$$\sigma y_i = q_1 \lambda_1^i + q_2 \lambda_2^i + q_3 \lambda_3^i, \quad (i = 0, 1, 2, 3, 4) \quad . \quad . \quad . \quad (4)$$

hold, and now the equation sought for is found by eliminating the nine quantities $\lambda_1, \lambda_2, \lambda_3, p_1, p_2, p_3, q_1, q_2, q_3$ out of the ten equations (3) and (4). This takes place by inserting the values given by (3) and (4) in the left hand member of the second equation (2). For by this we find

$$\varrho^2 \sigma^2 \begin{vmatrix} x_0 & x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 & x_4 \\ y_0 & y_1 & y_2 & y_3 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 \end{vmatrix} . \begin{vmatrix} p_1 & p_2 & p_3 \\ p_1\lambda_1 & p_2\lambda_2 & p_3\lambda_3 \\ q_1 & q_2 & q_3 \\ q_1\lambda_1 & q_2\lambda_2 & q_3\lambda_3 \end{vmatrix} = 0.$$

We considered in the above cited communication equations forming the extension of the first of the equations (2) to the curve k^{2n} of the space Sp_{2n}. In connection with this we shall notice that the second of the equations (2) admits of corresponding extensions, in which those of the first are included. However, these will be developed elsewhere.

Mathematics. — "*The* PLÜCKER *equivalents of a cyclic point of a twisted curve.*" By W. A. VERSLUYS. (Communicated by Prof. P. H. SCHOUTE.

If a twisted curve C admits of a higher singularity (cyclic point) of order n, of rank r and of class m, it is to be represented according to HALPHEN [1]) in the vicinity of this singular point M by the following developments in series:

$$x = t^n,$$
$$y = t^{n+r}\,[t],$$
$$z = t^{n+r+m}\,[t],$$

where $[t]$ represents an arbitrary power series of t, starting with a constant term.

If n, r and m satisfy the conditions that

1°	n and r,	
2°	r and m,	
3°	n and $r+m$,	(A)
4°	$n+r$ and m	

are mutually prime, then this higher singularity $M\ (n, r, m)$ for

[1]) Bull. d. l. Soc. Mat. d. France t. VI p. 10.

the formulae of CAYLEY-PLÜCKER and for the genus is equivalent to the following numbers of ordinary singularities:

$$n-1 \text{ cusps } \beta,$$
$$\frac{(n-1)(n+r-3)}{2} \text{ nodes } H,$$
$$m-1 \text{ stationary planes } \alpha,$$
$$\frac{(m-1)(m+r-3)}{2} \text{ double planes } G, \qquad (B)$$
$$r-1 \text{ stationary tangents } \theta,$$
$$\frac{(r-1)(r+m-3)}{2} \text{ double generatrices } \omega_1,$$
$$\frac{(r-1)(r+n-3)}{2} \text{ double tangents } \omega_2.$$

For a curve with only ordinary singularities we always have $\omega_1 = \omega_2$.

If the curve admits of higher singularities, then the tangents in these singular points will not have to count for as many double tangents to the curve as they must count for double generatrices of the developable belonging to the curve. The number ω will then be different for the formulae of CAYLEY-PLÜCKER, relating to a section and for those formulae relating to a projection, i. o. w. the singularity ω of a twisted curve appearing in a term $(x + \omega)$ is not always the same as the one appearing in the term $(y + \omega)$.

So the formula

$$y - x = \nu - \mu \;{}^{1})$$

is no longer correct as soon as the curve has higher singularities for which order and class are unequal.

The above as well as the following results do not hold for a common cusp $\beta\,(2, 1, 1)$ and for a common stationary plane $\alpha\,(1, 1, 2)$, the conditions (A) not being satisfied for these cyclic points.

Through the singular point $M\ (n, r, m)$ pass

$$\frac{n\,(n+2r+m-4)}{2}$$

branches of the nodal curve of the developable O belonging to the curve C.

All these branches touch the curve C in M and have in M with the common tangent

$$\frac{(n+r)\,(n+2r+m-4)}{2}$$

coinciding points in common.

[1]) SALMON. 3 Dim. § 327.

These branches have in M the same osculating plane as C and with this osculating plane they have in M

$$\frac{(n+r+m)(n+2r+m-4)}{2}$$

coinciding points in common.

From the conditions (A) ensues that $(n+2r+m)$ is even, so that the three above numbers are integers.

The second polar surface of O according to an arbitrary point meets in the point $M(n, r, m)$ the cuspidal curve

$$(n+r-2)(n+r+m)$$

times and the nodal curve

$$\frac{n+2r+m-4}{2}(n+r-2)(n+r+m)$$

times.

Each point R, where the tangent in M still meets a sheet of the surface O, counts for

$$r^2+rm-m-r$$

points of intersection of the nodal curve with the second polar surface.

In the equation of CREMONA [1]) serving to determine λ (number of cusps of the nodal curve) we must add for every singular point $M(n, r, m)$ in the second member of the equation a term

$$(n+r-2)(n+r+m).$$

In the equation of CREMONA [2]), serving to determine τ (number of triple points of the nodal curve) we must add for every singular point to the second member of the equation a term

$$\frac{n+2r+m-4}{2}(n+r-2)(n+r+m)$$

and for the corresponding points R a term

$$(n+2r+m-4)(r+m)(r-1).$$

The decrease of λ and τ arising from the presence of a point $M(n, r, m)$ is not equal to the decrease of λ and τ caused by the ordinary singularities necessary to form a singularity $M(n, r, m)$. So the equivalence of the values expressed in (B) does not extend to numbers which are found by means of a second polar surface.

Delft, November 1905.

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[1]) CREMONA-CURTZE, Oberflächen § 104.
[2]) loc. cit. § 109.

Astronomy. — "*Preliminary Report on the Dutch expedition to Burgos for the observation of the total solar eclipse of August* 30, 1905," communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN, in behalf of the Eclipse Committee.

In March 1904 the Eclipse Committee determined to fit out on a small scale an expedition to Spain to observe the total solar eclipse of August 30, 1905. The means for it were found from some liberal gifts of private persons and of societies (Provinciaal Utrechtsch Genootschap, Teyler's Stichting, Utrechtsch Oud-Studentenfonds, Natuur en Geneeskundig Congres). As observers the same persons were appointed who had been sent to Sumatra in 1901: Messrs. W. H. JULIUS, J. H. WILTERDINK and the undersigned. The observations were to include the spectrography of the corona and of the sun's limb and, provided a fourth observer should offer himself to join as a volunteer, the radiation of heat of the corona.

A volunteer was soon found in the person of Mr. MOLL, assistant for physics at Utrecht, and so the entire programme could be worked out.

The outfit of the expedition consisted of:

a siderostat with a coelostat apparatus;

two slit-spectrographs, to be directed on the coelostat mirror;

a prismatic camera, to be directed on the northern polar mirror;

a heat actinometer;

a pyrheliometer;

a sextant with accessories;

three chronometers and other auxiliary apparatus.

As the principal instruments were also used for the eclipse of 1901, I refer for the description of them to previous publications (These Proc. III p. 529).

The sextant and two of the three chronometers were kindly placed at our disposal by His Excellency "de Minister van Marine" out of the collection of instruments at Leyden.

On the 13th of August the party arrived at Burgos. This town had been chosen for the observations not only on account of its favourable situation and other outward advantages, but also because, as far as was known at the time, it would not be visited by other expeditions. These advantages were lost through the visit of H.M. the King of Spain, on which occasion the town council of Burgos organised a series of festivities which seriously interfered with the astronomical work. For it is chiefly owing to those feasts that in spite of all