§ 7. The curve of inflections (I) and the bitangential curve (D) have in each of the $3(n-1)^2$ nodes of (c^n) in common a number of $2(n-3)(n+2)$ points.

For, out of a node we can draw to the c^n to which it belongs (n^2-n-6) tangents, to be regarded as double tangents, whilst each node of a c^n is at the same time node of (I).

In each basepoint lie moreover $3(n+4)(n-3)$ points of intersection (§ 4). The remaining points common to (D) and (I) are the inflections of which the tangent touches the c^n once more (§ 4) and the undulation-points (§ 6) where the two curves touch each other.

Indeed, we have

$$6(n-1)^2(n-3)(n+2)+3n^2(n+4)(n-3)+3(n-4)(n-3)(n^2+6n-4)+$$
$$+12(n-3)(3n-2)=6(n-1)^2(n-3)(n+2)+3(n-3)(2n^3+6n^2-16n+8)=$$
$$=6(n-1)(n-3)(2n^2+5n-6),$$

and this is the product of the orders of (I) and (D).

Physiology. "*On the strength of reflex stimuli as weak as possible.*" By Prof. H. ZWAARDEMAKER. (Report of a research made by D. I. A. VAN REEKUM).

(Communicated in the meeting of March 31, 1906).

Investigated were chemical, thermal, mechanical and electrical stimuli, which partly acted upon the skin partly on the sensible nerves of the animals, which were experimented on.

§ 1. The chemical stimuli were applied by immerging the hind-leg of a winterfrog in a little bowl with a solution of sulphuric acid varying from $^1/_4$ to $^1/_{32}\%$ $\left(\frac{n}{20}\text{ to }\frac{n}{160}\right)$. The spinal cord system was withdrawn in the usual way from the influence of the cerebrum. After the experiment the legs were washed with distilled water and the experiment repeated after a pause of 5 minutes. Neglecting the preliminary reflex, only a complete reflex was considered as a positive result. After-reflexes and general movements did only show themselves when rather strong concentrations were used.

As a rule a $^1/_{17}\%$ $\left(\frac{n}{85}\right)$ solution of sulphuric acid may be accepted as the minimum stimulus which still produces reflexes. The reflex-

time at an immerging of the two legs was 10 seconds, at an immerging of one leg 22 seconds.

It was calculated how much sulphuric acid disappeared in the skin of the frog, when $^1/_{17}$ % sulphuric acid $\left(\frac{n}{85}\right)$ was used, respectively how much was fixed by the excretion-products. This occurred by titrating the immerging liquid with caustic soda (methylene orange as indicator) before and after a series of 20 singular reflexes.

Then it appears that about $^1/_{15}$ of the total quantity of the used sulphuric acid has been bound. Supposing the heat of reaction of 2 aequivalents natron and 1 aequivalent sulphuric acid to be 31,4 great calories and supposing that our sulphuric acid has been bound in a reaction of this kind then the heat of reaction of the chemical process pro singular reflex, reckoned over the whole immerged surface of the skin, amounts to 1,37 gram-calorie. It is evident that only a small part of this supposed reaction can have taken place in or near the terminations of the nerves and that this value of 1,37 gram-calorie must be also a limit under which is situated the heat of reaction.

This amount may surpass the real value of the reflex-stimulus perhaps a million of times. By measuring the electrical conductivity of the stimulating solution before and after the reflexes it was controlled if anything else had passed into the immerging liquid in the place of the disappeared sulphuric acid. This proved to be the case for the increase of resistance of the liquid experimented with, was greater than would follow from the decrease of the sulphuric acid.

§ 2. As a thermal stimulus served immersion in cold or warm water. The most favourable result was obtained by a decreasing difference of temperature between animal and water of 10° C. and by an increasing difference of temperature of 15° C. The reservoir, isolated by an asbestos envelope, in which the immersion of the frog took place contained 50 ccm. The immersion was performed once and after that the reflex was waited for. Then it could be stated that the temperature of the water increased on an average of 8 centigrades by the immersion of the heated frog and decreased on an average of 22 centigrades by the immersion of the leg of a frog which was cooled down. Some experiments already gave a reflex before it had come to this. A sufficient quantity of reflexes large enough to avoid casualties, were accompanied by an increase of temperature of 7 centigrades resp. a decrease of temperature of 19

centigrades. Consequently at these last experiments a quantity of heat of 3,5 gr. calorie must have been withdrawn from the leg of the frog, and 9,5 calorie have been added. This heat divided itself during a reflex-time of average $7^1/_2$ sec. resp. 9 sec. over the whole immerged part of the skin. Only a very small part will have come to the benefit of the terminations of the nerves and what appears as a reflex-stimulus may very well be millions of times smaller than the total quantity of the heat which is given or taken up. The above mentioned values have again only the significance of limit values beneath which the heat resp. cold stimulus, which causes a reflex movement, must be necessarily situated.

§ 3. To produce mechanical reflex-stimuli first falling mercury drops were made use of [1]), afterwards a little ball of resin fastened to a pig's-bristle, which by an electrically moved tuning fork of 16 double vibrations was kept in a forced vibration of fixed amplitude. In both cases as much as possible the lateral side of the foot, where the corpuscula tactus are situated, was taken.

The mercury drops were all of the same size (average 100 mgr.) and were used to the number of 1 to 15, trickling down one after the other. The height from which the drops were falling varied from 1 to 20 cm. At each experiment the vis viva was calculated with which the drop came down on the skin of the animal. It was obvious that for causing a reflex the vis viva had to be in minimo 686 ergs which amount was obtained by dropping 7 drops one after the other from a falling height of 1 cm. Once it was possible to obtain a reflex by the fall of one drop from a height of 7 cm. which shows the same quantity of energy now contained in one single stimulus without any summation.

The smallest results according to vis viva which still produce a reflex were obtained with a little ball of resin of 7 milligram which vibrated with an excursion of 5 mm. After a reflextime of on an average 3 sec. the reflex movement was obtained. The quantity of energy which was added to the skin in this way in summing contains 212 ergs.

The result of the mechanical stimulation is quantitatively considerably lower than the above mentioned chemical and caloric stimulation. It leads to a minimum, which however put together in a restrict spot still possesses the peculiarity of having been communicated to a part of the skin which probably is considerably

[1]) E. A. Schäfer, Proc. Physiol. Soc. 26 Jan. 1901.

larger than the surface of a corpusculum tactus. The divergency between the quantity of energy applied and that which is used for reflex-stimulus is in this last case not by far so great as in the thermal forms of reflex stimulus. The simplest relation might be expected in the very favourable case already mentioned, in which only one drop of mercury falling from a height of 7 cm. was used: Meanwhile, with the ball of resin, still smaller values were obtained, notwithstanding the summation was taken into the account, so that we may accept, this most simple case has not at all been a most favourable one.

§ 4. The electrical stimulation brought about by discharges of a condensator which was immediately before charged with a voltage varying between 0 and 2 volts. The capacity of the condensators, which were constructed in the laboratory from mica of different thickness and coverings of tinfoil different in surface varied from 15×10^{-5} to 4×10^{-3} m. F. They were wholly closed in by paraffine and verified by comparing with an air-condensator. The following stimuli were used: firstly on the skin of the leg of the frog by means of little catches of steel which surround the leg: secondly on the posterior roots of the lumbal-cord, by means of platinum-electrodes set in paraffine, thirdly on the nervus vagus of a rabbit by means of platinum-electrodes set in ebonite. The stimuli were for the greater part supplied in series with an interval of $^1/_2$ sec. in a number varying between 1 and 15. All those regulations took place automatically by properly isolated swings and keys. The best results gave a condensator of 59×10^{-5} m. F.

Skinreflexes (not ordened series)

(with condensator of 59 10^{-5} m.F.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	number of stimuli
121	103	158	98	76	31	31	40	9	20	6	6	2	5	10	number of observat
0 87	0 81	0 83	0 77	0.77	0 75	0 71	0 79	0 94	0 86	0.85	0.75	0 65	0 62	0.67	average voltage
22.32	19 35	20 32	17 49	17 49	16.59	16 15	18 41	26 07	21 81	21.31	16.56	12.46	11.31	13.21	energy in 10^{-4}

The above mentioned experiments were taken without a system. Observing a more judicious succession of the stimuli more favourable conditions of stimulation were obtained in the following series.

From this table it distinctly appears that the stimulus is limited to the smallest quantity of energy when a condensator 0,00035 m. F. is used. Then $1{,}4 \times 10^{-4}$ ergs is sufficient on condition that the stimulus is repeated three times with an interval of $^1/_2$ sec. Consulting the experiments about reflexes which are not mentioned

Skinreflexes (ordened series)

(the average for the different condensators).

Capacity in m.F.	Voltage	Number of stimuli	Energy of each stimulus in 10^{-4} ergs.
0 00025	0.40	2	2.0
0.00035	0.28	3	1.4
0.00059	0 24	8	1.7
0.0013	0.24	3	3.7
0 004	0.34	15	23.1

in the tables a minimum value is obtained which is only slightly larger, namely an amount of 5×10^{-4} ergs.

The result got at the last root of the lumbal region with frogs cannot be given in one table as the individual experiments differed too much and have not been numerous enough to fix the average. In a very sensitive preparation when the above mentioned condensator of 0,00035 m. F. was used, a distinct reflex was obtained with a single discharge of only $8{,}6 \times 10^{-6}$ ergs, a result which shows clearly that in the experiments of Mr. VAN REEKUM the reflex sensitiveness has been considerably greater from the root than that from the skin. In a single case there was even found a value still three times smaller. The above stated number however was not obtained accidentally but represents a whole series of observations (12 in number).

By central stimulation of the cervical part of the nervus vagus of a rabbit reflex-changes of the breathing were caused, which could be registered by means of the aerodromograph [1]). The said reflex consists according to the intensity of the stimulus 1. if stimulating with very weak discharges in a slight increase of frequency of breathing and in an increase of the rapidity of the current of air in in- and expiration 2. if stimulating with somewhat greater discharges, an increase of the rapidity of the stream of air notwithstanding decrease of frequency 3. stimulating with sufficient great discharges a distinct decrease in rapidity of the stream of air and frequency both. If we only examine the result mentioned in the third case as the reflex on which we want to base our measurements, the results of the experiments may be taken together as follows:

[1]) H. ZWAARDEMAKER und C. D. OUWEHAND. Arch. f. Physiol. 1901. p. 241.

Breath-reflexes.

capacity in m.F.	15 successive discharges		1 discharge	
	voltage	energy of the stimulus in 10^{-4} ergs	voltage	energy in 10^{-4} ergs
0.00015	0.17	0.24	0.23	0 40
0.00025	0.13	0 21	0 21	0.55
0.00035	0 10	0.17	0 17	0 51
0 0059	0 09	0 24	0.16	0 76
0 0013	0 11	0.79	0 19	2.35
0.004	0.12	2 88	0.18	6.48

CONCLUSION.

The reflex stimuli of different kinds used as weak as possible on cold- resp. warmblooded animals have in minimo very different value. Thus one and the same effect was brought about by applicating on the skin of a frog of an electric stimulus of $3{,}15 \times 10^{-4}$ ergs by a mechanical stimulus of 212 ergs, by a thermal stimulus of 11,5 mega-ergs and by a chemical stimulus of 57 mega-ergs. So of all these forms of stimulus the electrical is the most favourable. It may be still more favourable when we let the stimulus act not on the skin but on a posterior lumbal root of the frog. Then 3×10^{6} ergs is sufficient to cause a typical reflex and so the amount approaches to that which occurs with weak sensorial stimuli (light stimuli vary in general between 1×10^{-10} as lowest and 6×10^{5} as highest value; acoustical stimuli between $0{,}3 \times 3^{-8}$ as lowest and 1×10^{8} as highest value [1]). What holds true for frogs, as a rule holds true for mammals. From the nervus vagus there can be brought about by central stimulation with an electrical stimulus of $0{,}17 \times 10^{-4}$ ergs a very marked change of breathing, whereas a few times smaller value causes an indistinct but yet an unmistakable acceleration of breathing. Here also the minimum reflex stimuli have a limit value of the order 1×10^{-6} ergs.

[1]) Die physiol. wahrnehmbaren Energiewänderungen, Ergebnisse der Physiologie Bd. IV. 1906. p. 423.

(May 25, 1906).