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n-rays should be promoted to objective truth, our hypothesis would receive a direct experimental support.

At all events, the above has demonstrated that the principal measurable phenomena, noticed in the enzyme action are in harmony with our hypothesis.

Meteorology. — “*On a twenty-six-day period in daily means of the barometric height.*” By Dr. J. P. VAN DER STOK.

1. A few years ago ¹⁾ Prof. A. SCHUSTER investigated the problem, how to detect the presence of a periodical oscillation, the amplitude of which is small in comparison with large superposed fluctuations which may be considered as fortuitous with respect to the purely periodical motion.

Starting from an analogy which may be seen between this question and the problem of disturbances by vibrations in the aether — a problem treated by Lord RAYLEIGH ²⁾ in 1880 — Prof. SCHUSTER has endeavoured to apply the theory of probability to the determination of the first couple of coefficients of a FOURIER series, and the method he arrives at, and strongly advocates, is applied to records of magnetic declination observed at Greenwich during a period of 25 years.

The choice of this material, in Prof. SCHUSTER's opinion not favourable for the discovery of small effects, is justified by the remark that “the only real pieces of evidence so far (1899) produced in favour of a period approximately coincident with that of solar rotation were derived from magnetic declination and the occurrence of thunderstorms.”

In this and in an earlier paper ³⁾ the author emphasizes that, in inquiries of this kind, it is not at all sufficient to come to some result, but that it is necessary to apply a reliable criterion by which a judgment may be formed about the value to be attached to the result arrived at.

His mathematical investigation, however, does not, lead to an outcome which in every respect can be regarded as satisfactory, in so far that a method of determining the mean and probable error of the result from the series of observations themselves is not given and,

¹⁾ Trans. Cambr. Phil. Soc. Vol. XVIII. 1899.

²⁾ Phil. Mag. Vol. X. II, 1880.

³⁾ Terrestrial Magnetism Vol. III, 1898.

as a surrogate, the author suggests the repeated rearrangement of the records according to different periods, not much differing from the period in question. Thus, in a purely empirical way ("by trial"), a standard may be obtained by which a correct estimate can be formed in how far the outcome arrived at must be considered as a merely accidental one.

Now the same problem was treated some 15 years ago ¹⁾ by the author of this paper after a different method, applied not only to magnetical but as well to meteorological data of different description, and Prof. SCHUSTER's important investigation gives a ready occasion for taking this problem in hand again.

It is only natural to choose in the first place for this inquiry the series of barometric observations made at Batavia which now extends over a period of 36 years (1866—1901).

An investigation into a possible synchronism between the frequency of sunspots and atmospheric temperature, commenced in 1873 ²⁾ and recently conducted up to date ³⁾, gives some ground to the expectation that, for inquiries of this kind, observations made at tropical stations are of more value than those made in regions where the atmospheric disturbances are such as experienced in higher latitudes.

In the second place it seems desirable to look for a shorter way for coming to a reliable criterion than the tedious process of the calculation of SCHUSTER's periodograph.

2. An arrangement of quantities according to a given period T may be executed by measuring out the successive data from a point O taken as origin and along straight lines drawn through this point at equal angular distances $\frac{2\pi}{T}$.

If we assume the unity of mass attached to the ends of those radii, it is evident that a judgment may be formed about the degree of symmetry in the distribution of the masses with respect to point O , by simply calculating the average value of all these vectors, or in other words to determine the situation of the centre of parallel forces supposed acting on the masses.

If the quantities T show a well marked periodicity as e.g. tidal

¹⁾ *Observ. Magn. Meteor. Observ. Batavia. X, 1888, Append. II also Natuurk. Tijdschr. XLVIII. 1889.*

Verh. Kon. Akad. v. Wet. Amsterdam. XXVIII, 1890.

²⁾ *KÖPFEN. Zeitschr. Oesterr. Gesellsch. f. Meteor. VIII, 1873.*

³⁾ *C. NORDMANN. Essai sur le rôle des ondes Hertiennes and : Astron. Phys. et sur diverses questions qui s'y rattachent, Thèse, Paris, 1903.*

observations do, it will be possible to draw a line through O in such a manner, that on the one side all the vectors are greater than the corresponding opposite vectors on the other side of the line.

If, therefore, we arrange in this way a great number of quantities which show a slight tendency to asymmetry, the radial momentum will steadily increase, as the mass concentrated in the centre of parallel forces is equal to the total number of observations, whilst the distribution of accidental quantities will tend to a symmetrical distribution.

Assuming two rectangular axes going through O , we find for the coordinates, by which the centre of gravity is determined, N being the number of observations :

$$x_1 = \frac{1}{N} \sum \rho \cos \theta \quad y_1 = \frac{1}{N} \sum \rho \sin \theta \quad . \quad . \quad . \quad (1)$$

The calculation, therefore, comes to the same as the determination of the first couple of FOURIER coefficients:

$$a_1 = \frac{2}{N} \sum \rho \cos \theta \quad b_1 = \frac{2}{N} \sum \rho \sin \theta \quad . \quad . \quad . \quad (2)$$

and, if the periodical movement is represented by the expression:

$$A \cos (nt - C).$$

$$A^2 = a_1^2 + b_1^2 \quad \text{tang } C = \frac{b_1}{a_1} \quad n = \frac{2\pi}{T} \quad . \quad . \quad . \quad (3)$$

This way of representing the arrangement seems preferable to the development in a FOURIER series: firstly because the development of a function in a series, as a representation of the function, derives its value from the composition of a great number of terms, so that, in calculating one term only, we are hardly justified in speaking of a Fourierisation of the function.

In the second place, because by this way it becomes at once evident that the problem is fully equivalent to that of the determination of a point in a plane by means of a great many inaccurate observations.

This problem has been treated by several mathematicians, but certainly in the most complete manner by the late Prof. SCHOLS¹⁾, whose original conception of the question leads to the detection of some laws, which are independent of the assumption of any law

¹⁾ Over de theorie der fouten in de ruimte en in het platte vlak, Amsterdam, Verh. K. Akad. v. Wet 1^e Sect. XV, 1875, and: Théorie des erreurs dans le plan et dans l'espace, Delft, Ann. II, 1886.

of errors and to a remarkable analogy between this problem and that of the moments of inertia in dynamics.

If we take N , the number of observations, equal to unity, the relative frequency of the ends or representative points of the vectors may be represented by the density of these points per unity of surface. This function of probability is called by SCHOLS "the module", the "specific probability" or the "facilité de l'erreur".

We thus obtain a mechanical image of a surface of probability, the density of which will, in general, be a function of the length and direction of the vectors.

The determination of what SCHOLS calls the constant part of the error — the probability of which is $N = 1$ — is identical with the determination of the situation of the centre of gravity, and the calculation of the mean (not average) error:

$$M = \sqrt{\frac{\sum \rho^2}{N}}$$

with that of the moment of inertia, which leads to the determination of two (in the plane) principal axes of inertia, which, in our case, may be called axes of probability.

Assuming that these errors in the plane are due to the cooperation of a great number of elementary errors, SCHOLS has proved that the projections of the errors on an arbitrary axis follow the exponential law of errors in a line and that the law of the resulting error can be found by supposing the error to originate in the coincidence of projections of the error upon the axes of probability, these projections being regarded as independent of each other.

The application of this theory to our case can be reduced to very simple calculations.

Errors arising from individual or instrumental causes are always distributed in a more or less systematical way, but there is no reason to suppose that the fluctuations e.g. of barometric heights within an arbitrary length of time, and cleared from their constant part, will show any tendency to systematic distribution when arranged around a point in the way described above.

SCHOLS' specific probability of an error in the plane is given by the expression:

$$F = e^{-\frac{1}{2} \left(\frac{x^2}{M_x^2} + \frac{y^2}{M_y^2} \right)} \dots \dots \dots (4)$$

$$F = \frac{e^{-\frac{1}{2} \left(\frac{x^2}{M_x^2} + \frac{y^2}{M_y^2} \right)}}{2\pi M_x M_y} \dots \dots \dots (4)$$

in which x and y are the coordinates of the error (polar coord. ρ and θ)

and M_x and M_y denote the principal axes of probability, so that:

$$M^2 = M_x^2 + M_y^2 = M_a^2 + M_b^2.$$

The mean error, therefore, can be calculated without any knowledge of the situation of the principal axes, when the mean error of the components relative to arbitrary rectangular axes is known.

If F is independent of θ :

$$M_x = M_y = \frac{M}{\sqrt{2}}$$

or, putting :

$$\frac{1}{M^2} = h^2$$
$$F = \frac{h^2}{\pi} e^{-h^2 \rho^2} (5)$$

The specific probability of an error, independent of the direction, is :

$$\frac{h^2}{\pi} \int_0^{2\pi} \rho e^{-h^2 \rho^2} d\varphi = 2 h^2 \rho e^{-h^2 \rho^2} (6)$$

From this it appears that the probability of an error zero is not, as in the case of linear errors, a maximum, but a minimum, that the curve of the spec. prob. (6) (given in SCHOLS' paper) shews a maximum for the value of ρ :

$$\rho_m = \frac{1}{h \sqrt{2}} = \frac{1}{2} M \sqrt{2} (7)$$

and, also, that the computation of the probable error will lead to a coefficient of M considerably different from that found for linear errors.

We have then to ask for what value r of ρ :

$$2 h^2 \int_0^r \rho e^{-h^2 \rho^2} d\rho = \frac{1}{2}$$
$$r = 0.83256 M (8)$$

This value of the coefficient of the probable error, considerably greater than is found for linear errors, 0.6745, clearly shows that and to what degree results, obtained in investigations of this kind, have to be put to an unusual severe test, and also that there is some reason to adhere to the use of the probable error, which of late years has been somewhat neglected.

A reduction of the mean error has no sense if this reduction is

always in the same proportion, but it becomes important if this proportion depends on the nature of the problem.

If the distribution of errors is not independent of the direction, the coefficient of M is determined by the quantity :

$$N = \frac{M_x^2 - M_y^2}{M_x^2 + M_y^2}$$

The coefficient of the probable error, for which SCHOLS gives the approximate value :

$$r = 0.8326 - 0.1581 N^2 (9)$$

is a maximum for errors independent of the direction and a minimum for linear errors, when $N = 1$.

By the assumption, therefore, that all directions are equally probable the most unfavourable case is chosen, which, in doubtful cases, is, of course, the safest way of forming a judgment.

Whether the operations, which are to be applied to the data, are considered as a determination of the first couple of constants of a FOURIER series (the very first, $\frac{1}{2} b_0$, is left out of consideration), or as a calculation of the average or most probable position of the end-points of the vectors, or as a determination of the situation of the centre of gravity — in all cases the result is a quantity determined by two coordinates and the operations we have to perform are :

- 1stly. to separate the constant part;
- 2ndly. if necessary to determine the situation of the axes of probability;
- 3rdly. to calculate the mean and probable error, in this case better called incertitude.

The same method can, of course, be applied to groups of periods, which gives a considerable saving of labour, but also leaves some want of clearness in the result.

3. The investigation of the series of daily means of barometric observations made at Batavia has been conducted in the same manner as it was commenced in 1888. The arrangement has been performed according to a period of 25.8 days, and groups of 30 rows have been taken together so that, out of the 510 periods, 17 groups have been formed.

The result of this operation is given in Table I.

If, therefore, an oscillation, periodic in 25.8 days, really exists, its amplitude is not more than :

$$\frac{1.66}{30} = 0.055 \text{ mm.}$$

T A B L E I.

	Amplitude.	Argument.	Components.		Differences.	
	<i>A</i>	<i>C</i>	<i>a</i> ₁	<i>b</i> ₁	Δ_a	Δ_b
	<i>mm.</i>		<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>
1	0.69	242°	-0.32	-0.61	-0.86	0.96
2	7.63	287°	2.17	-7.32	1.63	-5.75
3	3.45	285°	0.87	-3.34	0.33	-1.77
4	3.14	68°	1.16	2.91	0.62	4.48
5	1.52	215°	-1.24	-0.88	-1.78	0.69
6	2.08	204°	-1.90	-0.84	-2.44	0.73
7	4.52	345°	4.38	-1.14	3.84	0.43
8	1.21	104°	-0.30	1.17	-0.84	2.74
9	1.78	270°	0.01	-1.78	-0.53	-0.21
10	6.31	318°	4.71	-4.20	4.17	-2.63
11	5.00	194°	-4.86	-1.17	-5.40	0.40
12	3.25	266°	-0.20	-3.24	-0.74	-1.67
13	6.00	255°	-1.54	-5.80	-2.08	-4.23
14	2.18	317°	1.59	-1.49	1.05	0.08
15	3.34	195°	-3.23	-0.84	-3.77	0.73
16	2.60	83°	0.34	2.57	-0.20	4.14
17	7.62	354°	7.58	-0.76	7.04	0.81
Mean	1.66	289°	0.54	-1.57		

By subtracting this restant, which has to be regarded as a constant part, from the corresponding values, the differences exhibited in the last columns have been found, which are to be regarded as fortuitous disturbances.

4. The value to be attached to the result may be estimated in different ways.

The first and most simple manner is to split up the series into two or more groups. From the data given in Table I we easily find:

Group	A	C	Number of periods.
1— 6	1.68 mm.	274°	180
„ 7—11	1.62	299°	150
„ 12—17	1.76	296°	180
„ 1— 9	1.42	292°	270
„ 10—17	1.95	286°	240
„ 1—17	1.66	289°	510

From this it appears that there is certainly some indication for the existence of a periodical oscillation, and also that the arrangement has been made according to a period which practically leads to a maximum value of the amplitude.

The probability that three points, taken successively at random, are situated within an angular space of 30° is $\left(\frac{1}{12}\right)^2$ and the probability of mere chance would have been even less if we had taken into account that the amplitudes too are in good accordance.

5. A second, equally simple method is afforded by a direct view of the outcome of the arrangement itself, split up into two or more groups.

Fig. 1 gives a graphical representation of the differences given in the three last columns of Table II.

Fig. 1 shows that the curves of the two series agree satisfactorily and also that a tendency to a double period, with a maximum on the 8—9th day, which in the first group is still well marked, vanishes when the arrangement is continued.

If these results are considered as fairly conclusive, so as to justify a more exact determination of the length of the period, this may be easily done by varying the arguments C of Table I successively by $\frac{1}{2}x$, $\frac{3}{2}x$, $\frac{5}{2}x$ etc., x denoting the variation of each group-argument which leads to the most constant value of C .

In this way 17 equations are obtained from which the most probable values of C and the period T can be calculated. If to each equation the weight is given of the corresponding amplitude, the equations will assume the form:

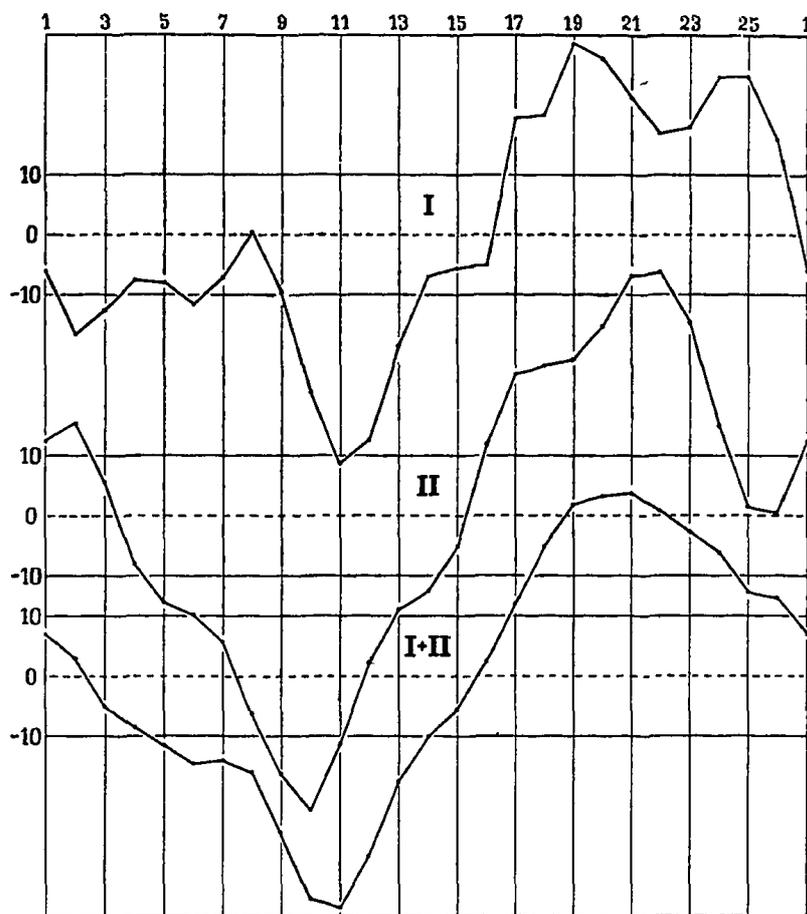
$$\begin{aligned} 0.69 (-118^\circ + \frac{1}{2}x) &= 0.69 C \\ 7.63 (-73^\circ + \frac{3}{2}x) &= 7.63 C \text{ etc.} \end{aligned}$$

T A B L E II.

Results of the arrangement. Average values, the general mean value being subtracted.

Number of Groups.	I	II	I+II	Three subsequent values taken together.		
				I	II	I+II
	270	240	510	270	240	510
	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>	<i>mm.</i>
1	-0.016	0.034	0.036	-0.012	0.025	0.014
2	-0.055	0.047	-0.008	-0.033	0.031	0.006
3	-0.026	0.012	-0.009	-0.025	0.011	-0.010
4	0.006	-0.028	-0.011	-0.015	-0.016	-0.017
5	-0.026	-0.033	-0.030	-0.016	-0.029	-0.023
6	-0.028	-0.025	-0.028	-0.023	-0.033	-0.029
7	-0.014	-0.042	-0.028	-0.014	-0.042	-0.028
8	0.010	-0.060	-0.029	0.001	-0.066	-0.032
9	0.015	-0.096	-0.038	-0.019	-0.086	-0.052
10	-0.072	-0.103	-0.088	-0.052	-0.098	-0.074
11	-0.098	-0.094	-0.097	-0.076	-0.076	-0.077
12	-0.059	-0.032	-0.047	-0.068	-0.049	-0.060
13	-0.047	-0.021	-0.036	-0.037	-0.031	-0.035
14	-0.004	-0.041	-0.022	-0.014	-0.025	-0.020
15	0.009	-0.013	-0.003	-0.011	-0.010	-0.011
16	-0.037	0.024	-0.009	-0.010	0.024	0.003
17	-0.001	0.061	0.027	0.039	0.047	0.024
18	0.054	0.058	0.055	0.040	0.050	0.043
19	0.067	0.031	0.049	0.064	0.052	0.057
20	0.071	0.069	0.069	0.059	0.063	0.060
21	0.039	0.091	0.062	0.046	0.080	0.061
22	0.029	0.079	0.051	0.034	0.081	0.055
23	0.035	0.074	0.052	0.036	0.064	0.048
24	0.046	0.038	0.041	0.053	0.030	0.041
25	0.077	-0.024	0.028	0.053	0.003	0.028
26	0.036	-0.006	0.015	0.032	0.001	0.026
Mean	0.097	0.103	0.097			

Fig. 1.



As might have been expected the result of this calculation shews little or no difference from that of the arrangement itself.

$$x = -1^{\circ},09 \quad C = 291^{\circ} \quad T = 25.803^d.$$

As one day corresponds to $360^{\circ}/25.8$, a variation of x degrees for each group is equivalent to:

$$\frac{1.09}{30} \cdot \frac{258}{3600} = 0.003^d.$$

6. By application of the method discussed in § 2 to the differences Δ_a and Δ_b of Table I we find:

$$\Sigma M^2_x = 145.05 \quad \Sigma M^2_y = 212.02 \quad \Sigma M^2_{xy} = -12.44.$$

From the well known formula :

$$\text{tang } 2 \psi = \frac{2M^2_{xy}}{M^2_x - M^2_y} \dots \dots \dots (10)$$

for the situation of the principal axes of inertia, deduced from the

condition that, when the axes of coordinates coincide with the principal axes, the moment of deviation or centrifugal force M_{xy}^2 will vanish, we find :

$$\psi = -18^{\circ}35'$$

and further

$$M_x^2 = 6.45 \quad M_y^2 = 8.72 \quad N = 0.15$$

and from formula (9)

$$r = 0.829.$$

It appears, therefore, that, in this case, all directions of the accidental quantities are equally probable, so that we are fully justified in putting :

$$r = 0.833.$$

The mean and the probable error for each group are then :

$$M = 3.89 \quad W = 3.24$$

and the final result for each group:

$$1.76 \text{ mM. . . . probable error } 0.810$$

and for each row :

$$0.055 \text{ mM. . . . probable error } 0.027$$

so that the probable incertitude of the final outcome amounts to almost exactly half the amplitude.

7. The question may also be put, what will happen if the arguments of Table I are varied in such a manner, that the variations are equivalent to arrangements according to other periods slightly different from 25.8 days.

The amount of the variation is limited by the number of rows taken together in one group, which can be shifted only as a whole, and the variation ceases to have any sense as soon as the sums of each group would be sensibly affected by the actual arrangement according to the new period.

If quantities, periodical within a length of time T , are arranged according to a period T' in m columns, the value at the origin of time being represented by :

$$A \cos C,$$

the record to be inscribed in the t^{th} column of the p^{th} row (t and p counted from nought) will be :

$$A \cos \left(\frac{2\pi}{m} \cdot \frac{n}{n'} - C + 2\pi p \frac{n}{n'} \right)$$

$$n = \frac{2\pi}{T} \quad n' = \frac{2\pi}{T'}$$

The sum of R rows is then :

$$n' - n = \delta \qquad a = \frac{T\delta}{2}$$

$$A \frac{\sin Ra}{\sin \alpha} \cos \left[\frac{2\pi}{m} T + \frac{\delta T'}{m} T - C + (R - 1) \alpha \right].$$

When δ is small this expression can be simplified by putting :

$$T' = T \qquad T = \frac{m}{2}$$

in the second term under the cosine. The sum of the first, second etc. group of R rows is then:

$$A \frac{\sin Ra}{\sin \alpha} \cos \left(\frac{2\pi}{m} T - C + Ra \right) (11)$$

$$A \frac{\sin Ra}{\sin \alpha} \cos \left(\frac{2\pi}{m} T - C + 3 Ra \right) \text{ etc.}$$

If the oscillation is of a purely periodical description and of equal amplitudes the sum will show a principal maximum, RA , for $\alpha = 0$, and further secondary maxima for all values of α which satisfy the equation:

$$R \tan \alpha = \tan Ra$$

i.e., when $R = 510$, for values of α corresponding with periods of:

$$\left. \begin{array}{l} 25.872^d \\ 25.728 \end{array} \right\} \qquad \left. \begin{array}{l} 25.925^d \\ 25.675 \end{array} \right\}$$

but the amplitudes of these maxima will be resp. 5 and 8 times smaller than the principal maximum.

The amplitude will vanish whenever

$$Ra = \pi, 2\pi, 3\pi \text{ etc.}$$

i.e. for periods of

$$\left. \begin{array}{l} 25.850^d \\ 25.750 \end{array} \right\} \qquad \left. \begin{array}{l} 25.900^d \\ 25.790 \end{array} \right\}$$

The upper curve of fig. 2 gives an image of the fluctuations of these theoretical amplitudes.

If we put:

$$T = 25.8 \qquad T' = 25.8 \pm x$$

the amount of shifting to be given to each group corresponding with 0.01 day, is:

$$30 \alpha = \frac{30 \cdot \pi \cdot x}{25.8 \pm x} = 2^\circ.094,$$

x in the denominator being neglected.

The variation has been carried on, as utmost allowable limit, to

$x = \pm 0.15$, corresponding with a group-variation of about 31° . The group-amplitude is only slightly affected by this variation as:

$$\frac{\sin R\alpha}{\sin \alpha} = 28.58$$

instead of 30.

When R , the total number of periods, increases, the secondary maxima will become smaller and smaller, and at the same time maxima and minima will approach nearer to the principal maximum.

T A B L E III.

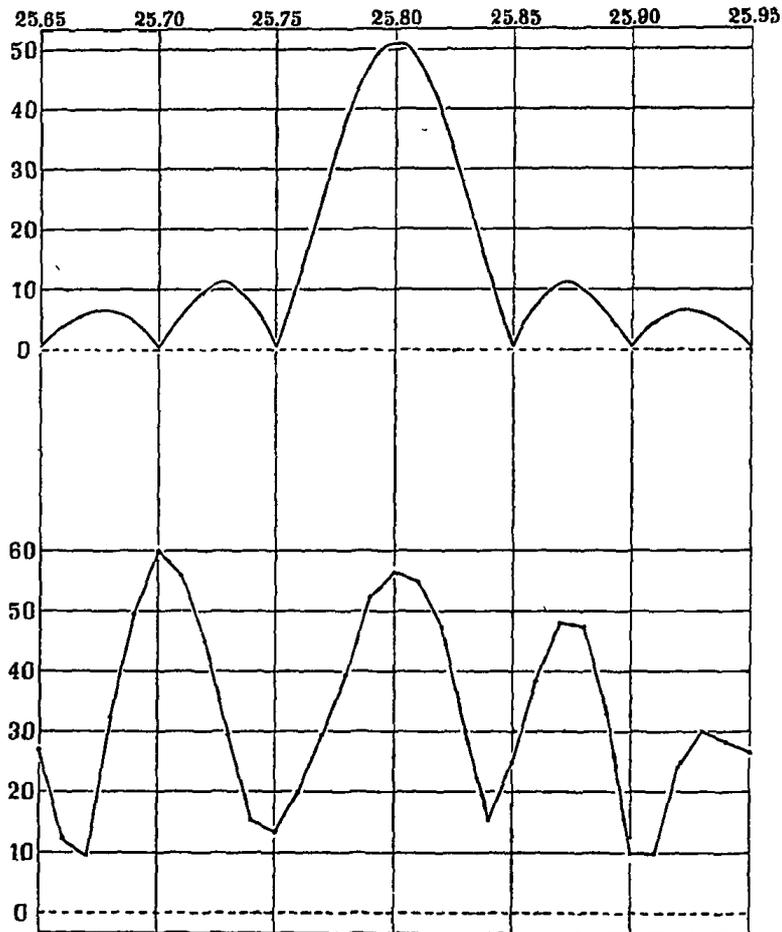
Results of the arrangement according to different periods by the shifting-process.

Period	A	C	Period	A	C
	<i>mm.</i>			<i>mm.</i>	
d. 25.65	13.4	351°	d. 25.81	27.4	247°
25.66	6.4	298°	82	23.7	217°
25.67	4.8*	148°	83	15.0	169°
25.68	16.4	76°	84	7.6*	84°
25.69	24.8	34°	85	12.6	344°
25.70	<u>30.0</u>	345°	86	19.2	286°
25.71	27.9	315°	87	<u>24.0</u>	244°
25.72	22.5	265°	88	23.8	207°
25.73	14.6	231°	89	16.5	174°
25.74	7.8	171°	90	5.1*	132°
25.75	6.7*	88°	91	4.9	301°
25.76	10.1	33°	92	11.9	262°
25.77	14.7	9°	93	<u>15.0</u>	232°
25.78	19.8	346°	94	13.6	212°
25.79	26.4	322°	95	13.3	300°
25.80	<u>28.3</u>	289°			

Table III exhibits the outcome of this shifting of the groups and fig. 2 shews both the theoretical and actual curves.

For the sake of comparison the data of Table III have been multiplied by 2 and the amplitude of the theoretical period has been taken equal to 0.1.

Fig. 2.



It appears then that, in fact, secondary maxima and minima occur and, at least as regards the first minima, in the right places, but that the secondary maxima, instead of being small as compared with the principal maximum, as might have been expected, are of about equal intensity and, most so on the left side, not at all agreeing with the theoretical lengths of period.

This result may be interpreted in three ways:

a. We may assume that every one of the three periods 25.80, 25.70 and 25.87 is due to a purely accidental distribution of the quantities under consideration.

b. We may concede that at least for the period 25.80 there is some indication, but that the two adventitious periods are the consequence of the unequal distribution of the group-amplitudes so that they will disappear when the arrangement is continued over a longer series of observations.

c. We can assume that the evidence is equally good for the three periods, and will be enhanced by continued arrangement.

Of course only an actual continuation of arrangement for another series of twenty years will enable us to answer these questions. The only test which at present can be applied is to form two or more groups as has been done above for the arrangement according to 25.8 days.

Period		25.70 ^d		25.87 ^d	
		A	C	A	C
Group	1—6	1.60 mM.	6°	1.40 mM.	252°
	„ 7—11	3.35 „	292°	0.73 „	165°
	„ 12—17	2.48 „	27°	2.52 „	253°
	„ 1—9	1.11 „	335°	1.22 „	216°
	„ 10—17	2.53 „	349°	1.88 „	263°

So far as this criterion allows a conclusion to be drawn, it appears from this result that the evidence for real existence of the periods 25.70 and 25.87 is considerably less than of the period 25.80.

In the latter case the arguments for three groups did not differ more than 25°, against differences of resp. 88° and 95° for periods of 25.70 and 25.87 days. The probabilities of mere chance, therefore, are, taking 30° and 90° :

$$\frac{1}{144} \text{ and } \frac{1}{16}$$

i. e. more than 8 times as great. If we take also into account that the amplitudes of the three groups are accordant for 25.80 and widely different for the adventitious periods, we can estimate the probability of chance at 10 times as great.

The computation of the probable error (incertitude) of the result for each group also gives an indication for this greater probability, but not in the same degree.

	Amplitude.	Probable error.
25.80	1.76 mm.	0.810
24.70	1.76 „	0.916
25.87	1.41 „	0.830

7. If we apply, in so far as possible, the different criteria to the data published by Prof SCHUSTER concerning daily means of magnetic declination for Greenwich, arranged according to 26 and 27 days, we find for the sums of groups, each of which contains resp. 14 and 13.5 rows.

		26 ^{d.}		27 ^{d.}	
		<i>A</i> ₁	<i>C</i>	<i>A</i> ₁	<i>C</i>
Group	1— 5	6'.19	267°	3'.53	351°
	6—10	4.08	243°	2.19	88°
	11—15	3.09	351°	4.75	354°
	16—20	1.45	152°	7.05	203°
	21—25	2.88	229°	7.56	298°

The probability, therefore, of mere chance is :

$$26 \text{ days } \dots \dots \dots \left(\frac{283}{360}\right)^4 = 0.38$$

$$27 \text{ days } \dots \dots \dots \left(\frac{246}{360}\right)^4 = 0.20.$$

and the final outcome

$$26 \text{ days } 0'.480 \dots \dots \dots \text{prob. error } 0'.343$$

$$27 \text{ ,, } 0'.405 \dots \dots \dots \text{,, ,, } 0'.415$$

If we vary the arguments given in Table VIII of SCHUSTER's paper for a period of 26 days in such a way that the result is equivalent to an arrangement according to 25.80 days we find :

		<i>A</i> ₁	<i>C</i>
Group	1— 5	10'.83	54°
	6—10	5.72	104°
	11—15	4.88	77°
	16—20	4.61	44°
	21—25	4.03	89°

As these arguments do not differ more than 60 degrees, the probability of chance is in this case :

$$\frac{1}{6^4}$$

The final result, calculated for a group of 14 rows,

$$1'.118 \dots \dots \dots \text{prob. error } 0'.292.$$

The accurate length of the period and the most probable value of *C*, calculated after the method discussed sub 5, are then :

$$25.804 \text{ days } \dots \dots \dots C = 55.^\circ 6.$$

It appears from these calculations that an arrangement according to 26 and 27 days leads to results the probable incertitude of which has about the same value as the amplitude itself. On arranging according to 25.8 days we find a probable incertitude about four times less than the amplitude.

Further, from this investigation, as compared with SCHUSTER's inquiry, we may draw the conclusion, that elements of terrestrial magnetism, as observed in higher latitudes, allow a more decided judgment to be formed concerning the real existence of periodical

oscillations of this kind than meteorological observations made at tropical stations.

If the outcome arrived at by the arrangement of barometric daily means for Batavia is considered to afford some evidence or indication for an oscillation periodic in 25.80 days, a much greater probability must be attached to the real existence of this fluctuation in the observations of magnetic declination made at Greenwich.

Anatomy. — “*On the Form of the Trunk-myotome.*” (First Communication). By Prof. J. W. LANGELAAN. (Communicated by Prof. T. PLACE).

The segmented plan of construction of the vertebrate animals, most marked in the muscular system, has led to the conception of the myotome.

Two methods are chiefly employed in establishing the form of this myotome. The first method is based on the hypothesis of the primary connection between muscle and nerve; the second, a more direct one, is based on the dissection of the intersegmental tissue. Both methods seem equally restricted in their application, as can be concluded from the researches of BARDEEN¹⁾, moreover there is reason to believe, that they will not always yield concordant results.

The second method is followed in this research.

I. *Trunk-myotome of Petromyzon fluviatilis.* (Fig. 1).

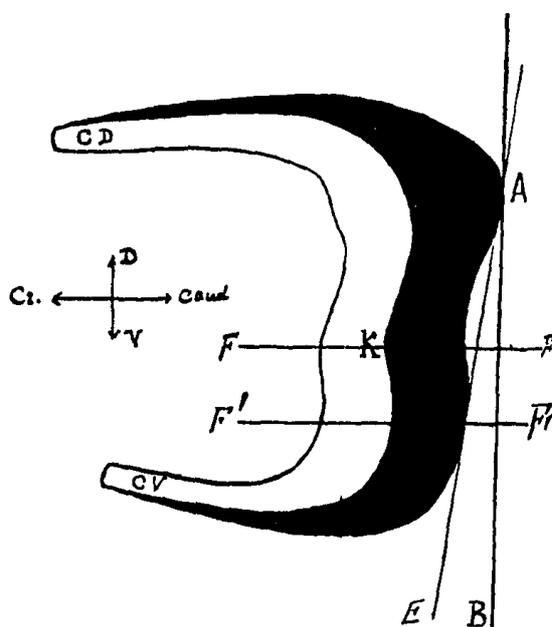


Fig II.
torquated in respect to each other.

¹⁾ Anat. Anz. Bd. XXIII, N^o. 10/11.

The trunk-myotome of the adult animal has in general the form of a crescent, the cornua being directed towards the cranial end of the body and slightly inclined to each other. The dorsal cornu (fig. II *CD*) reaches to the mid-dorsal line, while the ventral cornu (fig. II *CV*) ends at the mid-ventral line of the body. Both cornua differ in length, the dorsal being about $\frac{1}{7}$ longer than the ventral, and while both reach to the mid-plane of the body, they are slightly