Physiology. — *"On a new method of damping oscillatory deflections of a galvanometer"*. By Prof. W. EINTHOVEN.

(Communicated in the meeting of September 24, 1904).

In a number of investigations, requiring the use of a galvanometer or electrometer, it is desirable to damp the oscillatory deflections shown by most of these instruments under many circumstances. Either mechanical damping is applied or electromagnetic damping or both are combined in order to obtain a stronger effect.

In some instruments, e.g. the DEPREZ-D'ARSONVAL galvanometer, in which the coil is movable in a stationary magnetic field, the electromagnetic damping may without any special arrangement be so great that the deflections have lost their oscillatory character and have become quite dead-beat. The movements are thereby retarded. This retardation may be very considerable and so become troublesome, even to such an extent that the instrument becomes impracticable. Means of diminishing the damping are then applied, e.g. by increasing the resistance in the galvanometer.

In order to apply electromagnetic damping in a needle-galvanometer the rotating magnetic system is to a greater or less extent enveloped by a mass of pure copper in which during the motion of the needles damping vortex currents are raised.

Mechanical damping is applied as liquid or air damping, thin plates of aluminium or mica or insect wings being often used.

The method of damping to be described in this paper is entirely different from the methods just mentioned. It consists in inserting a condenser between the ends of the galvanometer wire as is indicated in fig. 1. In the figure E represents a source of current by means of which an arbitrary potential difference can be established between P and P_1. G is the galvanometer and C the condenser.

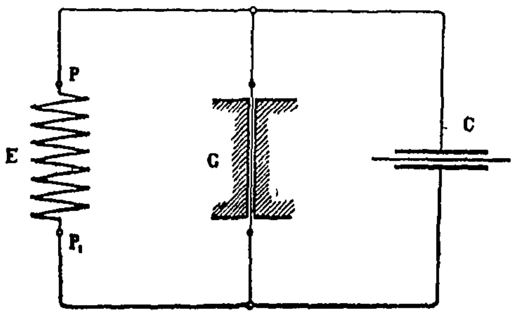

Fig. 1.

The action of the condenser is most easily understood by assuming the mass of the moving parts of the galvanometer to be zero and the eventual causes by which the motion is damped to tend to zero. If under these conditions the capacity of the condenser is zero, when a potential difference between P and P_1 is suddenly established, the galvanometer will also at once assume the corresponding position of equilibrium. If on the other hand there is a certain capacity, the deflection will require some time.

The way in which the image of the mirror or in the string-galvanometer the quartz thread then moves, is entirely determined by the way in which a condenser is charged or discharged. Calling a the deflection of the galvanometer at the time t after the potential difference is established and A the permanent deflection, we have

$$a = A\left(1 - e^{-\frac{t}{w'c}}\right)$$

where e is the base of natural logarithms, c the capacity of the condenser and w' a resistance of which it is easy to give a nearer definition.

In the closed circuit containing the source and the galvanometer the external resistance be W_e, the resistance of the galvanometer be W_i, then, if we neglect the resistance of the wires joining the condenser and the galvanometer, we have

$$w' = \frac{W_i\ W_e}{W_i + W_e} \quad \text{. . . . .} \quad (1)$$

The value $w'c$ is the time constant of the deflection

$$w'c = T.$$

Expressing w' in Ohms and c in Farads, T is given in seconds.

When the deflection of the galvanometer is recorded on a uniformly moving plane, a curve will be obtained which is the expression of an exponential function and which agrees entirely with the wellknown normal or standardising curves of the capillary electrometer[1]).

The constants of the curve, besides being determined by the rate of motion of the recording plane and the amplitude of the deflection, will depend only on the value of T. By changing w' and c we can regulate the value of T at will. This means that we are able to retard or damp the deflection of the galvanometer to any extent.

The reasoning given is confirmed by the observations. As an example we reproduce three curves, figs. 1—3 of the plate, recorded by the string-galvanometer[2]). The connections are schematically

1) See e. g. W. EINTHOVEN. PFLÜGER's Arch. f. d. gesammte Physiol. Bd. 56, p. 528. 1894. And „Onderzoekingen" Physiol. laborat. Leyden, 2nd series I.

2) See W. EINTHOVEN, Ann. der Phys. 12. p. 1059. 1903 and 14. p. 182. 1904. Also in Kon. Akad. v. Wetensch. te Amsterdam, Report of the meeting of June 27, 1903 and March 30, 1904.

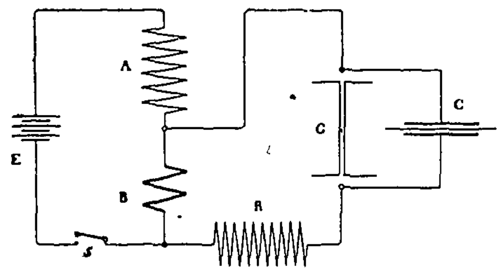

Fig. 2.

represented in fig. 2. Here E is a battery, S a key, G the string-galvanometer and C the condenser, A, B and R representing resistances. The sensitiveness of the galvanometer has been kept about equal in the three cases so that a deflection of 1 mm. corresponds to a current of 2×10^{-7} Amp., the electromotive force E of the battery and the resistances A, B and R being so chosen that when the current is passed a permanent deflection of 20 mm. is obtained. The rate of motion of the recording plane is 500 mm. per second. Hence in the net of square millimetres on the plates [1]) 1 mm. of absciss $= 0.002$ sec. and 1 mm. ordinate $= 2 \times 10^{-7}$ Amp. The circuit was automatically made and broken at S by an arrangement attached to the recording plane.

For R a carbon resistance was taken with large resistance and B was small compared with R. W_e could be put equal to R without an appreciable error. In figs. 1 and 2 of the plate W_e was 1.11 megohm, whereas W_e in fig. 3 amounted to 117000 ohms. The resistance of the galvanometer W_i was 8600 ohms.

In fig. 1 of the plate the capacity of the condenser is 0. The string is seen to make oscillatory movements with a period of about 1,3 mm. $= 2.6\,\sigma$ [2]).

These movements are damped by inserting a certain capacity in the condenser. In fig. 2 of the plate that capacity is 0.94 microfarad, in fig. 3 of the plate 0.2 microfarad.

Calculating the value of w' from W_i and W_e by formula (1) and then the time constant $T = w'c$, the time constant of fig. 2 is found to be $8.0\,\sigma$, and that of fig. 3 $1.6\,\sigma$ and it is clear that the amount of retardation or of damping is determined by the value of the time constant.

For clearness' sake we started in the above reasoning from the simplest case and assumed that the mass m of the string and the forces which independently of the condenser damp its motion and which we will collectively indicate by r, may be neglected. This hypothetical case will the more closely agree with reality, the larger, other conditions being equal, T is taken. Hence in this respect fig. 2

1) On the way of recording and the net of square millimetres see Annalen der Phys. l. c.

2) $1\,\sigma = 0.001$ sec.

of the plate answers better the conditions required than fig. 3, but the great practical importance of the method is exactly that it is possible to damp the oscillations and at the same time to retard the deflection as little as possible. When measurements are made one will always try to choose T such that exactly the limit between oscillatory and aperiodical motion is attained. In this case T is relatively small and m and r may no longer be neglected.

The question now arises how for known values of m and r the value of T must be calculated in order to obtain the limiting case mentioned.

In passing it be remembered that with the capillary electrometer the damping of the motion of the mercury meniscus is also composed of mechanical friction and of retardation by capacity.[1]) And from the combination of these two results a motion which can be expressed by a simple exponential function either quite accurately or with only small deviations. The resistance of air or liquid damping as well as electromagnetic damping influence the motion of a body having mass, in exactly the same way as conductive resistance influences the motion of electricity when a condenser is charged or discharged.

A simple reasoning will show, however, that adding a condenser to the galvanometer has not always an influence on the movements of the string of the same nature as an increase of the damping forces which we called r.

For the addition of the condenser has the effect of a temporary change of the active force. And the way in which the force is increased or decreased from moment to moment is not determined by the motion of the string, as the mechanical and electromagnetic damping, but by the product of the conductive resistance and the capacity $w' c = T$.

When applying the condenser method, the character of the motion of the string near the limiting case of aperiodicity can only be represented by a more or less complicated formula. I have therefore for this limiting case preferred direct experimental determination of the value of T to calculation.

Some curves have been reproduced which exemplify the motion

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1) Some investigators have been of opinion that the motion in the capillary electrometer is dependent on the charge of the mercury meniscus only. But in reality damping by mechanical friction is much more active here. See PFLÜGER's Arch. f. d. ges. Physiol. Bd 79, p. 1. 1900; and „Onderzoekingen" Physiol. laborat. Leyden, 2nd series, 4.

of the string in the limiting case in question[1]). Figs 4, 5 and 6 of the plate were taken with the same string as the former figures. For the connections we refer to figure 2 in the text. The deviation is now 30 mm. Again 1 mm. of absciss is 0.002 sec. and 1 mm. of ordinate $= 2 \times 10^{-7}$ Amp.

R = 1300 Ohms.
B = 27 „
W_i = 8600 „ , from which we calculate
W_e = 1327 and w' = 1148 Ohms.

In fig, 4 the capacity of the condenser = 0, hence $T = 0$.
„ „ 5 „ „ „ „ „ = 0,6$\mu f.$, „ $T = 0{,}69\ \sigma$.
„ „ 6 „ „ „ „ „ = 0,7$\mu f.$, „ $T = 0{,}80\ \sigma$.

One sees that the oscillatory motion, the period of which is about 2,7 σ, is damped by the application of the condenser method and that the time constants T of 0.69 and 0,80 σ, obtained by means of capacities of 0.6 and 0.7 microfarad, are required in order to reach the desired limit of aperiodicity.

In fig. 5, where a capacity of 0,6 $\mu f.$ is used, the limit has not yet fully been reached, in fig. 6 the limiting value has already been passed with a capacity of 0.7 $\mu f.$

The two last-mentioned plates show that the motion of the string in the neighbourhood of this limit is not very simple. In the small oscillation which has remained in fig. 5 the string, after having deflected through 30 mm., passes the new position of equilibrium by 0.5 mm. and then returns to a point which is still 0.3 mm. lower than the position of equilibrium. The ratio of the values of these deflections does not agree with the laws which damped motions generally obey. Moreover the first turning-point is reached after 2 σ, the second after 1 σ, whereas with damped vibrations, such as generally occur, these times are equal.

In fig. 6 the string comes to rest after about 0.002 sec. at a distance of 0.3 mm. from the new position of equilibrium and reaches its equilibrium after a small movement in the opposite direction. If in the measurement of a current one is contented with an accuracy of 2% the result is known in about 1,5 σ.

Another example is found in figs. 7 and 8 of the plate. These photograms were taken in the same way as those immediately

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1) The process by which the photograms of the plate have been reproduced does not reveal the minor details of the curves. I shall be pleased to send direct photographic copies of the original negatives to those who are interested in them.

preceding, but the string is lighter here, has a greater conductive resistance and is slightly more stretched.

1 mm. absciss = 0,002 sec., 1 mm. ordinate = 3×10^{-7} Amp. $W_i = 17800$, $W_e = 20000$, hence $w' = 9420$ Ohms.

In fig. 7 the capacity is 0; in fig. 8 it is 0,05 μf, hence $T = 0{,}47\ \sigma$. In this latter photogram the string shows a turning-point after about 1,1 σ exactly on the new position of equilibrium. It moves back through 0.9 mm. and then reaches its equilibrium again and finally.

If in the measurement of a current one is contented with an accuracy of $3^0/_0$, the result is obtained in 0.8 σ. If an accuracy of $0.3^0/_0$ is wanted, the result is only obtained in 2.2 σ.

These examples may suffice to see what can be expected of the method. It is obvious that when seeking the exact value of T for reaching the limit, we were led by theoretical considerations although we could not use a rigorous formula. One of these considerations was that for a given string and constant resistances, the capacity required for the limit must be the smaller the more strongly the string is stretched. For with greater tension of the string the period t of its oscillations becomes smaller and we may expect the wanted value of the time constant T to change in the same sense as the period t.

This consideration leads to some paradoxically sounding predictions. So, for example, it is to be expected that the motion of a strongly stretched string that has been made dead-beat by applying the condenser method, will become oscillatory again as soon as the tension is diminished and thereby the motion is retarded. Such an expectation seems at variance with the experience gained with other galvanometers, we might say, gained without exception with all instruments in which vibratory motions are observed.

The result, which was expected with some anxiety, completely confirmed the prediction. A quartz-thread of such a tension that a permanent deflection of 1 mm. corresponded to a current of 2×10^{-7} Amp., showed, when a current was suddenly passed or interrupted, (see figure 2 in the text) a number of oscillations. By inserting a capacity $c = 0.135\ \mu f$ the motion was damped to such an extent that the limit of aperiodicity was reached. Next the tension of the string was exactly 4 times relaxed so that a deflection of 1 mm. was caused by 5×10^{-8} Amp. The oscillations then re-appeared, and could not be checked again until the capacity was increased to 0.40 μf. With a 4 times smaller tension, i.e. with a 4 times greater sensitiveness the capacity and at the same time the value of T had to be increased 2.96 times in order to reach the limit of aperiodicity.

Observations with other quartz-threads, the tension of which was varied, always gave corresponding results: with strong tension a small value, with a more feebly stretched string a larger value of $w'c$ is required in order to check oscillations.

If w' is kept unchanged, one has an easy means of accurately regulating the desired degree of damping in a commercial condenser, in which capacities are shunted in by means of plugs in the same way as the resistances of a resistance-box. And it is remarkable that less of the means of damping is sufficient as the oscillations pass the zero-point farther and last longer and consequently the need of damping is greater. The phenomenon that, leaving the other circumstances unchanged, diminution of the tension only, i.e. — with the same deflection —, diminution of the moving force, changes an aperiodic motion into an oscillatory one, stands quite isolated and has, as far as is known to me, no mechanical or electrical analogon, no more in scientific instruments than in industry.

We shall now give some results of measurements which although they cannot compensate the lack of a simple formula, may yet be helpful to form an idea of the method in practical work.

1. When the damping influences already existing are increased, for example when the electromagnetic damping is strengthened by diminishing the resistance in the galvanometer circuit, a smaller value of T will suffice in order to reach the limit of aperiodicity when the quartz-thread is the same and the tension is not changed.

2. If the change in the electromagnetic damping which is caused by varying the value of W_e is taken into account, it makes no difference how the single factors w' and c are chosen. If only their product $w'c = T$ retains the same value, also the damping influence will remain the same. This latter is only determined by the product T.

3. If the motion of the quartz-thread is oscillatory it will be observed when the condenser method is applied, beginning with small values of T and gradually rising until the limit of aperiodicity is reached, that increasing T does not always cause a regular increase of the damping. Especially with feeble tension of the quartz-thread, when only a few small oscillations are normally produced, one sees an irregularity appear. The addition of a very small capacity can then even slightly enlarge the existing vibrations.

When such a value of T has once been taken that the limit of aperiodicity is reached, T has only little to be raised in order to obtain a regularly shaped curve. With further raising of T the motion is more and more retarded, the regular form of the curve being retained.

4. That we may form some opinion about the value of the time constant T that is required in various circumstances in order to reach the limit of aperiodicity, we give the following table, containing the results of measurements, part of which have already been mentioned above.

W_i in Ohms.	W_e in Ohms.	w' in Ohms.	c in micro-farads.	$T = w'c$ in thousandths of a second.	t in thousandths of a second.	k Damping-ratio.
8600	117000	8000	0.40	3.2	7.7	7.6
8600	117000	8000	0.135	1.08	2.7	3.1
8600	1.11×10^6	8520	0.12	1.02	2.64	3.1
8600	1327	1148	0.65	0.75	2.7	4.5
17800	20000	9420	0.05	0.47	1.41	3.16

The first five columns of this table need no nearer explanation; they give the conductive resistances, the capacities and the values of the time constant T. For the values of T mentioned the limit of aperiodicity was just reached.

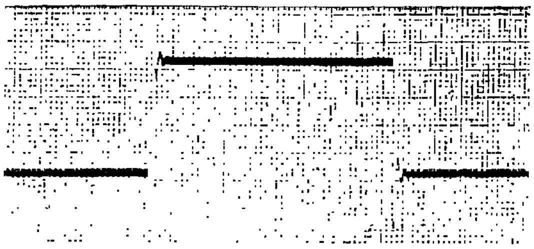
The two last columns indicate how the string vibrates when the capacity of the condenser and together with it T is zero. In the last column but one we find the period t expressed in thousandths of a second, while the last column gives the damping ratio k. The observations have been arranged according to the values of T.

Finally some remarks may find a place here on the circumstances under which the condenser method will be useful in practice. For the present the applications will presumably be restricted to such measuring instruments as possess a great internal resistance and a short period of oscillation. A galvanometer for thermo-electric currents with a small internal resistance and a great period of oscillation would for damping by the condenser method require an enormous capacity. The mica or paper condensers, which admit of easy regulation, would be out of the question here, since even the largest sizes of the trade would turn out to be still a hundred thousand times too small. So one would have to have recourse to another kind of condensers, e.g. electrolytic ones, and it would require a separate investigation how far these can indeed be rendered practicable for the purpose in view.

Absc. 1 m.M. = 2 σ, Ordin. 1 mm. = 2×10^{-7} Amp.

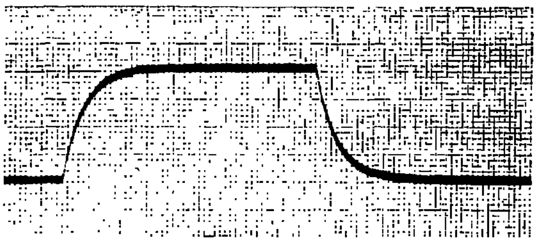
1

T=0.



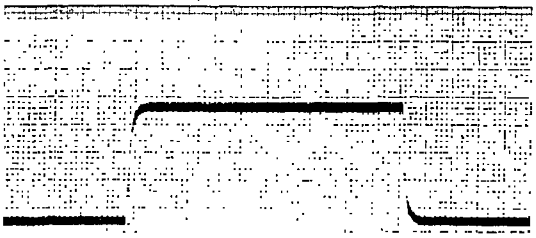
2

T=8.0 σ



3

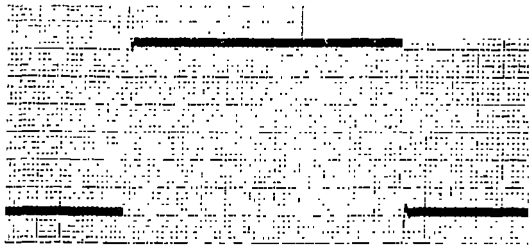
T=1.6 σ



Absc. 1 m.M. = 2 σ, Ordin. 1 mm. = 2×10^{-7} Amp.

4

T=0



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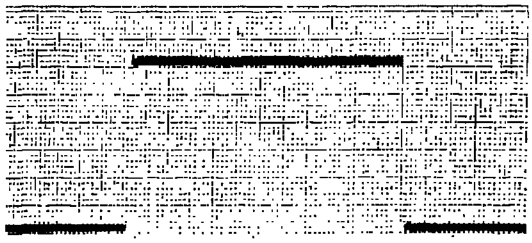
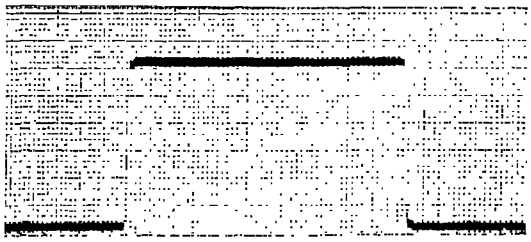
T= 0.69 σ



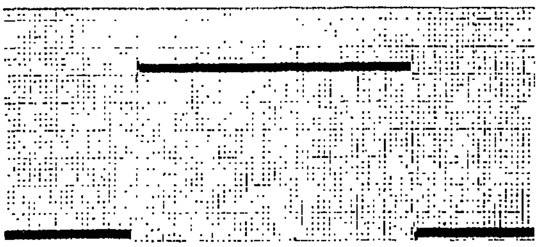
6

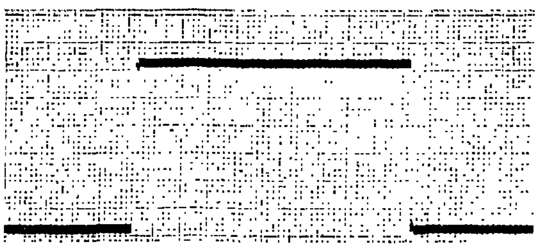
T 0.80 σ



7 Absc. 1 mm. = 2 σ, Ordin. 1 mm. = 3×10^{-7} Amp. **8**

T=0





T=0.47 σ

The conditions of a short period of oscillation combined with a relatively high internal resistance are fulfilled by only one instrument besides the string galvanometer, as far as is known to me, namely by the oscillograph. Here the damping is effected by means of oil which is heated [1]).

The temperature of the oil determines its viscosity and the regulation of the degree of damping is consequently obtained in the oscillograph by regulating the temperature of the oil. It is doubtful whether the instrument would greatly gain in practical usefulness if the oil with the heating arrangement were done away with and replaced by a condenser.

In the string galvanometer the condenser method will be successfully applied in cases where it is desired to measure variations of current of very short duration. Taking a very short and strongly stretched quartz-thread, it will be possible to obtain deflections whose quickness leaves little to be desired. Without a condenser these would be useless for many purposes on account of the oscillations, whereas now they may become useful for a number of physical and electrotechnical investigations by a judicious damping. In these cases the string galvanometer will for equal quickness of deflection appear to be a much more sensitive apparatus than the oscillograph.

Also in a number of electrophysiological investigations we can avail ourselves of the condenser method, while the study of sounds will be particularly facilitated by it. I hope to make a nearer communication on this subject in a following paper.

Physics. — "*Dispersion bands in the spectra of δ Orionis and Nova Persei*". By Prof. W. H. Julius.

When light, giving a continuous spectrum, passes through a selectively absorbing, non-homogeneous mass of gas, the spectrum of the transmitted light contains places which, according to circumstances, may contrast as bright or as dark regions with their surroundings [2]). Though resembling emission and absorption lines, these bands have a wholly different origin. They are due to anomalous dispersion and, therefore, the name *dispersion bands* has been suggested for them [3]).

1) Also a mixture of two liquids is used, of which one has a great, the other a small viscosity. The mixture is so chosen that the desired viscosity is just obtained.

2) Proc. Roy. Acad. Amst. II, p. 580 (1900).

3) Proc. Roy. Acad. Amst. VII, p. 134—140 (1904).