determinations which were made for each tone with two chief intensities have evidently taken place "wissentlich" in this way. At last a pressure and volume determination of the supplied air is made for the found diaphragm widths. The first takes place by means of a watermanometer, which for increasing sensibility has been put sloping; the second with an aerodromometer<sup>1)</sup>. The energy offered to the organ-pipe could be calculated with the usual formula  $e = \text{air-volume} \times \text{pressure} \times 981$ . This number, multiplied by a constant factor, different for each pipe, indicates the acoustic energy.

As in the expression of the "prozentische Unterschiedsschwelle"

$\frac{\Delta R}{R}$  the constant factor occurs both in the numerator and the denominator, the constant factor of the organ-pipe falls away from the further calculation and we can also come to a trustworthy result of the "prozentische Unterschiedsschwelle" without its preceding

<sup>1)</sup> Arch. f. (Anat. u.) Physiologie 1902 supplement. p. 417.

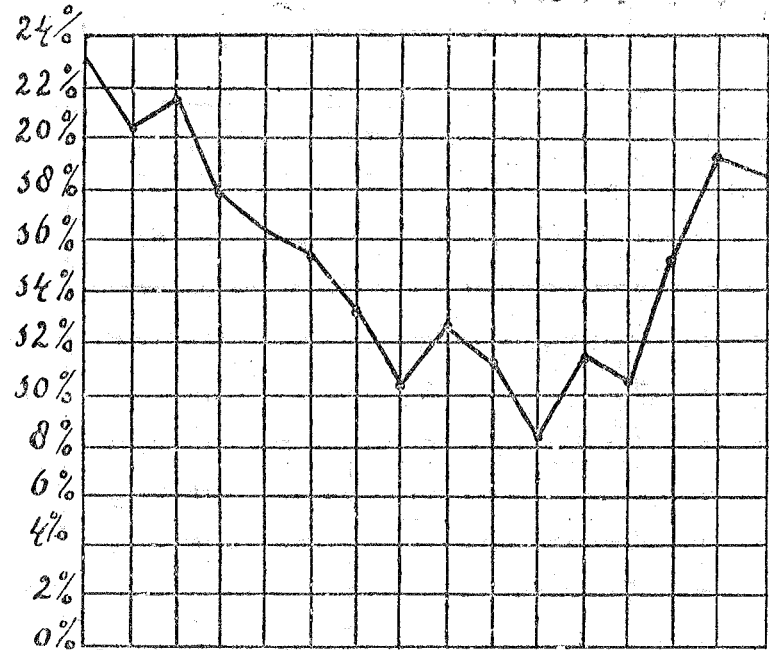
determination. The final result for each tone is in this way the average from 40 determinations.

TABLE II.

Tone level	Relative intensity of the chief stimulation <sup>1)</sup>	Unterschiedsschwelle.			
		$\frac{\Delta^o}{r}$	$\frac{\Delta^u}{r^u}$	$\frac{\Delta^r}{r}$	in %
<i>C</i>	1392.408 1120.55	0.237 0.236	0.219 0.237	{ 0.232	23.2
<i>G</i>	1560.168 1243.35	0.199 0.211	0.199 0.210	{ 0.204	20.4
<i>c</i>	1213.63 861.798	0.201 0.224	0.221 0.227	{ 0.218	21.8
<i>g</i>	1442.200 788.97	0.183 0.173	0.179 0.184	{ 0.179	17.9
<i>c<sub>1</sub></i>	107.112 86.129	0.162 0.166	0.158 0.168	{ 0.163	16.3
<i>g<sub>1</sub></i>	132.411 104.725	0.142 0.157	0.152 0.166	{ 0.154	15.4
<i>c<sub>2</sub></i>	140.800 114.444	0.143 0.137	0.108 0.134	{ 0.131	13.1
<i>g<sub>2</sub></i>	139.96 101.152	0.098 0.108	0.112 0.105	{ 0.105	10.5
<i>c<sub>3</sub></i>	135.976 101.764	0.104 0.126	0.132 0.138	{ 0.125	12.5
<i>g<sub>3</sub></i>	134.552 98.424	0.114 0.120	0.108 0.108	{ 0.112	11.2
<i>c<sub>4</sub></i>	251.01 139.438	0.077 0.081	0.082 0.101	{ 0.085	8.5
<i>g<sub>4</sub></i>	332.072 230.888	0.117 0.107	0.122 0.121	{ 0.117	11.7
<i>c<sub>5</sub></i>	424.636 280.908	0.108 0.100	0.114 0.107	{ 0.107	10.7
<i>g<sub>5</sub></i>	295.68 248.621	0.160 0.157	0.155 0.145	{ 0.154	15.4
<i>c<sub>6</sub></i>	260.100 183.272	0.188 0.204	0.200 0.178	{ 0.192	19.2
<i>g<sub>6</sub></i>	580.190 480.720	0.164 0.171	0.191 0.229	{ 0.188	18.8

<sup>1)</sup> For the calculation of the absolute intensity the number of the second column must still be multiplied by a constant factor which however falls away in the calculation of the "Unterschiedsschwelle" and is of no consequence.

Differences of intensities.



*c.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g. e.g.*  
1. 2. 3. 4. 5. 6.

Smallest perceptible difference of intensity by the scale.

CONCLUSION.

1. From the results of the experiments with the tuning-fork proceeds that the law of WEBER is valuable, when taken in a general way, but not exactly for the investigated middle-strong and weak intensities.
2. From the results of the organ-pipes proceeds that the most favourable "Unterschiedsschwelle" is found with  $c^4$  and that from there to the ends the power of distinguishing differences in intensities decreases rather regularly.

ERRATUM.

p. 380 line 7 for 0,990 1,03 read 1,02 1,04.

(November 22, 1905).

KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN  
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING  
of Saturday November 25, 1905.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige  
Afdeling van Zaterdag 25 November 1905, Dl. XIV).

CONTENTS.

- EUG. DUBOIS: "The geographical and geological signification of the Hondsrug, and the examination of the erratics in the Northern Diluvium of Holland", (Communicated by Prof. K. MARTIN), p. 427.
- D. J. KORTEWEG: "HUYGENS' sympathetic clocks and related phenomena in connection with the principal and the compound oscillations presenting themselves when two pendulums are suspended to a mechanism with one degree of freedom", p. 436.
- H. W. BAKHUIS ROOZEBOOM: "The different branches of the three-phases for solid, liquid, vapour in binary systems in which a compound occurs", p. 455.
- F. M. JAEGER: "On Diphenylhydrazine, Hydrazobenzene and Benzylaniline, and on the miscibility of the last two with Azobenzene, Stilbene and Dibenzyl in the solid aggregate condition". (Communicated by Prof. H. W. BAKHUIS ROOZEBOOM), p. 466.
- A. P. N. FRANCHIMONT and H. FRIEDMANN: "The amides of  $\alpha$ - and  $\beta$ -aminopropionic acid", p. 475.
- J. D. VAN DER WAALS JR.: "Remarks concerning the dynamics of the electron". (Communicated by Prof. J. D. VAN DER WAALS), p. 477.
- R. SISSINGH: "Derivation of the fundamental equations of metallic reflection from CAUCHY'S theory". (Communicated by Prof. H. A. LORENTZ), p. 486.
- P. H. SCHOUTE: "A tortuous surface of order six and of genus zero in space  $S_4$  of four dimensions", p. 489.
- W. VERSLUYS: "The PLÜCKER equivalents of a cyclic point of a twisted curve" (Communicated by Prof. P. H. SCHOUTE), p. 498.
- "Preliminary Report on the Dutch expedition to Burgos for the observation of the total solar eclipse of August 30, 1905," communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN, in behalf of the Eclipse Committee, p. 501.

**Geology.** — "The geographical and geological signification of the Hondsrug, and the examination of the erratics in the Northern Diluvium of Holland." By Prof. Eug. Dubois. (Communicated by Prof. K. MARTIN).

(Communicated in the meeting of September 30, 1905).

To those who do not know the Hondsrug from a personal visit the name generally suggests an imposing hilly ridge, or perhaps even a small mountain range. Visiting it for the first time, one is disappointed in finding it to be no more than a nearly imper-