The neutral ester is saponified by methyl alcohol and then yields an acid one like all sulphonic esters. With ammonia it yields an ammonium salt of the sulphonic ester function, which is also an ester of the carboxylic acid.

The acid ester, namely the carboxylic ester of the sulphonic acid, was also obtained from the sodium salt of sulpho-isobutyric acid by means of hydrogen chloride and methyl alcohol and is hygroscopic. Its isomer, the carboxylic acid of the sulphonic ester, which was prepared from the acid silver salt with methyl iodide, is not hygroscopic, it crystallises from benzene and melts at 90°. Dr. Moll van Charante's experiences with the esters of sulpho-isobutyric acid agree fairly well with those of Wegscheider with metasulphobenzoic acid.

The melting points of the compounds obtained behave as might be expected; those of the sulphonic acid chlorides are more elevated than those of the sulphonic esters; those of the carboxylic chlorides are lower than those of the carboxylic esters. The melting points of the esters as well as those of the chlorides of the carboxylic acids are lower than those of the carboxylic acids themselves.

Mathematics. — "*The relation between the radius of curvature of a twisted curve in a point P of the curve and the radius of curvature in P of the section of its developable with its osculating plane in point P.*" By W. A. Versluys. (Communicated by Prof. P. H. Schoute).

(Communicated in the meeting of September 24, 1904.)

§ 1. Theorem. *For each twisted cubic C^3 the ratio is constant of the radius of curvature in any point P to the radius of curvature of the section of the osculating plane in the point P with the developable O_4 belonging to C^3.*

Proof. If we take P to be origin of coordinates and the tangent, principal normal and binormal of the curve C^3 in the point P to be the axes of coordinates, then C^3 is the cuspidal curve of the surface O_4 enveloped by the plane

$$A t^3 - 3 B t^2 + 3 C t - D = 0,$$

where

$$\begin{aligned} D &= z, \\ C &= c_2 y, \\ B &= b_1 x + b_2 y + b_3 z, \\ A &= a_1 x + a_2 y + a_3 z + a_4. \end{aligned}$$

The coordinates of the points of the curve C^3 satisfy the conditions:

$$t\,C = z, \qquad t^2\,B = z, \qquad t^3\,A = z,$$

whence

$$z = \frac{a_4\,b_1\,c_2\,t^3}{b_1\,c_2 - a_1\,c_1\,t + (a_1\,b_2 - a_2\,b_1)\,t^2 - a_3\,b_1\,c_2\,t^3} = \frac{a_4\,b_1\,c_2\,t^3}{N},$$

$$y = \frac{a_4\,b_1\,t^2}{N}, \qquad x = \frac{a_4\,t\,(c_2 - b_2\,t - b_3\,c_2\,t^2)}{N}.$$

Now the radius of curvature R_0 of the twisted curve C^3 in the point P is the same as the radius of curvature of its orthogonal projection on its osculating plane in P, the curve with its projection in P having three consecutive points in common. The parameter expressions for the coordinates of this projection are

$$y = \frac{a_4\,b_1\,t^2}{N} \quad \text{and} \quad x = \frac{a_4\,t\,(c_2 - b_2\,t - b_3\,c_2\,t^2)}{N}$$

From the value of y we find $\dfrac{dy}{dt} = 0$ for $t = 0$; so for the general formula

$$R = \frac{(dx^2 + dy^2)^{3/2}}{dx\,d^2y - dy\,d^2x},$$

giving the radius of curvature of a plane curve, can be substituted the simpler expression :

$$R_0 = \frac{dx^2}{d^2y_{t=0}} = \frac{\left\{\dfrac{b_1\,c_2 \times a_4\,c_2}{(b_1\,c_2)^2}\right\}^2}{\dfrac{2\,a_4\,b_1}{b_1\,c_2}} = \frac{a_4\,c_2}{2\,b_1^2}.$$

The equation of the surface O_4 enveloped by the plane

$$A\,t^3 - 3\,B\,t^2 + 3\,C\,t - D = 0$$

is

$$A^2\,D^2 - 6\,A\,B\,C\,D + 4\,A\,C^3 + 4\,B^3\,D - 3\,B^2\,C^2 = 0.$$

The curve of intersection with the osculating plane $D = z = 0$ is:

$$C^2\,(4\,A\,C - 3\,B^2) = 0.$$

So the equation of the conic d_2 lying in the osculating plane is:

$$4\,(a_1\,x + a_2\,y + a_4)\,c_2\,y - 3\,(b_1\,x + b_2\,y)^2 = 0.$$

The equation of the parabola osculating this conic d_2 in the origin is:

$$4\,a_4\,c_2\,y - 3\,b_1^2\,x^2 = 0.$$

This parabola has in the origin the same radius of curvature r_0 as the conic d_2. The radius of curvature in the vertex of the parabola is the parameter. So the radius of curvature r_0 of the conic d_2 in the origin is $\frac{2\,a_4\,c_2}{3\,b_1^{\,2}}$.

From the values:

$$R_0 = \frac{a_4\,c_2}{2\,b_1^{\,2}} \text{ and } r_0 = \frac{2\,a_4\,c_2}{3\,b_1^{\,2}}$$

now follows:

$$R_0 : r_0 = 3 : 4. \qquad Q.\,E.\,D.$$

§ 2. The theorem can be easily expanded to a general twisted curve C.

Let P be an ordinary point of C, the tangent and the osculating plane in P showing no particularities. Through point P and five consecutive points of C a twisted cubic C^3 can always be laid. The radius of curvature R_0 in the point P is the same for the curves C and C^3, having six consecutive points in common. The osculating planes of the curves C and C^3 in the point P will coincide too. This common osculating plane O intersects the developables belonging to C and C^3 according to the tangent in P counting double and moreover according to two plane curves d and d_2.

If the curves C and C^3 had but a three-point contact in P, the curves d and d_2 would have a common tangent in the common point P, so that the curves d and d_2 would have in P at least two consecutive points in common. If the curves C and C^3 were to have a five-point contact, a common generatrix of the two developables not lying in the common osculating plane O would meet the osculating plane O in a third common point of the curves d and d_2. Now that the curves C and C^3 have a six-point contact in P the curves d and d_2 will have at least four consecutive points in common. These two sections d and d_2 have thus in P the same radius of curvature r_0. Consequently in the ordinary point P of the twisted curve C we have:

$$R_0 : r_0 = 3 : 4.$$

§ 3. When two arbitrary twisted curves have in a point P a three-point contact, they have in that point the same radius of curvature R. If now the common osculating plane O in P of the two curves cuts the two developables belonging to the curves in the plane curves d and d' then the radii of curvature in P of these sections d and d' are both $\frac{4}{3}R$ and therefore equal. The curves d and d' have thus in P also a three-point contact. From this follows the theorem:

If two twisted curves have in P three consecutive points in common this will be also the case with the plane curves forming part of the sections of the common osculating plane with the developables belonging to the twisted curves.

The radius of curvature of the section d in the point P being four thirds of the radius of curvature of the cuspidal curve C in this same point, the curves d and C have in P but two points in common.

From the theorem proved here, follows once again the theorem communicated by me before, concerning the situation of the three points which a twisted curve has in common with its osculating plane. (see These Proc., Febr. 27th, 1904).

§ 4. By expansion of the coordinates of an arbitrary algebraic or transcendent twisted curve in the proximity of an ordinary point P into convergent power series of a parameter t, the theorem of § 1 can be proved also directly for such a twisted curve without using the twisted cubic.

Let P be an ordinary point of the curve C; if the tangent, the principal normal and the binormal in P are taken respectively as X-axis, Y-axis and Z-axis, then the coordinates of the twisted curve C become:

$$\begin{aligned} x &= a_1 t + a_2 t^2 + \ldots \ldots , \\ y &= b_2 t^2 + b_3 t^3 + \ldots \ldots , \\ z &= c_3 t^3 + c_4 t^4 + \ldots \ldots . \end{aligned}$$

The point P corresponds to the value zero of the parameter t. If P is an ordinary point the coefficients a_1, b_2 and c_3 cannot be zero. Let R_0 be the radius of curvature of C in point P, thus the value obtained by the radius of curvature R for $t=0$. The radius of curvature in P of the projection of C on the osculating plane $z=0$ is also R_0, this projection having in P three consecutive points in common with C.

The coordinates of the points of this projection are:

$$\begin{aligned} x &= a_1 t + a_2 t^2 + \ldots \ldots , \\ y &= b_2 t^2 + b_3 t^3 + \ldots \ldots . \end{aligned}$$

As $\dfrac{dy}{dt}$ is equal to 0 for $t=0$ the general formula for the radius of curvature

$$R = \frac{(dx^2+dy^2)^{3/2}}{dx\, d^2y - dy\, d^2x},$$

transforms itself into the simpler one

$$R_0 = \frac{dx^2}{d^2 y_{t=0}} .$$

It is easy to find

$$R_0 = \frac{a_1^2}{2b_1} .$$

The coordinates ξ, η and ζ of an arbitrary point Q on the developable belonging to C can be expressed in the parameters t and r where r represents the distance from point Q (ξ, η, ζ) to point (x, y, z) of the cuspidal curve measured along the tangent of C passing through Q. The coordinates of Q are:

$$\xi = x + r \frac{dt}{ds} \frac{dx}{dt} ,$$

$$\eta = y + r \frac{dt}{ds} \frac{dy}{dt} ,$$

$$\zeta = z + r \frac{dt}{ds} \frac{dz}{dt} .$$

For the points Q situated in the osculating plane $\zeta = 0$ the relation

$$0 = z + r \frac{dt}{ds} \frac{dz}{dt}$$

must exist between the parameters r and t. By eliminating r out of this relation and the equations for ξ and η we find expressed in functions of t the coordinates of the points Q situated in the plane $\zeta = 0$. These coordinates of the points of the curve of intersection d are

$$\xi = x - z \frac{dx}{dt} : \frac{dz}{dt} ,$$

$$\eta = y - z \frac{dy}{dt} : \frac{dz}{dt} ,$$

or

$$\xi = a_1 t + a_2 t^2 + \ldots - \frac{(c_3 t^3 + c_4 t^4 + \ldots)(a_1 + 2a_2 t + \ldots)}{3c_3 t^2 + 4c_4 t^3 + \ldots} = \frac{2}{3} a_1 t + \ldots ,$$

$$\eta = b_2 t^2 + b_3 t^3 + \ldots - \frac{(c_3 t^3 + c_4 t^4 + \ldots)(2b_2 t + 3b_3 t^2 + \ldots)}{3c_3 t^2 + 4c_4 t^3 + \ldots} = \frac{1}{3} b_2 t^2 + \ldots .$$

As here too $\frac{d\eta}{dt}$ is equal to 0 for $t = 0$ we find as above that the radius of curvature r_0 in point P of the curve d is:

$$r_0 = \frac{d\xi^2}{d^2 \eta} .$$

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This formula gives for r_0 the value:

$$r_0 = \frac{\frac{4}{9} a_1^2}{\frac{2}{3} b_2} = \frac{2 a_1^2}{3 b_2}.$$

From the obtained values $R_0 = \frac{a_1^2}{2 b_2}$ and $r_0 = \frac{2 a_1^2}{3 b_2}$ we get

$$R_0 : r_0 = 3 : 4.$$

Delft, Sept. 1904.

Physiology. — "*Degenerations in the central nervous system after removal of the flocculus cerebelli*". By Dr. L. J. J. Muskens. (Communicated by Prof. C. Winkler).

(Communicated in the meeting of September 24, 1904).

In 6 rabbits the flocculus of the right side was extirpated. This organ lies, as is well known, in these animals in a separate bony hole, so that we here have the possibility to remove a part of the cerebellum without disturbing the nervous structures of the neighbourhood in their conditions of nutrition as well as of pressure. The animals were killed after 8 days to 5 weeks and complete series stained after Marchi, were prepared.

The degenerations of fibres after this lesion in 4 of the 6 cases were found exclusively directed upward i. e. to the superior crus-cerebelli and to the pons.

In one case there was a fine degeneration all over the restiform body; in this case however it could not be made out with certainty whether we had to deal with really descending degeneration, because firstly all through the cord fine, black spots were found, and secondly the black spots were of so little dimensions, that there is much doubt about the genuineness of such a fine degeneration. In this animal the staining was insufficient, irregular and not limited to degenerated nerve-fibres, for an unknown reason, so that we do not think much value can be attached to this single case, in which downward degeneration was found.

In another wellstained case in the restiform body a number of degenerate fibres on the operated side was found; also in the longitudinal posterior fascicle and in the field of the tecto-spinal bundle,