if n is an odd number, where

$$\lambda'_p = \frac{(2n-p-1)(2n-p-2)\ldots(2n-2p)}{p!} b_{n-p+2} \ldots b_{2n-p-1}$$

$$\varkappa'_p = \frac{(2n-p-2)(2n-p-3)\ldots(2n-2p-1)}{p!} b_{n-p+2} \ldots b_{2n-p-2}$$

$$\iota'_p = \frac{(2n-p-3)(2n-p-4)\ldots(2n-2p-2)}{p!} b_{n-p+2} \ldots b_{2n-p-3}$$

. . . .

$$\varepsilon'_p = \frac{(n-p+1)(n-p)\ldots(n-2p+2)}{p!}$$

en

$$\lambda_p = \frac{(2n-p-1)(2n-p-2)\ldots(2n-2p)}{p!} b_{p+1} \ldots b_{2n-p-1}$$

$$\varkappa_p = \frac{(2n-p-2)(2n-p-3)\ldots(2n-2p-1)}{p!} b_{p+1} \ldots b_{2n-p-2}$$

$$\iota_p = \frac{(2n-p-3)(2n-p-4)\ldots(2n-2p-2)}{p!} b_{p+1} \ldots b_{2n-p-3}$$

. . . .

$$\varepsilon_p = \frac{(n-p+1)(n-p)\ldots(n-2p+2)}{p!} b_{p+1} \ldots b_{n-p+1}.$$

It is of importance to remark that

$$\lambda_{n-1} = nb_n, \quad \varkappa_{n-1} = 1, \quad \iota_{n-2} = (n-1)\, b_{n-1}, \quad \theta_{n-2} = 1 \text{ etc.}$$

and that the determinants in the second members of the equations (I) and (II) after the substitution $b_p = \nu + p$, are respectively polynomia of degrees $\frac{n(n-2)}{4}$ and $\frac{(n-1)^2}{4}$ in ν.

Meteorology. – "*On frequency curves of barometric heights.*" By Dr. J. P. van der Stok.

1. The records of barometric heights, corrected for temperature, observed at Helder three times a day during the years August 1843 to July 1904, have been chosen as an appropriate material for this inquiry into the nature of barometric frequency curves. The number of observations for each month amounts to:

January	5673	July	5673
February	5169	August	5766
March	5646	September	5560
April	5490	October	5766
May	5673	November	5580
June	5490	December	5766
		Total	67252

TABLE I. Frequencies in 10.000 of deviations of barometric heights; positive and negative being taken together.

ε	Jan.	Febr.	March	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Nov. — Febr.	Mrch.—Apr. Sept.—Oct.	May — Aug.
0 — 0.5 mm.	381	420	417	493	612	705	767	704	559	419	357	377	384	472	697
0.5— 1.5	680	837	798	1028	1176	1486	1493	1405	1110	865	726	755	749	950	1390
1.5— 2.5	712	779	821	1040	1170	1367	1456	1369	1048	861	750	706	737	943	1340
2.5— 3.5	735	752	841	1010	1164	1191	1285	1292	1000	837	737	680	726	922	1233
3.5— 4.5	703	799	826	893	1020	1110	1121	1156	950	771	684	662	712	860	1102
4.5— 5.5	679	775	702	907	952	933	998	1016	876	775	686	724	716	845	975
5.5— 6.5	660	683	751	825	809	804	807	806	783	801	715	660	679	790	806
6.5— 7.5	647	615	637	733	767	716	602	651	713	728	655	601	630	703	684
7.5— 8.5	636	605	606	633	607	572	428	509	684	638	668	637	637	640	529
8.5— 9.5	564	564	579	533	486	402	305	393	550	582	610	553	573	561	396
9.5—10.5	528	493	532	459	391	262	233	230	468	574	557	514	523	508	279
10.5—11.5	498	425	459	362	261	171	179	166	351	452	546	473	485	406	194
11.5—12.5	421	353	406	281	199	105	128	111	283	347	460	445	420	329	136
12.5—13.5	338	342	341	225	126	66	78	73	203	351	395	415	372	280	86
13.5—14.5	267	302	249	166	73	41	42	48	123	257	312	322	301	199	51
14.5—15.5	243	270	235	102	59	32	28	28	88	226	249	307	267	163	37
15.5—16.5	238	213	238	91	60	22	22	12	50	133	195	268	229	128	29
16.5—17.5	225	154	152	59	32	9	13	9	36	118	151	201	183	91	16
17.5—18.5	188	127	99	42	12	3	7	8	35	67	115	147	144	61	8
18.5—19.5	129	116	84	37	10	3	5	4	29	43	119	118	120	48	5
19.5—20.5	94	80	59	23	7		3	5	17	53	86	91	88	38	4
20.5—21.5	76	64	29	17	4			4	8	30	62	70	68	21	2
21.5—22.5	67	56	42	12	3			1	10	16	41	60	56	20	1
22.5—23.5	69	35	16	8					6	17	29	53	46	12	
23.5—24.5	59	19	18	5					4	10	16	28	31	9	
24.5—25.5	42	23	13	7					3	14	16	34	29	9	
25.5—26.5	29	18	11	2					4	2	10	20	19	5	
26.5—27.5	29	22	6	2					3	9	14	21	21	5	
27.5—28.5	17	14	11	1					2	2	17	13	15	4	
28.5—29.5	8	11	6	4					2	2	10	10	10	3	
29.5—30.5	7	11	9						2		6	12	9	3	
30.5—31.5	10	11	3								2	3	6	1	
31.5—32.5	6	7	4								2	5	5	1	
32 5—33.5	4	3									2	3	3		
33.5—34.5	1	1										2	1		
34.5—35.5	1	0										2	1		
35.5—36.5	4	0										3	2		
36.5—37.5	3	0										0	1		
37.5—38.5	1	0										0	0		
38.5—39.5	1	1										5	2		

TABLE II. Differences, W—B, between observed frequencies and frequ. calculated according to exponential law; in 10.000 for every month, in 40.000 for the seasons.

ε	Jan.	Febr.	March	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Sums. Nov.—Febr.	Sums. Mrch.—Apr. Sept.—Oct.	Sums. May—Aug.
0—0.5	−3	+3	−23	−59	−30	−27	+11	−40	−16	−57	−60	−18	−78	−155	−86
0.5—1.5	−94	+7	−13	−56	−82	+24	+10	−45	−17	−86	−116	−30	−233	−172	−93
1.5—2.5	−52	−49	−48	−13	−50	−12	+48	−20	−56	−60	−66	−69	−236	−177	−34
2.5—3.5	−11	−42	−2	+5	+20	−75	+4	+19	−38	−62	−68	−75	−196	−97	−32
3.5—4.5	−18	+35	+19	−47	−17	−12	−15	+37	−16	−74	−79	−78	−140	−118	−7
4.5—5.5	−12	+47	−60	+37	+21	−36	+29	+50	−14	−27	−51	+25	+9	−64	+64
5.5—6.5	+4	−10	+47	+46	+7	+9	+15	+13	−3	+68	+32	−1	+25	+158	+44
6.5—7.5	+30	−22	−20	+43	+83	+79	−28	+13	+17	+52	+10	−19	−1	+92	+147
7.5—8.5	+70	+18	+7	+28	+44	+80	−51	+16	+94	+38	+83	+60	+231	+167	+79
8.5—9.5	+33	+30	+40	+24	+37	+35	−43	+29	+54	+50	+70	+23	+156	+168	+58
9.5—10.5	+84	+5	+53	+33	+37	−6	−20	−35	+56	+104	+78	+30	+197	+246	−24
10.5—11.5	+59	+2	+39	+12	−7	−14	+3	−16	+23	+48	+113	+37	+211	+122	−34
11.5—12.5	+28	−30	+42	−1	−3	−19	+16	−13	+20	+7	+85	+51	+134	+68	−19
12.5—13.5	−12	+9	+33	+2	−19	−15	+7	−9	+3	+64	+63	+71	+131	+102	−36
13.5—14.5	−40	+19	−15	−6	−30	−10	−6	+2	−30	+23	+31	+21	+31	−28	−44
14.5—15.5	−25	+28	+15	−31	−11	+1	0	−5	−25	+33	+6	+45	+54	−8	−15
15.5—16.5	+7	+9	+56	−7	+14	+4	+6	−6	−32	−19	−7	+44	+53	−2	+18
16.5—17.5	+31	−18	+4	−13	−1	−3	+5	−1	−22	−3	−19	+10	+4	−34	0
17.5—18.5	+20	−13	−17	−11	−7	−2	+3	+3	−10	−28	−24	−14	−31	−66	−3
18.5—19.5	−12	+2	−10	+3	−3	0	+2	+1	+2	−30	+5	−16	−21	−35	0
19.5—20.5	−23	−12	−15	−3	0	−2	+1	+3	−1	−2	−5	−20	−50	−21	+2
20.5—21.5	−20	−11	−29	−1				+3	−5	−12	−11	−21	−63	−47	+3
21.5—22.5	−11	−2	0	+1				+1	+3	−13	−16	−14	−43	−9	+1
22.5—23.5	+5	−10	−18	+1						−6	−16	−5	−26	−23	
23.5—24.5	+9	−16	−8	0						−5	+18	−19	−8	−13	
24.5—25.5	0	−3	−5	+4						+4	+11	−3	+5	+3	
25.5—26.5	−2	−4	−3	0						−7	+9	−9	−6	−10	
26.5—27.5	+7	+8	−4	+1						+4	+2	−1	+16	+1	
27.5—28.5	−2	+2	+5	0						−1	−7	−3	−10	+4	
28.5—29.5	−7	+4	+1	+3						−1	−2	−3	−8	+3	

TABLE III. Skew-differences, *P—N*, of positive and negative deviations.

ε	Jan.	Febr.	March	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Sums. Nov.—Febr.	Sums. Mrch.—Apr. Sept.—Oct.	Sums. May—Aug
0.5— 1 5	6	21	30	0	16	80	43	1	— 46	23	12	15	54	7	140
1 5— 2.5	10	— 3	55	106	18	139	114	77	48	71	22	62	91	280	348
2 5— 3 5	25	56	81	122	54	115	79	164	68	37	69	60	210	308	412
3.5— 4.5	93	75	16	135	44	114	75	92	90	61	50	20	238	302	325
4 5— 5.5	95	89	42	147	80	79	136	50	86	57	86	60	330	332	345
5 5— 6.5	118	105	79	121	61	78	133	126	85	83	91	86	400	368	398
6 5— 7.5	133	127	111	151	37	48	74	97	95	138	51	45	356	395	256
7.5— 8.5	102	83	16	113	43	— 2	44	41	138	94	60	89	334	411	126
8.5— 9 5	128	108	13	23	26	— 20	5	19	144	62	104	97	437	242	30
9.5—10.5	74	77	80	15	19	— 10	— 31	— 28	38	40	109	108	368	173	— 50
10.5—11.5	72	33	75	— 56	19	15	— 31	— 42	19	60	154	75	334	98	— 39
11.5—12.5	77	25	24	— 45	— 17	— 11	— 48	— 45	3	3	112	47	261	— 15	— 121
12 5—13.5	50	32	17	25	6	— 42	— 46	— 33	— 9	7	93	57	232	40	— 115
13 5—14.5	33	12	25	— 14	— 19	— 35	— 30	— 32	— 43	35	20	38	103	3	— 116
14.5—15.5	41	34	7	— 16	— 13	— 32	— 26	— 18	— 62	— 14	— 33	39	81	— 57	— 89
15.5—16.5	44	21	24	— 33	— 38	— 20	— 22	— 12	— 34	— 57	— 81	36	20	— 100	— 92
16.5—17.5	5	— 2	— 2	— 23	— 30	— 7	— 13	— 9	— 36	— 38	— 37	23	— 11	— 99	— 59
17.5—18.5	— 26	— 25	— 19	— 26	— 12	— 3	— 7	— 8	— 31	— 43	— 39	— 17	— 107	— 119	— 30
18.5—19.5	— 25	— 20	— 22	— 29	— 10	— 3	— 5	— 4	— 29	— 23	— 29	— 42	— 116	— 103	— 22
19.5—20.5	— 40	— 14	— 39	— 19	— 7		— 3	— 5	— 17	— 33	— 26	— 27	— 107	— 108	— 15
20.5—21.5	— 32	— 26	— 23	— 17	— 4			— 4	— 8	— 16	— 38	— 30	— 126	— 64	— 8
21.5—22 5	— 41	— 32	— 26	— 12	— 3			— 1	— 10	— 16	— 31	— 38	— 142	— 64	— 4
22.5—23.5	— 47	— 29	— 16	— 8					— 6	— 17	— 25	— 47	— 148	— 47	
23.5—24.5	— 39	— 19	— 18	— 5					— 4	— 10	— 16	— 22	— 96	— 37	
24 5—25.5	— 38	— 23	— 13	— 7					— 3	— 14	— 16	— 30	— 107	— 37	
25.5 - 26.5	— 27	— 18	— 11	— 2					— 4	— 2	— 10	— 20	— 75	— 19	
26.5—27.5	— 29	— 22	— 6	— 2					— 3	— 9	— 14	— 21	— 86	— 20	
27.5—28.5	— 17	— 14	— 11	— 1					— 2	— 2	— 17	— 13	— 61	— 16	
28.5—29.5	— 8	11	— 6	— 4					— 2	— 2	— 10	— 10	— 39	— 14	
29 5—30.5	— 7	- 11	— 9						— 2		— 6	— 12	— 36	— 11	
30.5—31.5	— 10	— 11	— 3								— 2	— 3	— 26	— 3	
31 5—32 5	— 6	— 7	— 4								— 2	— 5	— 20	— 4	
32.5—33 5	— 4	— 3									— 2	— 3	— 12		
33.5—34.5	— 1	— 1										— 2	— 4		
34.5—35.5	— 1	0										— 2	— 3		
35.5—33.5	— 4	0										— 3	— 7		
36.5—37.5	— 3	0										0	— 3		
37.5—38.5	— 1	0										0	— 1		
38.5 enz.	— 1	— 1										— 5	— 7		

In registering the observations the decimals have been omitted, so that the number of occurrences corresponding with a height of P mm. includes all values between P + 0.5 and P — 0.5 mm.

Owing to this simplification the amount of labour is less than would appear from the great number. of data. The next work to do was to multiply the frequency numbers with a factor such that the total number for each month amounted to 10.000. The frequencies thus obtained correspond with expressions for the probability of occurrence expressed in 10.000$^{\text{ths}}$ parts of unity. Then the average height was calculated and, by means of simple, linear interpolation the whole curve shifted in such a manner that the new frequencies correspond with deviations from the average value expressed in multiples of whole numbers. This has been done not only with a view of abridging the computations of the moments of the second and third order but principally in order to obtain an evaluation of the skewness of the curves, which may be defined as the inequality of frequency for equal positive and negative deviations from the arithmetical mean. If of such a series of data the frequencies corresponding with equal deviations are taken together, no account being taken of their sign, the skewness is eliminated, and the numbers obtained in this way may be considered as belonging to a symmetrical curve (Table I).

For this curve we calculate the factor of precision (stability) and investigate in how far the actual curve agrees or disagrees with the curve of the normal exponential law (Table II).

As has been mentioned above, the inequalities of frequencies for equal deviations of opposite sign have been taken as a measure of the skewness.

Tables I—III show, separately for each month, the sums and differences thus formed. The numbers of Table I added to those of Table III will give twice the number of frequencies corresponding to positive deviations, their differences being twice that corresponding to negative deviations. The values given for Winter, Summer and Spring-Autumn are obtained by taking together the corresponding numbers in the same Tables; consequently they are not quite identical with the numbers which would have been obtained if the frequencies for these seasons had been calculated from the absolute heights, instead of, as has been done here, from the deviations; in the latter the annual variation has been left out of consideration. The annual variation, however, being very small, this will not influence the results to an appreciable degree.

2. Table IV shows the results of the treatment of the frequencies given in Table I, as indicated. If the deviation from the arithmetical mean is denoted by ε, then:

$$M = \sqrt{\frac{|\varepsilon^2|}{n-1}}, \quad \vartheta = \frac{|\varepsilon|}{n}, \quad h = \frac{1}{M\sqrt{2}}, \quad h' = \frac{1}{\vartheta\sqrt{\pi}}, \quad \pi = \frac{2M^2}{\vartheta^2}.$$

TABLE IV.

	M	ϑ	h	h'	π
Jan.	10.261 mM.	8.272 mM.	0.0689	0.0682	3 081
Febr.	9.522	7.597	0.0743	0.0743	3.141
Mrch.	8.969	7.194	0.0788	0.0784	3.109
Apr.	7.280	5.864	0.0971	0.0962	3 083
May	6.218	5.022	0.1136	0.1124	3.067
June	5 391	4.322	0 1312	0.1305	3.112
July	5.276	4.169	0 1340	0.1354	3.204
Aug.	5.374	4.300	0 1316	0.1312	3.125
Sept.	6 972	5.602	0 1014	0.1007	3'098
Oct.	8.372	6.832	0.0845	0 0826	3.003
Nov.	9.490	9.006	0.0745	0.0725	2.974
Dec.	10.085	8.173	0.0701	0.0690	3.045

From this summary it appears that the frequency curve of barometric heights, as derived from observations made at Helder, shows systematic departures from the nórmal curve corresponding to the exponential law. For all months (except February and July) h is greater than h'; in February these factors are equal and the curve is nearly a normal one, in July $h' > h$.

In agreement with this result the calculated value of π is always (except in the two months mentioned) less than its true value; the departures from the normal law are greatest in winter, smallest in summer time.

It may be noticed that the departures from the normal curve, given in table II, are generally of an opposite sign to those which are found in the great majority of series of errors: whereas for the latter the rule holds that small deviations occur oftener than is required by the normal law (in which case $h' > h$ and π calc. $> \pi$), here the reverse obtains, the frequency of barometric heights showing a deficit for small and a surplus for moderate deviations.

In an earlier paper (this volume p. 314) I have shown that, in taking together series with different factors of steadiness, each series occurring with equal subfrequency, we must expect to find too great a number of small deviations.

From this follows the apparently somewhat paradoxical conclusion, that a sum of frequency numbers as those of barometric deviations, all showing negative differences for small deviations, may, when taken together, lead to a resulting curve in which these differences have vanished or even turned positive.

This conclusion is of some importance because an investigation into the frequency of barometric heights, in which the different months are not treated separately, may lead to normal curves (the skewness being left out of account) whereas in fact no normal curve exists and appears only as an artificial consequence of the combination of incomparable frequency numbers.

The exceptional behaviour of the months of February and July might then be explained by assuming that the different series of barometric curves corresponding with different winds (barometric windrose) are more differentiated in these two months than in the other ones.

A second remark is that frequency numbers as given in Table I cannot be accepted as a measure for the variability of the atmospheric pressure in the course of a month, at least not if we adhere to the conception of this variability as generally admitted.

On the one hand we have here to do with the superposition of two kinds of variability, 1st the secular variability as shown by the variability from year to year of monthly means and 2nd the variability from day to day, which might be called the interior variability for the month in question; it is the latter definition which corresponds with the usual conception.

On the other hand, daily means or observations taken at fixed hours are by no means to be regarded as being independent of each other.

The questions, therefore, arise: how can we separate the two kinds of variability, and to what degree are daily mean values of barometric observations to be taken as dependent upon each other in the different months.

For a knowledge of the climate of a place the latter question is of importance; it might also be formulated thus: what is the average duration of a barometric disturbance, a question which can hardly be answered by means of direct investigation.

TABLE V.

ε	Observ. W.	Exp. L. B	W—B	ε	Observ. W.	Exp. L. B.	W—B
0— 0 5	515	496	+ 19	14 5—15 5	148	172	— 24
0 5— 1 5	1025	996	+ 29	15 5—16.5	125	134	— 9
1.5— 2 5	1001	962	+ 39	16.5—17.5	100	104	— 4
2 5— 3 5	956	935	+ 21	17.5—18 5	75	79	— 4
3 5— 4 5	888	875	+ 13	18.5—19 5	58	59	— 1
4.5— 5 5	846	826	+ 20	19.5—20.5	43	45	— 2
5.5— 6.5	756	757	— 1	20 5—21 5	33	29	+ 4
6.5— 7 5	674	676	— 2	21 5—22.5	25	23	+ 2
7 5— 8.5	591	608	— 17	22.5—23 5	21	16	+ 5
8.5— 9 5	519	526	— 7	23 5—24.5	13	12	+ 1
9 5—10 5	437	459	— 22	24 5—25.5	13	7	+ 6
10.5—11.5	363	384	— 21	25 5—26.5	10	5	+ 5
11 5—12.5	304	322	— 18	26 5—27 5	7	3	+ 4
12 5—13.5	244	268	— 24	27 5—28.5	6	2	+ 4
13 5—14.5	191	216	— 25	28 5—etc.	13	4	+ 9

The first problem is identical with the calculation of the probability of an event $a + b$, when a and b follow the normal law and are independent of each other.

This problem of the superposition of two laws of errors has been already treated by Bessel [1]) and subsequently d'Ocagne [2]) gave a general solution for the superposition of several groups of errors.

It appears that, if H be the factor of stability of the secular and h_1 that of the interior variability, the resulting deviations also follow the normal law, the new factor h being determined by:

$$h = \frac{H h_1}{\sqrt{H^2 + h_1^2}}.$$

From the values of h given in Table IV and those of H calculated from monthly means we can, therefore, deduce that of h_1:

1) Untersuchungen uber die Wahrscheinlichkeit der Beobachtungsfehler. Astr. Nachr. XV, 1838.

2) Sur la composition des lois d'erreurs de situation d'un point. C R. Acad. sc. CXVIII, 1894.

$$h_1 = \frac{Hh}{\sqrt{H^2 - h^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By the following reasoning the second problem may also be easily solved.

The total mean value for a given month, as calculated from n monthly means, must be the same as that deduced from N corresponding daily means.

The mean error (incertitude) of the total mean is, as monthly means may be considered to be independent of each other:

$$\sqrt{\frac{|\varepsilon^2|}{n(n-1)}} = \frac{M_1}{\sqrt{n}}.$$

For the mean incertitude deduced in the same manner from observations made three times each day:

$$\frac{M_2}{\sqrt{N}}$$

too small a value would be found as these observations are certainly not independent of each other; therefore, if the number of observations which, on the average, constitute an independent group be called p, we must have:

$$\frac{M_1}{\sqrt{n}} = \frac{M_2 \sqrt{p}}{\sqrt{N}}.$$

If we wish to express the average duration of a disturbance D in numbers of days, we have, in our case:

$$D = \frac{M_1^2}{M_2^2} \cdot \frac{N}{3n} = \frac{h^2}{H^2} \cdot \frac{N}{3n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Table VI shows the values of the interior variability h_1 thus calculated by means of form. (1) and the duration D of a barometric disturbance.

It appears from these results that, on the average, the duration of a barometric disturbance at Helder is in:

Winter	6.90	days
Summer	4.89	,,
Spring-Autumn	6.04	,,

or in round numbers resp. 7, 6 and 5 days in winter, spring-autumn and summer.

3. It would perhaps not be impossible, and it certainly must be

TABLE VI.

	H	h	h_1	D
Jan.	0.1411	0.0689	0.0787	7.52
Febr.	0.1458	0.0743	0.0864	7.46
March	0.1682	0.0788	0.0892	6.82
Apr.	0.2105	0.0971	0.1094	6.49
May	0.3019	0.1136	0.1226	4.46
June	0.3181	0.1312	0.1440	5.19
July	0.3392	0.1340	0.1459	4.92
Aug.	0.3330	0.1316	0.1432	5.00
Sept.	0.2350	0.1014	0.1123	5.74
Oct.	0.2113	0.0845	0.0926	5.12
Nov.	0.1892	0.0745	0.0810	4.81
Dec.	0.1419	0.0701	0.0806	7.82

the final aim in inquiries of this kind to come to a rational expression for the frequency of barometric deviations as a function of the distance of centres of depressions and of their average depth and extent but, even if we assume the most simple relations between pressure and distance of the centrum, we must expect to find rather complicated, exponential expressions, which can be treated only by expansion into series. It is, therefore, desirable to summarize the characteristics of the frequency curve in an empirical formula of the form:

$$e^{-H^2x^2}(A + Bx + Cx^2 + Dx^3 + Ex^4). \quad . \quad . \quad . \quad . \quad (3)$$

The constants of this formula can be easily determined and, if we succeed in establishing a rational expression, there will probably be no difficulty in indicating their meteorological meaning.

The frequency curve, positive and negative deviations being taken together (Table I), is then represented by the expression:

$$Z = 2e^{-H^2x^2}(A + Cx^2 + Ex^4), \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which represents a symmetrical curve, and the formula for the differences of Table III becomes:

$$Y = 2e^{-H^2x^2}(Bx + Dx^3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If, as in our case, the deviations x are departures from the arithmetical mean,

$$\int_0^\infty Z\,dx = 1 \quad , \quad \int_0^\infty Zx\,dx = \mu_1 = \vartheta \quad , \quad \int_0^\infty Zx^2\,dx = \mu_2 = M^2$$

$$\int_0^\infty Zx^3\,dx = \mu_3 \quad \text{etc.}\,; \quad \int_0^\infty Yx\,dx = 0\,. \quad . \quad . \quad . \quad . \quad (6)$$

For the determination of the constants of form. (4) we then have the four relations:

$$\left.\begin{aligned} A + \frac{C}{2H^2} + \frac{1.3\,E}{4H^4} &= \frac{H}{\sqrt{\pi}} \\ A + \frac{2C}{2H^2} + \frac{2.4\,E}{4H^4} &= H^2\,\mu_1 \\ A + \frac{3C}{2H^2} + \frac{3.5\,E}{4H^4} &= \frac{2H^3\mu_2}{\sqrt{\pi}} \\ A + \frac{4C}{2H^2} + \frac{4.6E}{4H^4} &= H^4\mu_3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Multiplying resp. by $1, -3, +3$ and -1 and adding, we find:

$$H^3 - a\,H^2 + b\,H - c = 0$$

$$a = \frac{6\mu_2}{\mu_3\sqrt{\pi}},\ b = \frac{3\mu_1}{\mu_3},\ c = \frac{1}{\mu_3\sqrt{\pi}},$$

or, because:

$$\mu_1 = \frac{1}{h'\sqrt{\pi}},\ \mu_2 = \frac{1}{2h^2},\ \mu_3 = \frac{1}{h''^3\sqrt{\pi}},$$

$$\frac{H^3}{h''^3} - \frac{3H^2}{h^2} + \frac{3H}{h'} - 1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From this equation possible values for H can be derived, but not in an advantageous manner as the quantities h, h' and h'' generally are only slightly different.

In practice, i. e. if we come to expression (4) by expansion of a theoretical formula, the problem will probably be less difficult, as the constants H and A or H and h will not be independent of each other, and it will be possible to reduce the four equations (7) to three or two.

In this preliminary investigation we confine ourselves to the most simple case that $H = h$ which, as it will appear, leads to satisfactory results.

Putting

$$\frac{h - h'}{h'} = K,$$

we find:

$$A\sqrt{\pi} = h(1-3K)$$
$$C\sqrt{\pi} = 12\,h^3 K$$
$$E\sqrt{\pi} = -\,4h^5 K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The position of the points of intersection of the observed frequency curve with that calculated by assuming the simple exponential law to hold good (the points where in Table II the numbers change their sign) is determined by the equation:

$$(A + Ca^2 + Ea^4)\sqrt{\pi} - h = 0,$$

or:

$$a^4 - \frac{3}{h^2}a^2 + \frac{3}{4h^4} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$a_1 = \frac{0.525}{h}\ ,\quad a_2 = \frac{1.651}{h}.$$

In fact Table II shows that there are no more than two well defined points of intersection, which justifies the omission of higher powers than the fourth in form. (4).

Tabel VII shows the values of the constants of (4) and the values of a calculated with the help of form. (9) and (10).

It is evident that, if form. (4) and the values of its constants determined in the way indicated give a good representation of the observed facts, the values of the coefficient A must be nearly equal

TABLE VII.

	A		C	E	a_1	a_2
	Calculated.	Observed.				
Jan.	377×10^{-1}	381×10^{-1}	227×10^{-7}	$-\ 36\times10^{-9}$	7.62	23.96
Febr.	419	420	0	0	7.07	22.22
March	438	417	142	− 35	6.66	20.95
Apr.	532	493	310	− 182	5.41	17 00
May	620	612	662	− 456	4.62	14.53
June	728	705	532	− 471	4.00	12.58
July	779	767	− 1581	+ 1008	3.92	12.32
Aug.	734	704	588	− 340	3.99	12.55
Sept.	560	559	401	− 168	5.18	16.28
Oct.	444	419	940	− 224	6.21	19.54
Nov.	386	357	772	− 143	7.05	22.16
Dec.	377	377	372	− 61	7.49	23.55

to the frequencies corresponding with the deviations 0—0.5 mm., as given in Table I, so that the greater or less degree of agreement between these values may be taken as a criterion for the proposed assumption $H = h$.

In order to show that this agreement is fairly satisfactory, the observed frequencies between the limits 0 and 0.5 are given once more besides the calculated values of A.

If we compare the situation of the intersection points as shown in Table II and as calculated according to form. (10), we see that the situation of the first point of intersection agrees well with the observed facts, but that the second points α_2, as calculated, correspond with greater deviations than occur in reality.

As this second point of intersection naturally coincides with small frequencies the degree of precision of which is questionable, it seems difficult to decide whether these differences may be ascribed to insufficiency of material, to the omission of a possible fourth term in form. (4), or to an error introduced by the supposition $H = h$; as the calculated values of α_2 are jointly too great, the latter cause has to be regarded as the most probable one.

4. The fact that in Table III, in which a measure is given for the skewness of the curves, except for $\varepsilon = 0$, only one zero-value occurs, proves that in form. (5) the addition of a third term is certainly not required. The calculation of the constants B and D as well as the determination of the point of intersection β can, therefore, easily be made.

As:

$$\int_0^\infty Yx\,dx = 0$$

we find immediately:

$$B + \frac{3}{2}\frac{D}{h^2} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

whereas:

$$\int_0^\infty Y\,dx = \frac{B}{h^2} + \frac{D}{h^4} = \nu = p - n \quad . \quad . \quad . \quad . \quad . \quad (12)$$

denotes the surplus of positive over negative deviations.

If we take the absolute sum of positive and negative deviations as a measure for the skewness s:

$$s = p + n = 2\int_0^\beta Y\,dx - \int_0^\infty Y\,dx = 2\int_0^\beta Y\,dx - \nu,$$

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or :

$$s + v = 2 \int_0^\beta Y\,dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The situation of the point of intersection β is determined by the equation :

$$B + D\beta^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

By (11) and (12) :

$$B = 3\,h^2\,v, \quad D = -\,2\,h^4\,v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\beta^2\,h^2 = {}^3/_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

With the help of (13) we find from these values :

$$s = v\,(1 + 4e^{-3/2}) \,,$$

$$\frac{s}{v} = \frac{p+n}{p-n} = 1.89 \,, \quad \frac{p}{n} = 3.25 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

By means of the values v or s, to be taken from Table III, the constants of form (5) as well as the position of the point of intersection can, therefore, be determined ; we choose v, so that a comparison of the calculated and observed values of $^s/_v$ or $^p/_n$ may serve as a criterion for the method followed in calculating the constants of the empirical formula.

TABLE VIII.

	Observed.		Calculated.		
	v	s	B	D	β
Jan.	707×10^{-4}	1505×10^{-4}	101×10^{-5}	$-\ 32 \times 10^{-7}$	17.8
Febr.	606	1184	100	— 37	16.5
March	467	923	87	— 36	15 5
Apr.	639	1277	181	— 114	12.6
May	423	576	163	— 141	10.5
June	483	668	249	— 286	9.3
July	486	998	262	— 313	9 1
Aug.	426	908	295	— 256	9 3
Sept.	463	1073	143	— 98	12.1
Oct.	429	748	92	— 44	14.5
Nov.	599	1467	100	— 37	16.4
Dec.	605	1309	89	— 29	17 5
Mean	528	1053			

The average values of ν and s show a satisfactory agreement with the form. (17):

$$\frac{s}{\nu} = \frac{1053}{528} = 1.99$$

From the aggregate values given in Table III for three seasons we find:

Sums.

	p	n	$p+n=s$	s	p/n
Winter	3849	1340	5189	12.97 %	2.87
Spring-Autumn	2959	937	3896	9.74	3.16
Summer	2380	747	3127	7.82	3.19

For the values of β in these three seasons:

	Observ. Tab. III	Calc. Tab. VII
Winter	17	17.05
Spring-Autumn	14	13.68
Summer	9.5	9.55

Anatomy. — *"Anatomical research about cerebellar connections."* By L. J. J. MUSKENS. (second communication). (Communicated by Prof. C. WINKLER).

A comparative examination into different species of mammals I have thought desirable in order to get information about the course of the axis-cylinders arising from the cortex cerebelli. The development of our knowledge in this matter in the last 15 years has resulted in that at the present time the following question has been placed in the center of discussion . do the strands of fibres, which form the superior Crus cerebelli, arise from the cortex cerebelli strictiore sensu or have we to regard the basal cerebellar nuclei as an undispensable intermediary for all these cortico-fugal nervefibres? On the one hand we find in some rodentia in the lobus petrosus cerebelli exclusively cortex and white matter (squirrel), on the other hand we find in others (rabbit) equally a part of the nucleus dentatus situated in the peduncle of that lobe. In both animals the lobus petrosus is situated in a separate bony hole. We find in this lobe therefore a very fortunate opportunity for operative procedure therein, leaving the other neighbouring central structures and also the semicircular canals intact. We can here in a comparative physiological way find an answer on the above question and at the same time avoid a large cranial aperture.