condensation, it is desirable that the capillary at the place where the temperature is between T and T''' should not be too narrow. Moreover the capillary is surrounded by an air jacket p_4, made of a glass tube tightly closed with india-rubber rings p_6 and fish-glue. To avoid diffusion the other portion of the capillary g''_1 must be narrow.

If by previous determinations with the dew-point apparatus we have determined x_{vpT} (as a first approximation it will in some cases be possible to use a preliminary ψ-surface as constructed in Supplement N°. 8 see p. 222), it is easy to apply the correction necessary to derive the composition of the investigated liquidphase at T from x_1, the original composition. On the piezometer divisions the volume of the vapour is read. Let V_v be this volume reduced to normal circumstances and corrected for the first virial coefficient B (comp. for instance the continuation of this paper), let V be the entire volume of vapour and liquid, measured and corrected in the same way, then $V_l = V - V_v$ is the volume of gas, measured and corrected in the same way, which would form the liquid phase.

Hence:

$$X_{lpT} = \frac{X_1\, V - X_{vpT}\, V_v}{V - V_v} = X_1 + \frac{V_v}{V}\,(X_1 - X_{vpT}) + \dots$$

If we operate under moderate pressures, the correction will be always small and even if x_{vpT} is not very accurately known, it can be applied in a satisfactory way.

Astronomy. — *"A new method of interpolation with compensation applied to the reduction of the corrections and the rates of the standard clock of the Observatory at Leyden,* HOHWÜ 17, *determined by the observations with the transit circle in* 1903." By J. WEEDER. Communicated by Dr. H. G. VAN DE SANDE BAKHUYZEN.

§ 4. In the Proceedings of Nov. 29, 1902 occurs a paper "On interpolation based on a supposed condition of minimum," of which the present paper is to be considered as a continuation; this explains the numbering of the sections. In order to interpolate between the ordinates S belonging to the abscissae $t = a, b \dots y, z$, I have there determined the interpolating curve for which the total sinuosity $I_S = \int_a^z \left(\frac{d^2S}{dt^2}\right)^2 dt$ has the least value. I found that between two successive points which are well defined by observations this curve satisfies an equation of the third degree $S_t = S_q + g_q T + c_q T^2 + e_n T^3$, where $T = t - q$ and $q < t < r$, and has moreover the properties that besides S also $\frac{dS}{dt}$ and $\frac{d^2S}{dt^2}$ are continuous, while at the ends of this curve $\frac{d^2S}{dt^2}$ is zero; these data appeared to be sufficient for a definite determination of such an interpolating curve.

In the same paper in § 3 I have referred to the problem of compensation for the case that the results of the observations S are affected with errors.

Now I shall describe how that problem was solved and I shall apply this method of compensation to the corrections and the rates of the clock Hohwü 17 during the period Jan. 14, 1903—Jan. 14, 1904.

For the sake of the compensation I have used instead of the supposition on which my interpolation was based, another hypothesis which covers more, and for this purpose I have accepted that the probability of a group of corrections S is proportional to $e^{-\lambda I_S}$, where the factor λ is independent of the intervals between the observations; the original condition of minimum is a result of this hypothesis, if we accept as interpolated for the time t that value which (considered as result of observation) makes the expression $e^{-\lambda I_S}$ as large as possible.

As formerly I have called the error of observation in S_q: f_q and the real clock correction L_q, so that $L_q = S_q - f_q$, meanwhile, I venture to use the same letters f_q and L_q in the sense of most probable values of the errors and measured quantities, although there is naturally a difference between the latter quantities and the real values.

As soon as we have to do with errors of observation we must substitute I_L for I_S in the expression for the probability of a group.

The probability that the errors $f_a, \ldots f_p, f_q, f_r, \ldots f_z$ respectively, occur in the observations, must in those cases where the continuity of the observed quantity agrees with the hypothesis made above, be proportional to the product of $e^{-\lambda I_{S-f}}$ and $e^{-\frac{1}{2}\Sigma \frac{f_q^2}{\mu_q^2}}$; in the latter expression μ_q represents the value of the mean error in S_q. The system of errors of greatest probability satisfies the condition:

$$\lambda I_{S-f} + \frac{1}{2} \Sigma \frac{f_q^2}{\mu_q^2} = \text{minimum},$$

whence for each of those most probable errors the relation:

$$\lambda \frac{\partial I_{S-f}}{\partial f_q} + \frac{f_q}{\mu_q^2} = 0.$$

Theoretically $\frac{\partial I_{S-f}}{\partial f_q}$ contains all the unknown quantities f, practically, however, besides f_q only the 3 or 4 preceding and following ones, since, owing to the small coefficients, the other terms may be neglected. The most probable errors occur in it in a linear form, and hence the complete system of error-equations is sufficient to determine them all.

§ 5. In this manner I have treated 61 clock corrections determined by observations made from Jan. 14, 1903 to Jan. 14, 1904 by E. F. VAN DE SANDE BAKHUYZEN and A. PANNEKOEK. This problem was somewhat complicated because the observed rates depend on the pressure and the temperature of the air in which the pendulum moves. An increase of atmospheric pressure of 1 mm. mercury gives to the clock rate a retardation of 0.015 seconds a day, while an increase of temperature of 1 deg. C. accelerates the rate by about 0.030 seconds a day.

I have found that also the differences of temperature observed in the clockcase have a perceptible influence on the rate[1]).

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[1]) This temperature gradient forms a new element in the reduction of the clock rates, in so far as at the Leyden Observatory it was not accounted for until 1903. Although in the clockcase of Hohwü 17 two thermometers have been suspended at different heights since 1866, the influence of the temperature gradient did not clearly appear from the earlier observations owing to the inaccuracy of those readings. Although Dr. E. F. VAN DE SANDE BAKHUYZEN detected a yearly periodicity in the differences between those thermometers, which might explain the phenomenon of the yearly periodicity in the rates (Proc. Kon. Ak. v. W. 1902) yet at that time he still doubted of their reality, because the thermometers were graduated according to Réaumur and the accuracy of the reader in estimating the tenth parts was insufficient to warrant an inequality of 0.1° R. in the monthly means of the difference in temperature.

In order to obtain certainty on this point, two thermometers were placed into the clockcase in January 1903, which were divided into tenth parts of a centigrade and had been compared with each other beforehand by the director of the Observatory; since that time they have been read with accurate estimation to a hundredth part of a centigrade.

The vertical distance between these thermometers is 63 cm. It soon appeared that in the clockcase there is a variable temperature gradient, which cannot but cause a considerable variation of rate.

In July 1903 it was derived from observations that a difference between the

A temperature gradient in vertical direction of 1 centigrade per meter causes, roughly calculated, a variation in rate of 0s.25.

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thermometers (upper—lower) of one centigrade corresponds with a retardation of 0s.30 a day.

This difference of temperature, apart from variations in connection with meteorological conditions, clearly shows a yearly periodicity; in winter it is small, in summer on some days it increases to above 0°.50 C. It can hardly be doubted that, at least to begin with December 1898, the yearly periodicity must be ascribed to the temperature gradient; for it is since that date that the standard clock has been placed in the vestibule in a niche cut out from the pier of the 10-inches refractor. Since about March 1899 the place on the Eastern pier in the transit room where the standard clock hung during the former periods 1862—'74 and 1877—'98, treated by E. F. van de Sande Bakhuyzen, has been occupied by the clock Hohwü 46 with a Riefler-pendulum.

In order to obtain accurate data about the temperature gradient also for this place, two thermometers, which had been compared with each other and were graduated to tenth parts of a centigrade were suspended in the case of Hohwü 46 at a vertical distance of 65 cms. on February 27, 1903. It appeared that here the differences in temperature were in general greater than in the pier-niche, and in July 1903 it was derived from the observations that a difference of temperature between the thermometers of 1 centigrade corresponded with a variation of 0s.40 in the daily rate of this clock.

A yearly periodicity in the temperature gradient appeared also distinctly in Hohwü 46, as will be seen from the following monthly means, which are given by the side of those of Hohwü 17.

		Hohwü 17.	Hohwü 46.
1903	March	+0°.14	+0°.25
„	April	+ 0.09	+ 0.16
„	May	+ 0.31	+ 0.42
„	June	+ 0.28	+ 0.37
„	July	+ 0.29	+ 0.32
„	August	+ 0.18	+ 0.22
„	September	+ 0.16	+ 0.26
„	October	+ 0.06	+ 0.12
„	November	+ 0.02	+ 0.10
„	December	+ 0.01	+ 0.03
1904	January	— 0.02	+ 0.08
„	February	+ 0.05	+ 0.12

Moreover in the temperature gradient in the clockcase in the transit room a daily inequality was observed, which was hardly perceptible in the niche.

In the case of Hohwü 46 the mean values of the temperature gradient from 0^h to 12^h mean time, are regularly greater than the mean values from 12^h to 24^h mean time. From the differences between these mean values in connection with the differences in the relative rates of the two clocks for the corresponding half days I could derive the influence of a variation in the temperature gradient on the rate of Hohwü 46 for a shorter period.

The investigation is not yet finished, but in connection with the theoretical

A 4th inequality in the rates was due to the difference between the personal equations of the observers. From direct determinations we have obtained the following differences in the clock corrections:

	B—P
1 April 1901	— 0s201
18 March 1903	— 0.275
15 March 1904	— 0.224

For the time being I have adopted for this difference during the period Jan. 14, 1903—Jan. 14, 1904: —0s.250 and have reduced the clock corrections determined by P. to B.'s system. Besides the corrections for atmospheric pressure, temperature and difference in temperature I introduced a correction for the assumed personal equation; these 4 unknown quantities are introduced into the equations and will be derived from the observations. I represented them by x, y, z and u and chose the following units so that these quantities should have about the same values: for atmospheric pressure (B) the unit $= {}^1/_4$ mm. mercury of 0° C., for temperature (ϑ) the unit $= {}^1/_{10}$ degree Celsius, for difference in temperature (V) the unit $= {}^1/_{100}$ degree Celsius, for clock rates per day the unit $= {}^1/_{1000}$ second of time; x, y and z represent the influence of each of the three units on the daily rate, expressed in thousandth parts of a second of time, while u is the 10th part of the correction for the assumed difference in the personal equations of the observers during the same unit of time, viz. $B - P = -250 - 10\,u$.

§ 6. The observed clock corrections were reduced beforehand to midnight of the data of observation. This was done with clock rates according to a preliminary formula with due regard to atmospheric pressure, temperature and temperature difference in the period from the instant for which the clock correction was determined to midnight. This formula is derived from observations during the first half of 1903, it has been regularly tested by each new determination of the clock error, and has proved very satisfactory for the purpose set here.

$$\text{Constans} = \text{daily rate} - 0\overset{s}{.}0157\,(\text{bar.} - 760) + 0\overset{s}{.}032\,(\text{temp.} - 10^\circ\,\text{C.}) - 0\overset{s}{.}31\,(\text{temp. difference}).$$

development of B. Wanach of Potsdam on the influence of the temperature-gradient on clocks (A. N. Nrs. 3967—68) I think it worth mentioning that in this way I found that per 1 centigrade difference in temperature between the thermometers, there was a variation in rate of about 0s.80, twice the amount derived from the before mentioned observations.

From readings of the barometer and the thermometers, daily means of atmospheric pressure, temperature and difference of temperature from midnight to midnight have been derived. Let the clock corrections at midnight be S, and the mean daily rates in the intervals between the successive determinations of the clock corrections be Q, in accordance with the letters previously used by me. From the said daily means, the mean values of the atmospheric pressure, of the temperature and of the difference in temperature were derived for the same intervals. In the interpolation these quantities correspond with Q, hence I call them Q^B, Q^ϑ, Q^V. For the barometer readings I used the deviations from 76 cms. The temperatures ϑ are those of the upper thermometer.

In this paper I shall also use the letters S^B, S^ϑ, S^V to indicate quantities that can be computed for each observation by taking the sum of the daily means of B, ϑ and V, to begin with a certain date, say Jan. 14, 1903 till midnight of the day for which the clock correction is determined. S^P denotes a value which is + 10 for each observation of PANNEKOEK, and zero for each observation of BAKHUYZEN, and the series Q^P relates to the series S^P in the same way as each series Q relates to the series S according to the formula $Q_n = \dfrac{S_r - S_q}{n}$. S_r and S_q are two successive quantities of the series, and n is the interval between them expressed in days.

$$L = S - x\,S^B - yS^\vartheta - z\,S^V - u\,S^P - f$$

must then be considered as a formula for a reduced and compensated clock correction.

Each of the letters L, S, f represents a series of discrete values, one for each observed clock correction, but by means of the interpolation along the least sinuous line they can also be taken as continuous variable quantities of which the derivatives of the first and second order also vary continuously, but of which the derivatives of the third order vary abruptly. To determine such a variable, say S^B, for the instant $t = q + T$ between the epochs q and r of the determinations of the clock correction, we use according to § 4 the formula

$$\overset{B}{S}_{q+T} = \overset{B}{S}_q + \overset{B}{g}_q T + \overset{B}{c}_q T^2 + \overset{B}{e}_n T^3,$$

where q is the epoch of the observation which immediately precedes t. The coefficients g^B c^B e^B can be derived from the series Q^B if we use the formulae C in § 2 of the previous paper on this method of interpolation.

Taking the quantities L, S, f in the sense as explained above we can develop the total sinuosity of the reduced and compensated correction as follows:

$$I_L = \int_a^z \left(\frac{d^2L}{dt^2}\right)^2 dt = \int_a^z \left[\frac{d^2(S-f)}{dt^2} - x\frac{d^2S^B}{dt^2} - y\frac{d^2S^\vartheta}{dt^2} - z\frac{d^2S^V}{dt^2} - u\frac{d^2S^P}{dt^2}\right]^2 dt.$$

In order to give the greatest probability to the series L we must choose x, y, z and u so, that the partial derivatives of I_L with regard to each of these are zero; i.e. that they satisfy the following relation:

$$\int_a^z \left[\frac{d^2(S-f)}{dt^2} - x\frac{d^2S^B}{dt^2} - y\frac{d^2S^\vartheta}{dt^2} - z\frac{d^2S^V}{dt^2} - u\frac{d^2S^P}{dt^2}\right]\frac{d^2S^B}{dt^2} = 0\,,$$

and 3 others, which we obtain by substituting for the last factor $\frac{d^2S^B}{dt^2}$, successively $\frac{d^2S^\vartheta}{dt^2}$, $\frac{d^2S^V}{dt^2}$ and $\frac{d^2S^P}{dt^2}$.

Definite integrals, such as $\int_a^z \frac{d^2\,S^B}{dt^2} \cdot \frac{d^2\,S^\vartheta}{dt^2}\,dt$, which here occur as coefficients, can be computed in the following way:

$$\int_a^z \frac{d^2\,S^B}{dt^2}\,d\,.\,\frac{dS^\vartheta}{dt} = \left[\frac{d^2\,S^B}{dt^2}\cdot\frac{dS^\vartheta}{dt}\right]_a^z - \int_a^z \frac{dS^\vartheta}{dt}\,\frac{d^3\,S^B}{dt^3}\,dt =$$

$$= 2\,[c^B\,g^\vartheta]_a^z - 6\,\Sigma\,\overset{B}{e_n}\,(S_r - S_q)^\vartheta = 2\,[c^B\,g^\vartheta]_a^z - 6\,\Sigma\,n\,\overset{B}{e_n}\,\overset{\vartheta}{Q_n}.$$

If in the last term we also substitute for

$$3\,n\,\overset{B}{e_n}\,,\ \overset{B}{(c_r - c_q)}\,,\ \text{we have}$$

$$^1/_2\int_a^z \frac{d^2\,S^B}{dt^2}\,\frac{d^2\,S^\vartheta}{dt^2}\,dt = [c^B\,g^\vartheta]^z + \Sigma\,\overset{\vartheta}{Q_n}\,\overset{B}{(c_q - c_r)}$$

and after interchanging the indices B and ϑ in the second member also

$$[c^\vartheta\,g^B]^z + \Sigma\,\overset{B}{Q_n}\,\overset{\vartheta}{(c_q - c_r)}.$$

I have computed these coefficients according to both formulae, and thus obtained a rigorous test.

At the beginning and the end of the interpolation the quantities c are always zero, so that products such as $c^B g^\vartheta$ would be zero at either end. With a view, however, to the continuation of this computation for next year, I have not closed the interpolation-computation for the atmospheric pressure, the temperature, etc. on January 14, 1904. Hence on Jan. 14, 1904 c^B, c^{ϑ}, c^V, c^P differ from zero and in the formula I had to retain the term for the end.

In integrals such as $\int_a^z \frac{d^2 S^B}{dt^2} \frac{d^2 (S-f)}{dt^2} dt$ I first put all errors equal to zero and computed the known terms of the four equations according to formulae of the form:

$$^1/_2 \int_a^z \frac{d^2 S^B}{dt^2} \frac{d^2 S}{dt^2} dt = [c^B g]^z_B + \Sigma (c_q - c_r) Q.$$

By solving these equations, I have obtained preliminary values for x, y, z and u, and in the way to be explained in § 7, also for the most probable errors f; from the mean daily rates computed with these preliminary values, using the formula

$$Q_L = Q - x\, Q^B - y\, Q^{\vartheta} - z\, Q^V - u\, Q^P - \frac{f_r - f_q}{n}$$

I have derived as a second approximation the corrections x, y, z and u to these preliminary values. The influence of those corrections on the most probable errors was of little importance.

Integrals such as $\int_a^z \left(\frac{d^2 S}{dt^2}\right)^2 dt$ have been computed twice, first

according to the formula for $^1/_2 \int_a^z \frac{d^2 S^B}{dt^2} \frac{d^2 S^{\vartheta}}{dt^2} dt$

$$^1/_2 \int_a^z \left(\frac{d^2 S}{dt^2}\right)^2 dt = [c\, g] + \Sigma\, Q_n (c_q - c_r)$$

and secondly according to the formula, deduced in § 2

$$\int_a^z \left(\frac{d^2 S}{dt^2}\right)^2 dt = \frac{4}{3} \Sigma\, n\, (c_q^2 + c_q c_r + c_r^2)$$

The following are the 4 equations expressed in numbers:

$$\begin{aligned}
5460\, x - 252\, y - 14\, z + 59\, u &= + 21664 - 145 \\
- 252\, x + 952\, y + 512\, z - 8\, u &= - 1757 - 88 \\
- 14\, x + 512\, y + 571\, z - 22\, u &= + 383 + 87 \\
+ 59\, x - 8\, y - 22\, z + 101\, u &= - 101 - 21
\end{aligned}$$

The second members are written in 2 parts. The first part is derived from the observed rates Q, the second from the preliminarily reduced compensated rates Q_L. The solution of the two sets yielded :

$$x = +3.90 - 0.04 = +3.86$$
$$y = -2.30 - 0.34 = -2.64$$
$$z = +2.72 + 0.46 = +3.18$$
$$u = -2.86 - 0.09 = -2.95$$

According to these results the influence on the daily clock rates of the atmospheric pressure per 1 mm. mercury of 0° C. is $+0^s0154$, of the temperature per 1 centigrade (upper thermometer) -0.0264, and of the difference in temperature per 1 centigrade (upper—lower) $+0,318$,

while for the personal difference BAKHUYZEN—PANNEKOEK in the clock corrections is found $-0^s.220$.

Table I contains the values required for the said computations; for each interval between two consecutive determinations of the clock correction : the number of days n, the observed clock rate Q, the values Q^B, Q^S, Q^V and Q^P, and also the values $(c_q - c_r)^B$, $(c_q - r_r)^S$, $(c_q - c_r)^V$, $(c_q - c_r)^P$. To this must be added in order to render the computation possible the values for the last epoch Jan. 14, 1904, :

$$g^B = -14.0 \qquad g^S = +57.7 \qquad g^V = -3.1 \qquad g^P = +2.4$$
$$c^B = +1.09 \qquad c^S = +2.01 \qquad c^V = +0.76 \qquad c^P \quad +0.54$$

$g = -146$, adopted in case we use the series of the observed clock rates Q.
$G = -194$, „ „ „ „ „ preliminarily reduced Q_L.

§ 7. Here follows the reduction of the relations between the most probable errors and the determination of the latter according to these relations.

Sub § 5 I have derived for each error f_q the relation :

$$\lambda \frac{\partial I_L}{\partial f_q} + \frac{f_q}{\mu_q^2} = 0 \quad \text{or} \quad \frac{\partial I_L}{\partial f_q} + \frac{1}{\lambda \mu_q^2} f_q = 0$$

I have thought myself justified in adopting the same value for the mean errors in the determinations of the clock correction, although for 1 of those corrections only 2 stars, for 13 corrections 4 stars, and for all the others 3 stars were observed, since the difference in accuracy resulting from the different numbers of stars is relatively

TABLE I.

Intervals in days.	Observed clock rates.	Atmospheric pressure (B).		Temperature (ϑ).		Temperature difference (V).		Personal equation (P).	
	Q	Q^B	$(c_q-c_r)^B$	Q^{ϑ}	$(c_q-c_r)^{\vartheta}$	Q^V	$(c_q-c_r)^V$	Q^P	$(c_q-c_r)^P$
3	$+0^s.230$	+52.9	+ 2.45	49.3	+1.88	+16.0	+3.62	—3.3	—1.78
3	+ .126	+40.5	+ 0.13	42.0	—2.12	+ 1.3	—3.46	+3.3	+2.33
8	+ .036	+23.3	— 4.15	49.6	—2.69	+ 1.1	—1.89	0.0	—0.14
3	+ .050	+25.5	+ 3.98	71.3	+3.23	+12.3	+2.83	—3.3	—1.57
6	— .034	+14.2	— 3.21	74.7	—1.44	+ 7.7	—1.22	+1.7	+1.91
10	+ .024	+31.2	— 3.17	80.9	+2.76	+ 7.9	—0.20	—1.0	—1.63
3	+ .164	+60.8	+11.39	72.0	—2.95	+10.7	+0.72	+3.3	+1.77
7	— .067	+ 4.9	— 2.99	80.4	+0.84	+ 8.9	—0.65	—1.4	—1.06
8	— .235	—31.0	—14.05	88.3	+0.54	+10.6	+0.18	0.0	—0.51
3	.000	+23.2	+14.22	87.7	+1.48	+12.3	—0.45	+3.3	+2.81
3	— .063	+10.7	— 0.90	81.7	—1.73	+13.0	+1.95	—3.3	—3.41
4	— .117	— 2.9	— 9.97	81.8	—0.34	+ 7.5	—1.91	0.0	+1.70
6	— .042	+17.4	+14.57	89.5	—1.36	+11.8	—1.00	0.0	—0.95
5	— .201	—23.1	—12.98	105.0	+1.62	+22.8	+1.99	+2.0	+1.16
7	— .139	— 2.7	+ 6.05	108.3	+1.19	+19.1	+0.38	—1.4	—0.98
7	— .150	+ 0.6	— 3.54	98.9	—0.21	+10.2	—1.17	0.0	+0.50
10	— .106	+ 4.2	+ 7.51	85.3	+0.38	+ 7.8	+1.32	0.0	—0.39
7	— .291	—46.1	— 6.63	77.0	—3.77	+ 2.9	—3.58	+1.4	+0.63
10	— .245	—37.9	— 0.51	107.8	+1.81	+27.2	+3.15	—1.0	—0.82
12	— .192	— 2.7	+ 0.67	126.6	+1.63	+25.3	—0.24	+0.8	+1.22
3	— .104	+22.1	— 5.19	125.7	—3.82	+21.7	—2.84	—3.3	—2.91
3	— .052	+40.1	+14.67	138.3	+1.92	+32.3	+1.41	+3.3	+3.22
5	— .190	+ 3.4	— 9.29	153.8	—2.61	+44.2	—1.20	0.0	—1.38
5	— .225	+ 5.0	— 4.61	174.0	+7.85	+53.2	+6.93	0.0	+0.29
4	— .139	+30.8	+12.40	157.3	—5.25	+30.2	—5.55	0.0	+0.30
10	— .311	—14.8	—10.09	156.7	+2.97	+27.0	+2.44	—1.0	—0.90
4	— .278	— 1.7	— 1.00	144.2	—2.21	+16.2	—2.17	+2.5	+1.25
4	— .153	+24.4	+ 4.82	147.8	—2.51	+20.8	—1.94	0.0	—0.81
4	— .158	+26.1	+ 0.10	167.0	+0.57	+36.2	+1.84	0.0	+0.09
4	— .209	+20.3	+ 0.39	180.8	+4.27	+40.5	+2.88	0.0	+0.42

Intervals in days.	Observed clock rates.	Atmospheric pressure (B).		Temperature (ϑ).		Temperature difference (V).		Personal equation (P).	
	Q	Q^B	$(c_q-c_r)^B$	Q^ϑ	$(c_q-c_r)^\vartheta$	Q^V	$(c_q-c_r)^V$	Q^P	$(c_q-c_r)^P$
6	—0.243 s	+ 9 1	+ 0 49	171 0	—1.93	+27 7	—2 55	—1.7	—0.65
7	— 324	— 4 9	— 2.28	168 6	—0 63	+27.4	+0 05	0 0	+0 36
8	— 344	— 5 1	+ 2.15	175 0	+0 61	+29.6	+1 07	0 0	—0.09
11	— 379	—12 0	— 3 95	175 9	+0.29	+21.7	—0 88	0 0	+0.02
8	— .338	+ 1 8	+ 7 33	173 3	—0.56	+20 8	+0 12	0 0	+0.02
6	— .436	—30 0	— 8.03	173.5	+1.16	+19.8	+0.80	0 0	—0.13
12	— .365	— 2 1	+ 3 51	167.1	—1.73	+14.6	—1.81	+0.8	+0.23
3	— 288	+10.5	— 2 83	174 0	—0 84	+22.7	+0 02	0 0	—0 11
4	— .271	+13 9	+ 8.68	180 5	+3 45	+27 0	+2 10	0 0	—0.16
8	— .363	—12.2	—13 53	163 4	+0 22	+17.1	—0 13	0.0	+0 61
4	— 196	+28 2	+ 9.11	143.8	—2 04	+ 6.8	—1.10	—2.5	—1.05
5	— 209	+26 9	— 1.18	146.2	—0 55	+ 7 6	—0 56	0 0	+0.81
4	— .237	+16 4	+ 2.75	152.8	—0.70	+13.0	—0.04	0.0	—0.49
7	— .373	—13 5	— 1 14	160 7	+2 46	+17 4	+1.53	+1 4	+0.42
8	— 429	—35.0	— 5.03	149 5	—0 06	+10.0	—0.34	0 0	+0.09
11	— 333	—16 4	+ 6.18	131.4	—0.72	+ 2 2	—0 45	—0.9	—0.91
3	— .447	—37 3	— 2 68	124 7	—1 02	+ 0 3	+0 24	+3 3	+1 87
2	— .395	—35 4	—11 48	126.0	+1 47	0 0	—0.90	0.0	—1.31
6	— 193	+19.6	+12.15	121 2	—0 37	+ 2 2	+1.25	0 0	—0 01
12	— 147	+22.9	— 0 56	111.9	+0.15	+ 0 8	—1 68	0 0	+0.53
3	— .135	+ 8 1	+ 1 55	104 0	—0.34	+ 7.7	+2 82	—3 3	—0 70
10	— 243	—18 0	— 0 58	95.2	+1 54	+ 0 2	—3 65	0.0	—0.05
3	— .274	— 40.5	— 1 52	77 0	+0 02	+ 9.0	+3.96	+3.3	+0 96
7	— .265	—43 7	— 0 09	66.7	—3 31	— 1 4	—2 59	0.0	—0 85
3	— .309	—41.5	— 3 32	73.7	+0 29	— 2.7	+0.81	0.0	+0.29
4	— 235	—20 1	— 1.54	80.5	+1 48	— 2.5	—0.86	0 0	—0.07
5	— 067	+12 2	+ 5 15	78.2	+1 12	+ 0.8	+0.58	0.0	+0 03
7	— .008	+18 1	— 0.81	61.3	+0 89	+ 2.6	—0.26	0.0	—0.09
5	+ .060	+15.9	+ 3 28	39.6	—2 13	+ 3.2	+1.31	0 0	+0 34
12	— .101	—11 3	— 3.04	42.2	—2 16	— 5.0	—1.79	— 0.8	—0 81

small as compared with that depending on other causes which are difficult to account for. But the computation would not become much more difficult if we should assign different weights to the determinations.

I now use the following thesis for the interpolation with regard to the smallest sinuosity that is proved in § 3 of my previous paper viz.: the partial derivative of the total sinuosity I_S of a series of clock corrections with regard to one of the corrections S_q, is equal to twice the abrupt variation which occurs in the derivative of the third order of the interpolation curve near the abscissa q and the ordinate S_q. Hence according to § 4

$$\frac{\partial I_S}{\partial S_q} = 12\,(e_n - e_m).$$

I apply this relation to the interpolating curve after compensation determined by the corrections L, and I obtain

$$\frac{\partial I_L}{\partial L_q} = 12\,(E_n - E_m) = 12\,\Sigma_q\,,$$

in case that each part of that curve is represented by an equation:

$$L_t = L_q + G_q\,t + C_q\,t^2 + E_n\,t^3.$$

In accordance with the previous paper I have used capitals for the interpolation coefficients belonging to the corrections which are freed from errors. Also the meaning of Σ for this interpolation corresponds entirely with that of σ used for the interpolation without compensation.

For $\frac{\partial I_L}{\partial L_q}$ we may substitute $-\frac{\partial I_L}{\partial f_q}$, because $L_q + f_q$ is invariable, hence $\partial L_q = -\partial f_q$.

After the substitution of $\frac{\partial I_L}{\partial f_q} = -12\,\Sigma_q$, each relation $\lambda\frac{\partial I_L}{\partial f_q} + \frac{f_q}{\mu_q^2} = 0$ takes the form: $\Sigma_q = \frac{1}{12\lambda\mu^2} f_q$.

The first member of this relation depends on x, y, z, u and the errors f, and all these quantities occur in it in a linear form:

$$\Sigma_q = \sigma_q - x\overset{B}{\sigma}_q - y\overset{\vartheta}{\sigma}_q - z\overset{V}{\sigma}_q - u\overset{P}{\sigma}_q - \overset{f}{\sigma}_q\,.$$

If we use the approximate values obtained for x, y, z and u in the supposition $f = 0$, we can compute the expression:

$$\psi_q = \sigma_q - x\overset{B}{\sigma}_q - y\overset{\vartheta}{\sigma}_q - z\overset{V}{\sigma}_q - u\overset{P}{\sigma}_q\,.$$

I have made this computation by determining the differences between the successive coefficients of t^3 for the reduced clock rates

$$Q - xQ^B - yQ^S - zQ^V - uQ_P,$$

according to the new method of interpolation.

σ_q^f may be developed in the following way:

$$\sigma_q^f = \ldots + K_q^o f_o + K_q^p f_p + K_q f_q + K_q^r f_r + K_q^s f_s + \ldots$$

We have also $\sigma_q^f = \frac{1}{12} \frac{\partial I_f}{\partial f_q}$ where $I_f = \int_a^z \left(\frac{d^2 f}{dt^2}\right)^2 dt.$

Hence:

$$K_q^o = \frac{\partial \sigma_q^f}{\partial f_o} = \frac{1}{12} \frac{\partial^2 I_f}{\partial f_q \partial f_o} = \frac{\partial}{\partial f_q}\left(\frac{1}{12}\frac{\partial I_f}{\partial f_o}\right) = \frac{\partial \sigma_o^f}{\partial f_q} = K_o^q$$

therefore the coefficient of f_o in the expression for σ_q^f is equal to the coefficient of f_q in the expression for σ_o^f.

Let us consider the case that $f_q = 1$ and all the other values of $f = 0$ then $\sigma_q = K_q$, $\sigma_p = K_p^q$ or $= K_q^p$, $\sigma_r = K_r^q$ or $= K_q^r$, etc. The series of the quantities σf for that case gives us directly all the coefficients K which are required for the development of σ_q^f. In this manner an interpolation was made between the numbers..., 0, 0, 100, 0, 0 ... as many times as there were observations, each time moving the number 100 further over one interval between two successive determinations of the clock correction.

If this computation is properly arranged, it can be made in a very short time and the accuracy can be tested by the results themselves as the coefficients are derived twice independently of each other.

I shall now describe in detail how I have arranged the computation of those coefficients for a determination of the clock correction.

According to § 2 of my paper "On interpolation etc." of Dec. 10, 1902 we have the relations;

$$\sigma_q = e_n - e_m \qquad e_n = \frac{g_q + g_r - 2\,Q_n}{n^2} \qquad e_m = \frac{g_p + g_q - 2\,Q_m}{m^2}$$

which allow us, once the series of the g being found, to deduce the series of the σ from the series of the g and Q.

The series of the g is determined by the equations (C) of § 2 viz.

$$g_q = \frac{n\,Q_m + m\,Q_n}{m + n} + \frac{n\,(Q_m - g_p)}{2\,(m + n)} + \frac{m\,(Q_n - g_r)}{2\,(m + n)},$$

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which for the first and the last epochs a and z become:

$$g_a = Q_\mu + \frac{Q_\mu - g_b}{2} \qquad g_z = Q_\nu + \frac{Q_\nu - g_y}{2}$$

where μ and ν refer to the intervals at both ends.

In the example given here, also one of the latter relations occurs, i. e. that for the observations of January 20, 1903. The successive intervals with the values of Q belonging to them are here:

∞	3	3	8	3	6	10	. . . (l, m, n, o)
	0	$+ {}^{100}/_{3}$	$- {}^{100}/_{8}$	0	0	0	. . . (Q)

The line separates what precedes this observation and what follows.

As a first approximation for the quantities g I have adopted $\frac{n\, Q_m + m\, Q_n}{m + n}$, only three of these values differ from zero, namely:

$$g_p = \frac{l}{m+l} Q_m \quad ; \quad g_q = Q_m - Q_n \quad ; \quad g_r = \frac{o}{n+o} Q_n.$$

In my example: $Q_m = +\,33$; $Q_n = -\,12$; $l = 3$, $m = 3$, $n = 8$, $o = 3$; hence $g_p = +\,17$; $g_q = +\,21$; $g_r = -\,3$. With these three values begins the annexed scheme, in which the successive corrections of the g are computed. (see p. 255 sq.)

Annexed to this paper is a table (II) (p. 256 sq.) containing the coefficients $100\,K$ for this computation; under the heading "Sum of the squares...." have been given the sums of the squares of the coefficients for each date, and to the first and second approximations the values of 100 ψ which belong to the observations.

Each equation of errors is reduced to the form:

$$\ldots . K_q^o f_o + K_q^p f_p + (K_q + \mathfrak{K}) f_q + K_q^r f_r + K_q^s f_s + \ldots . = \psi_q$$

where $\mathfrak{K} = \frac{1}{12\,\lambda\,\mu^2}$, is constant for all equations, and must be determined in order to enable a solution.

In the relation:

$$\Sigma_q + \ldots + K_q^o f_o + K_q^p f_q + K_q f_q + K_q^r f_r + K_q^s f_s + \ldots = \psi_q$$

which is independent of the hypothesis of the probability of a group of clock corrections, the quantities f represent real errors of observation, of which the mean value is μ. The error of interpolation φ_q in the interpolation with regard to the least sinuosity, i. e. the difference between the real clock correction L_q and the interpolated correction for the same instant is $\frac{\Sigma_q}{K_q}$. From the expression of pro-

		0	+17	+ 21	— 3	0			1. (g. first approximation)
			0	+33	—12	0			2 (Q).
		+17	—12	+ 9	0	0			3. (subtract obliquely to the left)
	8	3	3	8	3	6			4. (intervals)
			0	— 16	+33	— 3			5. (subtract 2 from 1 obliquely to the right)
			—36	+ 27	0	0			6 intervals × numbers to the right above
divide by 2 and change sign			0	—128	+99	—18			7 intervals × numbers to the left below
			—36	—101	+99	—18			8 take the sum of 6 and 7
		12	22	22	18				9. take the sum of twice two successive intervals
first corrections	— 8	+ 3	+ 5	— 4	+ 1	0	0		10. divide 8 by the numbers to the left below and change sign
		8	3	3	8	3	6	10.	11. intervals
			— 8	+ 3	+ 5	— 4	+ 1		12 row 10 moved to the right by two places
			+ 15	—12	+ 8	0	0		13. repeat these operations beginning with the multiplications, until the last corrections are small enough
			— 24	+24	+15	—24	+10		
			— 9	+12	+23	—24	+10		
		12	22	22	18	32			
second corrections	— 2	+ 1	— 1	— 1	+ 1	0			
first corrections	— 8	+ 3	+ 5	— 4	+ 1	0			
to the first approximation	0	+ 17	+21	— 3	0	0			
true values of g	—10	+ 21	+25	— 8	+ 2	0			

TABLE II.

	Coefficients for the compensation.									Sum of the squares of the preceding numbers for each date.	100 ψ in first approximation.	100 ψ in second approximation.
			100 K_q^o	100 K_q^p	100 K_q	100 K_q^r	100 K_q^s					
1903	+	—	+	—	+	—	+	—	+			
Jan. 14					1.0	2 1	1.2	0.2	0.1	6.8	— 30	— 20
» 17				2.1	5.1	3.5	0.9	0.4	0.0	43.1	+ 27	+ 4
» 20			1.2	3.5	2.9	1.3	0.8	0.1	0.0	24.2	+ 30	+ 43
» 28		0.1	0.9	1.3	2.5	2.4	0.6	0.2	0.1	14.6	— 80	— 79
» 31	0.1	0.4	0.8	2.4	2 8	1.0	0.4	0.2	0.0	15.7	+ 59	+ 57
Febr. 6	0.0	0.1	0.6	1.0	0.7	0.6	0.4	0 0	0.0	2.4	— 1	— 3
» 16	0.0	0.2	0.4	0.6	1.9	1.9	0.5	0 2	0.1	8.0	— 9	+ 2
» 19	0.1	0.2	0.4	1.9	2.3	0.9	0.4	0.3	0.1	10.3	— 15	— 25
» 26	0.0	0.0	0.5	0.9	0.7	0.8	0.6	0.1	0.0	2.6	+ 61	+ 57
Mrch. 6	0.0	0.2	0.4	0.8	2 8	3.8	1.8	0.3	0.1	26.9	— 301	— 320
» 9	0.1	0.3	0.6	3.8	7.2	5 1	1.5	0.3	0.1	95.7	+ 543	+ 580
» 12	0.1	0.2	1.8	5.1	5.6	2.7	0.8	0 2	0.0	68.3	— 380	— 405
» 16	0.0	0.3	1.5	2.7	2.3	1.2	0.5	0.1	0.0	16.8	+ 121	+ 122
» 22	0.0	0.2	0 8	1.2	1.5	1.0	0.3	0.1	0.0	5.5	— 48	— 40
» 27	0.1	0.2	0.5	1.0	1.2	0.6	0.2	0.0	0.0	3.2	+ 21	+ 14
Apr. 3	0.0	0.1	0.3	0.6	0.7	0.4	0.1	0.0	0.0	1.2	+ 14	+ 14
» 10	0.0	0.1	0.2	0.4	0.4	0.3	0.1	0.0	0.0	0.5	— 14	— 12
» 20	0.0	0.0	0.1	0.3	0.4	0.3	0.1	0.0	0.0	0.4	+ 9	+ 8
» 27	0.0	0.0	0.1	0.4	0.4	0.2	0.2	0.1	0.0	0.4	— 21	— 17
May 7	0.0	0.0	0 1	0.2	0 2	0.3	0.3	0.1	0.0	0.3	+ 20	+ 14
» 19	0.0	0.0	0.1	0.3	2.1	3.2	1.6	0.2	0.1	17.4	— 3	+ 4
» 22	0.0	0.1	0.3	3.2	6.6	4.5	1.2	0.3	0.1	76.2	+ 8	— 6
» 25	0.0	0.1	1.6	4.5	4.4	2.1	0.8	0.2	0.0	47.6	— 42	— 27
» 30	0.0	0.2	1.2	2.1	2.1	1.6	0.6	0.1	0.0	13.2	+ 70	+ 69
June 4	0.1	0.3	0.8	1.6	2.3	1.5	0.4	0.2	0.1	11.2	— 45	— 60
» 8	0.1	0.2	0.6	1.5	1.3	0.6	0.4	0.1	0.0	4.8	— 5	+ 7
» 18	0.0	0.1	0.4	0.6	1.3	1.7	0.9	0.2	0.0	6.3	+ 51	+ 46
» 22	0.0	0.2	0.4	1.7	3.3	2.7	1.1	0.3	0.0	22.4	— 82	— 80
» 26	0.1	0.1	0.9	2.7	3.7	2.7	1.0	0.2	0.0	30.0	+ 41	+ 47
» 30	0.0	0.2	1.1	2.7	3.5	2 2	0.6	0.1	0.0	26.0	+ 55	+ 48
July 4	0.0	0.3	1.0	2.2	2.1	1.0	0.3	0.1	0.0	11.4	— 101	— 102

	Coefficients for the compensation.									Sum of the squares of the preceding numbers for each date.	100 ψ in first approximation.	100 ψ in second approximation.
			100 K_q^o	100 K_q^p	100 K_q	100 K_q^r	100 K_q^s					
1903-04	+	—	+	—	+	—	+	—	+			
July 10	0.0	0.2	0.6	1.0	0.9	0.5	0.2	0.0	0.0	2.5	+ 63	+ 67
» 17	0.0	0.1	0.3	0.5	0.6	0.3	0.1	0.0	0.0	0.8	— 4	— 5
» 25	0.0	0.1	0.2	0.3	0.3	0.2	0.1	0.0	0 0	0.3	— 28	— 30
Aug. 5	0.0	0.0	0.1	0.2	0.3	0.4	0.2	0.0	0.0	0.3	+ 50	+ 51
» 13	0.0	0.0	0 1	0 3	0.6	0.5	0.2	0.2	0.0	0.8	— 91	— 89
» 19	0.0	0.0	0.2	0.5	0 5	0.4	0.4	0.0	0.0	0.8	+ 83	+ 80
» 31	0.0	0.0	0.2	0.4	1.9	2.6	1.0	0.2	0 1	11.6	— 64	— 58
Sept. 3	0.0	0.2	0.3	2.6	4.2	2.2	0.6	0.3	0.1	29.6	+ 35	+ 26
» 7	0.0	0.0	1.0	2.2	1.7	0.9	0.5	0.1	0 0	9.8	— 6	0
» 15	0 0	0.2	0.6	0.9	1.6	1.6	0.7	0.2	0.0	7.0	+ 27	+ 25
» 19	0.1	0.3	0.5	1.6	2.5	1.8	0.7	0.1	0.0	13.0	— 2	— 2
» 24	0.1	0.1	0.7	1.8	2.5	1.7	0.4	0.1	0.0	13.1	— 58	— 56
» 28	0.0	0.2	0.7	1.7	1.7	0.7	0.2	0.1	0.1	6.9	+ 63	+ 61
Oct. 5	0.0	0 1	0 4	0.7	0.6	0.3	0.2	0.2	0.0	1.2	— 25	— 24
» 13	0.0	0.1	0.2	0.4	0.4	0.5	0.4	0.1	0.0	0.7	— 16	— 17
» 24	0.0	0.1	0.2	0.5	2.5	4.5	2.5	0.1	0.0	32.8	+ 249	+ 254
» 27	0.1	0.2	0.4	4.5	11.6	8.3	1.0	0.2	0.2	225.7	— 619	— 629
» 29	0.0	0.1	2.5	8.3	7.1	1.4	0.4	0.2	0.0	128.8	+ 448	+ 455
Nov. 4	0.0	0.1	1.1	1.4	0.6	0.5	0.3	0.0	0.0	3.8	— 60	— 64
» 16	0.0	0.2	0.4	0.5	1.5	1.5	0.5	0.3	0.0	5.2	— 8	+ 10
» 19	0.1	0.2	0.3	1.5	1.7	0.9	0.6	0.1	0.0	6.2	+ 6	— 17
» 29	0.0	0.0	0.5	0.9	2.0	2.0	0.8	0.4	0.1	9.9	+ 70	+ 97
Dec. 2	0.0	0.3	0.6	2.0	2.5	1.6	0.9	0.2	0.0	14.4	— 123	— 142
» 9	0.0	0.1	0.8	1.6	3.1	3.4	1.2	0.2	0.0	25.7	+ 146	+ 150
» 12	0.0	0.4	1.0	3.4	4 9	2.8	0.8	0.2	0.0	44.9	— 54	— 65
» 16	0.1	0.2	1.2	2.8	2.7	1.4	0.4	0.1	0.0	19.0	— 95	— 90
» 21	0.0	0.2	0.8	1.4	1.3	0.8	0.3	0.0		4.9	+ 101	+ 101
» 28	0.0	0.2	0.4	0.8	1.0	0.6	0.1			2.3	— 96	— 84
Jan. 2	0.0	0.1	0.3	0.6	0.5	0.1				0.8	+ 64	+ 55
» 14	0.0	0.0	0.1	0.1	0.0					0.0	— 9	— 12

bability $e^{-\lambda I_L}$ we may deduce the mean value of φ_q. Let us suppose that all the real clock corrections remain unaltered with the exception of that for the instant q. Then I_L will be a minimum for $L_q - \varphi_q$; and for any other clock correction adopted for the same instant, which differs from this interpolated value by ε, I_L is equal to its least value augmented by $6\, K_q\, \varepsilon^2$.

Hence the probability of the group of clock corrections contains the factor $e^{-6\lambda K_q \varepsilon^2}$, whence follows that the mean value of ε, or also that of φ_q is equal to $\frac{1}{\sqrt{12\lambda K_q}}$. Then the mean value of $\varphi_q \sqrt{K_q}$ is constant and equal to $\frac{1}{\sqrt{12\lambda}}$; this constant is called ν.

Hence the 2nd power of the mean value of the first member of the above relation is:

$$K_q \nu^2 + \mu^2 [\ldots K_q^{o^2} + K_q^{p^2} + K_q^2 + K_q^{r^2} + K_q^{s^2} + \ldots].$$

The 2nd member ψ_q computed with approximative values of x, y, z and u is known. I had no direct data for the determination of μ^2. It consists of one part which is independent of the number of stars observed for the clock correction, and another part which is in inverse proportion to this number. For one star observed by BAKHUYZEN, the latter part amounts to about 900 and for PANNEKOEK it is a little less. I wished to avoid a too large value for μ^2 for fear of exaggerating the regularity of the clock at the cost of the accuracy of the observations, and therefore I have put for each of those parts of μ^2 300, together $\mu^2 = 600$.

According to the value given above for $\mathfrak{K}$ we have $\nu^2 : \mu^2 = \mathfrak{K}$ and the mean value of the expression:

$$\psi_q : \sqrt{K_q \mathfrak{K} + \ldots K_q^{o^2} + K_q^{p^2} + K_q^2 + K_q^{r^2} + K_q^{s^2} + \ldots}$$

will be equal to μ.

With different suppositions for $\mathfrak{K}$, I could derive from this relation the corresponding value μ^2. My result was that for $\mathfrak{K} = {}^1/_{60}$ the value of μ^2 is 592 and therefore I have retained this value of $\mathfrak{K}$ in the further computation.

The hypothetical expression of the probability was tested in two different ways.

In the first place I investigated whether indeed ν^2 might be regarded as equal for the different intervals of the period treated. Therefore I have arranged the observations according to their coefficients K_q, and have derived from the half with least K_q separately the value of μ^2 belonging to $\mathfrak{K} = {}^1/_{60}$. The result was $\mu^2 = 591$.

In the second place I have investigated whether also for intervals of longer duration, the constancy of v^2 remains the same. In that case I could derive v^2 from the total sinuosity I_L of the real clock corrections.

If we imagine that from the series of those clock corrections one is dropped, which deviates from the interpolated value by φ_q, then I_L is diminished by 6 $K_q \varphi_q^2$. 59 times we can drop successively one observation from the 61 observations at our disposal, and thus each time diminish the total sinuosity of the remaining clock corrections by 6 $K \varphi^2$ At the end I_L is zero and hence I_L may be considered as consisting of 59 parts, each of a mean value of 6 v^2. Hence the mean value of I_L is 354 v^2.

From the reduced rates $Q—x\,Q^B—y\,Q^S—z\,Q^V—u\,Q^P$ I deduced the total sinuosity of the clock corrections $L+f$ and obtained $I_{L+f}=8756$.

In my previous paper on this interpolation it is demonstrated that if the errors f and the clock corrections expressed by I are independent of each other $I_{L+f}=I_L+I_f$. Instead of the formula derived there:

$I_f = \Sigma 6 \varepsilon_n (f_q - f_r)$ I now write $I_f = \Sigma 6 f_q (\varepsilon_n - \varepsilon_m) = \Sigma 6 f_q \sigma_q^f$

and substitute for σ_q^f its value expressed in terms of the errors f.

In this way we get:

$$I_f = \Sigma\, 6 f_q \,[\ldots K_q^o f_o + K_q^p f_p + K_q f_q + K_q^r f_r + K_r^s f_s \ldots]$$

In the 2nd member occur under Σ many products of real errors. The mean value of these terms is zero. I omit them, substitute μ^2 for the square of each error and find as mean value for I_f 6 μ^2 Σ K_q.

The computation of Σ K_q yielded 1.39.

In this way I have found the following relation between v and μ:

$$8756 = 354\, v^2 + 8{,}34\, \mu^2$$

whence, if μ^2 is equal to 600, $v^2 = 11$, which result is in good harmony with the value first found for $\mathfrak{K}$.

The set of equations by which the most probable errors are connected is readily solved, if we use as a first approximation $f_q = 0.45 \dfrac{\psi_q}{K_q + \mathfrak{K}}$. After I had found by the substitution of these values that the 2nd member required still another correction $\Delta_1\, \psi$, I used $0.80 \dfrac{\Delta_1 \psi_q}{K_q + \mathfrak{K}}$ as a correction for the first approximation.

When computed according to the empirical formula

$$f_q = \frac{0.45\, \psi_q + 0.80\, \Delta_1\, \psi_q}{K_q + \mathfrak{K}}$$

the errors appeared to satisfy fairly well the set of equations. Where it was necessary I took away the last differences by adding $\frac{\Delta_2 \psi_q}{K_q + \mathfrak{K}}$ to the errors.

Table III contains the results of the compensation which are necessary to compute the clock correction at any instant between the observations. Let this instant be $q + T$, the epoch of the immediately preceding clock correction being q; if moreover during the interval from q to $q + T$:

the mean atmospheric pressure, expressed in mms. of mercury of 0° C. is $760 + B_T$,
the mean temperature in centigrades $10° \text{C} + \vartheta_T$,
the mean difference in temperature in centigrades V_T,
then we can compute the clock correction S_{q+T} according to the following formula:

$$S_{q+T} = S_q - f_q + \frac{T}{1000}(G_q + 15{,}4 B_T - 26{,}4 \vartheta_T + 318\, V_T + C_q T + E_n T^2).$$

The values of S, f, G, C, E, occurring in this formula can be derived from table III.

The 5[th] column shows the mean rates for each interval between two successive determinations of the clock correction after the reduction and the compensation. From this we may judge of the constancy of the rate of the clock. It must be remarked that the small yearly inequality occurring in these values is very probably due to a little inaccuracy in the coefficient of temperature obtained as described above.

The last column of table III shows the quantities Σ, the differences between the successive values of E. They give us a simple test for the computation of the compensation, because they must be equal to the errors f multiplied by 20, or $f_q = \Sigma_q : 20$. The adopted series of errors satisfies tolerably well this relation, if we admit small differences, which in thousandth parts of a second of time do not exceed the intervals m or n expressed in days, and hence give rise to a difference less than $0^s{,}001$ in the mean daily rates.

TABLE III.

Date	Observers	Clock corrections reduced to B S	Error according to the compensation f	Compensated daily rates Q_L	G	C	E	Σ
1903					—			
Jan. 14	P	— $3^m\ 28^s.222$	—$0^s.008$		171	0 0		— 0.14
				—$0^s.173$			— 0.14	
„ 17	B	27.561	+ 004		176	— 1.3		+ 0.12
				— .180			— 0.02	
„ 20	P	27 153	+ .010		184	— 1.5		+ 0.14
				— 188			+ 0 12	
„ 28	P	26 864	— .014		184	+ 1.5		— 0.21
				— .180			— 0 09	
„ 31	B	26.744	+ .006		177	+ 0.7		+ 0.04
				— .175			— 0.05	
Febr. 6	P	26.921	+ 004		175	— 0 2		+ 0.07
				— .174			+ 0 02	
„ 16	B	26 710	— 004		171	+ 0 5		— 0.08
				— .170			— 0 06	
„ 19	P	26 189	— 001		170	0 0		+ 0.02
				— .171			— 0 04	
„ 26	B	26.690	+ 005		174	— 0 8		+ 0.12
				— .176			+ 0.08	
Mrch. 6	B	28.568	— .029		173	+ 1 0		— 0.50
				— 174			— 0.42	
„ 9	P	28 539	+ .038		179	— 2 8		+ 0.80
				— 184			+ 0 38	
„ 12	B	28 757	— 021		186	+ 0 6		— 0.36
				— 183			+ 0 02	
„ 16	B	29 225	.000		180	+ 0 9		— 0.02
				— 174			0 00	
„ 22	B	29 475	— 004		168	+ 0 9		— 0 09
				— .166			— 0 09	
„ 27	P	30 449	+ 002		166	— 0 5		+ 0.05
				— .172			— 0 04	
Apr 3	B	31 453	+ 007		180	— 1 4		+ 0.12
				— .186			+ 0.08	
„ 10	B	32.506	— 004		189	+ 0 2		— 0.07
				— .186			+ 0.01	
„ 20	B	33.565	+ 001		183	+ 0 4		+ 0.02
				— 178			+ 0 03	
„ 27	P	35.575	— .007		172	+ 1.1		— 0.12
				— 169			— 0 09	
May 7	B	38.051	+ .007		175	— 1.6		+ 0.13
				— .189			+ 0.04	
„ 19	P	40 326	+ 002		197	— 0 3		— 0.01
				— 198			+ 0 03	
„ 22	B	40.667	— 004		198	0 0		— 0 10
				— 199			— 0.07	
„ 25	P	40 794	— 002		200	— 0 6		+ 0 02
				— 205			— 0 05	
„ 30	P	41 746	+ 014		211	— 1 4		+ 0.24
				— .213			+ 0.18	
June 4	P	42.872	— .010		210	+ 1.4		— 0.21
				— .205			— 0.02	
„ 8	P	43.427	— .002		200	+ 1 2		— 0.04
				— .194			— 0.06	
„ 18	B	46.570	+ 008		194	— 0.5		+ 0.17
				— .194			+ 0 11	
„ 22	P	47.654	— .012		193	+ 0.8		— 0.25
				— 192			- 0 14	
„ 26	P	48.265	+ .009		194	— 0.9		+ 0.20
				— .196			+ 0.06	
„ 30	P	48.896	+ .006		198	— 0.2		+ 0.09
				— .196			+ 0.15	

Date	Observers	Clock corrections reduced to B S	Error according to the compensation f	Compensated daily rates Q_L	G	C	E	Σ
1903-04					—			
July 4	P	— 3ᵐ 49ˢ.732	— 0ˢ.020	— 0ˢ.190	192	+ 1.6	— 0.22	— 0.37
„ 10	B	51 220	+ .016	— .209	196	— 2.3	+ 0.07	+ 0.29
„ 17	B	53.488	+ .002	— .219	219	— 0.9	+ 0.12	+ 0.05
„ 25	B	56.236	— .011	— .204	212	+ 1.9	— 0.11	— 0.23
Aug. 5	B	— 4ᵐ 0.408	+ .015	— .212	210	— 1.8	+ 0 19	+ 0.30
„ 13	B	3.115	— .026	— .198	202	+ 2 7	— 0.32	— 0.51
„ 19	B	5.734	+ .024	— .221	206	— 3.1	+ 0.15	+ 0.47
„ 31	P	10.082	— .012	— .209	215	+ 2.3	— 0.10	— 0.25
Sept. 3	P	10.946	— .001	— .200	204	+ 1.4	— 0.12	— 0.02
„ 7	P	12.029	+ .005	— .203	198	— 0.1	— 0.07	+ 0.05
„ 15	P	14.936	+ .008	— .216	212	— 1.7	+ 0.18	+ 0.25
„ 19	B	15.752	— .003	— .214	217	+ 0.4	+ 0.05	— 0.13
„ 24	B	16.795	— .010	— .208	210	+ 1.1	— 0.16	— 0.21
„ 28	B	17.744	+ .013	— .209	209	— 0.8	+ 0.11	+ 0.27
Oct. 5	P	20.328	— .009	— .196	204	+ 1.5	— 0 06	— 0.17
„ 13	P	23.762	+ .001	— .198	192	+ 0.1	— 0.06	0.00
„ 24	B	27.460	+ .020	— .214	212	— 1.9	+ 0.34	+ 0.40
„ 27	P	28.771	— .024	— .212	213	+ 1.2	— 0.35	— 0.69
„ 29	P	29.561	+ .021	— .215	212	— 0.9	+ 0.07	+ 0.42
Nov. 4	P	30.720	— .004	— .207	215	+ 0.4	+ 0.02	— 0.05
„ 16	P	32.482	— .004	— .192	195	+ 1.3	— 0.11	— 0.13
„ 19	B	32 918	+ .002	— .188	190	+ 0.3	— 0.02	+ 0.09
„ 29	B	35.348	+ .011	— .187	187	— 0.2	+ 0.14	+ 0.16
Dec. 2	P	36.141	— .020	— .186	185	+ 1.1	— 0.16	— 0.30
„ 9	P	37.998	+ .026	— .199	195	— 2.3	+ 0.27	+ 0.43
„ 12	P	38 926	— .005	— .197	201	+ 0.1	+ 0.22	— 0.05
„ 16	P	39.866	— .021	— .182	190	+ 2.8	— 0.27	— 0.49
„ 21	P	40.201	+ .019	— .183	181	— 1.2	+ 0.12	+ 0.39
„ 28	P	40.254	— .017	— .178	179	+ 1.3	— 0.20	— 0.32
Jan 2	P	39.953	+ .015	— .195	182	— 1.7	+ 0.05	+ 0.25
„ 14	B	41.200	— .005		201	0.0		— 0.05

(October 20, 1904).