VARIATIONS OF α, δ AND $\log \varrho$ FOR THE ALTERED TIME OF PASSAGE THROUGH THE PERIHELION.

1906	$T = -4$ days			$T = +4$ days		
	$\Delta\alpha$	$\Delta\delta$	$\Delta \log \rho$	$\Delta\alpha$	$\Delta\delta$	$\Delta \log \rho$
	m s	' ''		m s	' '	
May 5	+ 3 13.48	+ 38 55.2	+ 231	— 3 13.42	— 39 27.7	— 233
» 21	+ 3 22.15	+ 36 23.2	+ 294	— 3 22.12	— 37 3 8	— 297
June 6	+ 3 33.07	+ 33 10.6	+ 355	— 3 33.23	— 33 58.3	— 359
» 22	+ 3 46.12	+ 29 19.9	+ 413	— 3 46.62	— 30 13.4	— 418
July 8	+ 4 1.25	+ 24 55.8	+ 469	— 4 2.30	— 25 53.4	— 476
» 24	+ 4 18.58	+ 20 4.1	+ 521	— 4 20.42	— 21 4.4	— 529
Aug. 9	+ 4 38.61	+ 14 54.2	+ 567	— 4 41.55	— 15 55.9	— 576
» 25	+ 5 2 49	+ 9 39.3	+ 606	— 5 6.87	— 10 41.7	— 616
Sept. 10	+ 5 31.97	+ 4 41.9	+ 632	— 5 38.29	— 5 45.9	— 642
» 26	+ 6 9.74	+ 39 2	+ 640	— 6 18.02	— 1 49.1	— 649
Oct. 12	+ 6 55.91	— 1 27.1	+ 621	— 7 5.99	+ 4 1	— 627
» 28	+ 7 44.03	— 23.3	+ 566	— 7 54.54	— 1 20.9	— 569
Nov. 13	+ 8 15.71	+ 4 15.2	+ 475	— 8 23.95	— 6 19.7	— 474
» 29	+ 8 10.71	+ 10.42.4	+ 361	— 8 14.80	— 12 50.9	— 356
Dec. 15	+ 7 29.94	+ 15 44.7	+ 247	— 7 30.69	— 17 37.3	— 241

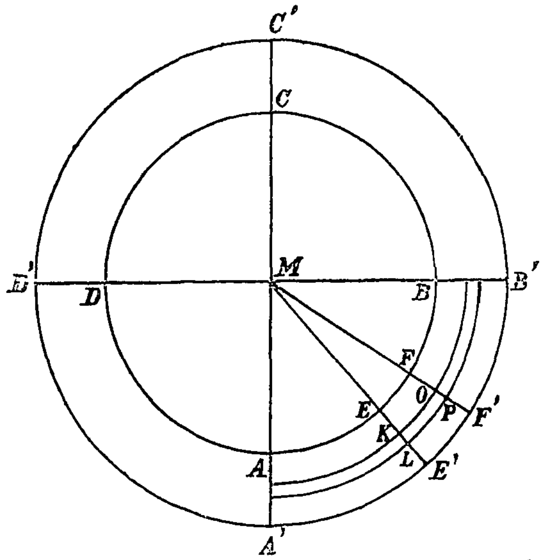
Leyden, January 1906.

Physics. — "*On the motion of a metal wire through a lump of ice*". By L. S. ORNSTEIN. (Communicated by Prof. H. A. LORENTZ).

In a well known experiment on the regelation of ice a metal wire charged with weights is placed on a lump of ice. It moves slowly through the ice, while on the upper side new ice is formed; after a short time the motion takes place with uniform velocity. This phenomenon is explained by the fact, that if we increase the pressure the meltingpoint is lowered.

In order to calculate the velocity of the wire I shall consider an infinite circular cylinder which is moved through an infinite lump of ice by a force perpendicular to its axis. The phenomenon is the same in each normal section. I suppose round the wire a layer of water whose thickness is small in comparison with the diameter of the wire. At the bounding surface of water and ice there is a pressure, which decreases from the lower to the upper side of the boundary. This pressure depends on the force by which the wire is acted on pro unit of length. As the motion is very slow the temperature in each point may be supposed to be the meltingpoint corresponding to the pressure existing in the point. The flow of heat, determined by the distribution of temperature is the same as if the wire were at rest. At the upperside of the bounding surface of ice and water heat flows away and water is frozen, at the lower side the ice is melted by the heat that is carried towards the surface. If we can determine the quantity that is melted we shall be able to determine the velocity acquired by the wire.

Let M be the centre of the circular section of the wire and R the radius, the boundary between ice and water being a circle of radius $R + d$.



The pressure at the circle $A'B'C'$ in any point E' may be represented by the formula

$$p = p_0 + b \cos \varphi ,$$

φ being the angle between the radius ME' and the line MA which has been taken for axis of ordinates. The corresponding temperature is

$$t = t_0 + b \left(\frac{dt}{dp}\right)_0 \cos \varphi ,$$

$\left(\frac{dt}{dp}\right)_0$ being the change of the meltingpoint per unit increase of pressure near 0° C.

Let k_1, be the coefficient of conductivity within the circle ABC, k_2 that of the layer of water, and k_3 that of the ice without $A'B'C'$.

The differential equation for the temperature is in every one of these fields

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 .$$

The conditions at the limits of the fields are:

1. at ABC $\quad t_1 = t_2, \quad k_1 \left(\frac{\partial t_1}{\partial n}\right)_1 = k_2 \left(\frac{\partial t_2}{\partial n}\right)_2,$

2. at $A'B'C'$ $\quad t_2 = t_3 = t_0 + b \left(\frac{dt}{dp}\right)_0 \cos\varphi,$

3. at infinite distance $t_3 = t_0$.

The normal at ABC coinciding with the radius.

The formulae:

$$t_1 = t_0 + B_1 r \cos\varphi \quad \text{in the wire,}$$

$$t_2 = t_0 + B_2 r \cos\varphi + \frac{C_2}{r}\cos\varphi \quad \text{in the layer of water,}$$

$$t_3 = t_0 \qquad + \frac{C_3}{r}\cos\varphi \quad \text{in the surrounding ice}$$

satisfy the equations r being the distance from the point M. For the coefficients I find the relations

$$B_1 = B_2 + \frac{C_2}{R^2},$$

$$k_1 B_1 = k_2 \left(B_2 - \frac{C_2}{R^2}\right)$$

$$B_2 + \frac{C_2}{(R+d)^2} = \frac{C_3}{(R+d)^2} = b\left(\frac{dt}{dp}\right)_0 \frac{1}{R+d}.$$

Neglecting powers of d/R I find

$$\frac{C_2}{R^2} = -\frac{b\left(\frac{dt}{dp}\right)_0 (k_1-k_2)}{2(R+d)\{k_2+{}^d/_R(k_1-k_2)\}},$$

$$B_2 = \frac{b\left(\frac{dt}{dp}\right)_0 (k_1+k_2)}{2(R+d)\{k_2+{}^d/_R(k_1-k_2)\}}.$$

For an element $E'F'$ of $A'B'C'$ the flow of heat into the ice towards the surfaces amounts to:

$$-k_3 \frac{C_3}{R+d^2}\cos\varphi\, d\varphi,$$

if we write $d\varphi$ for the angle $E'MF'$. Hence the total quantity of heat conducted through the ice towards the surface $A'B'$ per unit of time:

$$-k_3\, b\left(\frac{dt}{dp}\right)_0 \int_0^{\pi/2} \cos\varphi\, d\varphi = -k_3\, b\left(\frac{dt}{dp}\right)_0.$$

In the layer of water the flow of heat per unit of time is for $E'F'$

$$-(R+d)\,d\varphi\cos\varphi\,k_2\left(B_2-\frac{C_2}{(R+d)^2}\right)$$

and for $A'B'$ totally

$$-k_2(R+d)\left(B_2-\frac{C_2}{(R+d)^2}\right)=-k_2\,b\left(\frac{dt}{dp}\right)_0\frac{k_1-(k_1-k_2)^d/_R}{k_2+(k_1-k_2)^d/_R}.$$

Of course as much heat is lost at the surface $B'C'$ as is conducted towards $A'B'$; and the melted and frozen quantities of ice and water will therefore be equal. W being the quantity of heat that is required for the melting of a gramme of ice, the melted quantity is

$$-\frac{2\left[k_2\frac{k_1-{}^d/_R(k_1-k_2)}{k_2+{}^d/_R(k_1-k_2)}+k_3\right]b\left(\frac{dt}{dp}\right)_0}{W.},$$

If S_y is the specific gravity of ice, the volume of this quantity is:

$$-\frac{2\left[k_2\frac{k_1-{}^d/_R(k_1-k_2)}{k_2+{}^d/_R(k_1-k_2)}+k_3\right]\left(\frac{dt}{dp}\right)_0 b}{W\,S_y}.$$

On the other hand, if the cylinder moves with a uniform velocity v a volume

$$2\,R\,v.$$

is melted. So we find for the value of v

$$v=-\frac{\left[k_2\frac{k_1-{}^d/_R(k_1-k_2)}{k_2+{}^d/_R(k_1-k_2)}+k_3\right]\left(\frac{dt}{dp}\right)_0 b}{R\,W\,S_y}.$$

To express b in the force P acting per unit of length of the cylinder we have only to notice that an element $EF=Rd\varphi$ is acted on by a force per unit of surface $p\cos\varphi=(p_0+b\cos\varphi)\cos\varphi$. Hence:

$$P=2\int_0^\pi(p_0\cos\varphi+b\cos^2\varphi)\,Rd\varphi=\pi bR$$

The velocity C in case $P=1$ is found to be

$$C=-\frac{\left(\frac{dt}{dp}\right)_0\left[k_2\frac{k_1-{}^d/_R(k_1-k_2)}{k_2+{}^d/_R(k_1-k_2)}+k_3\right]}{\pi R^2\,W\,S_y}\quad\ldots\ldots\;(I).$$

We can find another expression for $^d/_R$, if we pay attention to the motion of the water. If we conceive the wire to be at rest but the ice moving along it, we shall see at the limit $A'B'$ water continually streaming into the channel $ABA'B'$ while it streams out of it and freezes at the part $B'C'$ of the surface. The velocity of the ice being v we find for the quantity of water entering through $E'F'$

$$(R+d)\, v \cos\varphi\, d\varphi.$$

This is also the difference between the quantities flowing across FF' and EE' upwards.

This quantity can also be determined by means of the hydrodynamical equations. Take for axis of ξ a circle with radius $R+\frac{1}{2}d$ and for axis of η a radius of the circle. The forces acting on an element $KLOP$ are in equilibrium. Writing u_ξ for the velocity parallel to the axis of ξ, μ for the coefficient of viscosity, neglecting the velocity u_η and taking the intersection of the ξ circle, with EE' for origin of coordinates we have:

$$\mu \frac{\partial^2 u_\xi}{\partial \eta^2} = -\frac{b \sin\varphi}{R}.$$

At the circle AB, $u_\xi = 0$, at $A'B'$, $u_\xi = v \sin\varphi$, therefore:

$$\mu\, u_\xi = -\frac{b \sin\varphi\, \eta^2}{2R} - \frac{v \sin\varphi}{d}\eta + \frac{\sin\varphi}{2}\left(v + \frac{b\, d^2}{4R}\right)$$

and the quantity streaming across EE' is

$$\int_{-d/2}^{+d/2} u_\xi\, d\eta = \frac{1}{\mu}\left\{\frac{bd^3}{12R} + \frac{vd}{2}\right\} \sin\varphi,$$

the difference between the quantities of water flowing across FF' and EE' will therefore be

$$\frac{1}{\mu}\left\{\frac{b}{12}\frac{d^3}{R} + \frac{vd}{2}\right\} \cos\varphi\, d\varphi$$

and we have, neglecting powers of d/R:

$$v = \frac{b\, d^3}{12\,\mu R^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II^a)$$

In the experiments the wires become curved. I suppose the wire to be perfectly flexible and the stress to have the same value S in all its parts; the force per unit of length perpendicular to the wire is given in each point by

$$S\frac{d\omega}{ds},$$

$d\omega$ being the angle between two consecutive tangents to the curve. The curvature being not large we can use the coefficient given by (I) to find the normal velocity arising from this force. This velocity is

$$C\, S\frac{d\omega}{ds}.$$

In a time dt the element ds of the wire describes a surface

$$C\,S\frac{d\omega}{ds}\,ds\,dt.$$

If the wire at the ends is vertical the whole wire will therefore describe an area

$$dt\int_0^{\pi} C\,S\frac{d\omega}{ds}\,ds = \pi\,CS\,dt.$$

Now if the velocity of the wire is v, and the distance between the vertical ends d_1, the same area will be vd_1 so that we have

$$v = \frac{\pi CS}{d_1}\quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (III)$$

or if the angle between the ends is 2α, and P the weight at each end,

$$v = \frac{2\alpha CP}{d_1 \sin\alpha}.\quad . \; . \; . \; . \; . \; . \; . \; . \quad (III_a)$$

We shall next consider the form taken by the wire if it descends as a whole with uniform velocity. It is determined by the condition

$$C\,S\frac{d\omega}{ds} = v\,\frac{dx}{ds},$$

or

$$\frac{d\omega}{ds} = \frac{\pi}{d_1}\frac{dx}{ds}$$

As $\varrho\,d\omega = ds$, ϱ being the radius of curvature, this equation becomes

$$\frac{\dfrac{d^2y}{dx^2}}{1+\left(\dfrac{dy}{dx}\right)^2} = \frac{\pi}{d_1}.$$

Taking the axis of x horizontal at the highest point of the line, the axis of y vertical downwards we have for $x = 0$,

$$y = 0\;\frac{dy}{dx} = 0$$

therefore

$$\frac{dy}{dx} = tg\,\frac{\pi}{d_1}\,x\,,$$

$$\cos\frac{\pi}{d_1}\,x = e^{-\frac{\pi}{d_1}y}.$$

The normal pressure at the highest point is

$$S_n = \frac{S\pi}{d_1}$$

In order to find the formula (II^a) for curved wires we can put, approximately, for b its value at the point $x=0$ $y=0$.

So that we may put for

$$b=\frac{S_n}{\pi R}=\frac{S}{Rd_1}.$$

By this the formula (II^a) gives

$$v=\frac{S}{12\mu d_1}\left(\frac{d}{R}\right)^3 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (II^b)$$

S being equal to the weight hanging at each end.

If the angle between the tangents at the ends is 2α, we have other formulae. The equation of the curve becomes

$$\cos\frac{2\alpha}{d_1}x=e^{-\frac{2\alpha}{d_1}y},$$

and the velocity, if P is again the weight at each end

$$v=\frac{2\alpha CP}{d_1 \sin\alpha} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (III^a)$$

By the hydrodynamical method the same velocity is found to be

$$v=\frac{2\alpha P}{12\pi\mu d_1 \sin\alpha}\left(\frac{d}{R}\right)^3 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (II^c)$$

Dr. J. H. MEERBURG has made a series of experiments, of which he will communicate the results at a later opportunity. The agreement with the theory is not very satisfactory. It must be noticed however that d is very small. The roughness of the surface of the wire will therefore greatly increase the resistance to the motion of the water, so that the result of the hydrodynamical method can no longer be considered as correct.

Zoology. — "*On the Polyandry of Scalpellum Stearnsi*" by P. P. C. HOEK.

One of the largest forms of the genus *Scalpellum* which is so rich in species is *Scalpellum Stearnsi*, PILSBRY from shallow water near the coast of Japan.

This species is represented by two varieties or sub-species in the collection of Cirripedes made by the Siboga Expedition in the waters of the Dutch East Indies and handed over to me for description. Both forms agree in the main with PILSBRY's species — they differ, however,