development from the brain of the Insectivores, it is clear that the Tarsius brain shows unmistakable genetic relations with this latter also.

A more detailed account with figures will be published in the Handbuch d. Entwicklungsgeschichte edited by HERTWIG.

Mathematics. — *"On the equation determining the angles of two polydimensional spaces".* By Prof. P. H. SCHOUTE.

The problem which we wish to solve is the following:

"In a space S_n with n dimensions a rectangular system of coordinates O $(X_1 X_2 \ldots X_n)$ has been taken and with respect to this system a space S_p passing through O has been given by the equations

$$x_{p+i} = a_{1,i}\, x_1 + a_{2,i}\, x_2 + \ldots + a_{p,i}\, x_p\,;$$
$$(i = 1, 2, \ldots, n-p)$$

supposing this space S_p to have with the space of coordinates O $(X_1 X_2 \ldots X_p)$ but one point O in common, the p angles $\alpha_1, \alpha_2, \ldots \alpha_p$ are to be determined between these two p-dimensional spaces."

By means of geometry we should set to work as follows. Suppose in the given space S_p a spherical space having O as centre and unity as radius and thus forming in S_p the locus of the points at distance unity from O; if this spherical space projects itself on the space of coordinate O $(X_1 X_2 \ldots X_p)$ as a quadratic space with the half axes $a_1, a_2, \ldots a_p$, we get

$$a_1 = \cos \alpha_1,\ a_2 = \cos \alpha_2,\ \ldots,\ a_p = \cos \alpha_p.$$

In an almost equally simple way the tangents of the demanded angles are connected analytically with the central radii-vectores of an other quadratic space. If P is an arbitrary point of S_p and Q its projection on the space of coordinate O $(X_1 X_2 \ldots X_p)$, then the angle $P\,O\,Q = \alpha$ is also determined by the relation

$$tg^2\,\alpha = \frac{OP^2 - OQ^2}{OQ^2} = \frac{\sum\limits_{i=1}^{n-p} (a_{1,i}\, x_1 + a_{2,i}\, x_2 \ldots + a_{p,i}\, x_p)^2}{\sum\limits_{i=1}^{p} x_i^2}.$$

If we consider in S_p the points P the coordinates of which are bound to the condition

$$\sum_{i=1}^{n-p} (a_{1,i}\, x_1 + a_{2,i}\, x_2 + \ldots + a_{p,i}\, x_p)^2 = 1 \quad \ldots\ldots\ (1),$$

containing only $x_1, x_2, \ldots x_p$ and thus expressing that the projection Q of P on $O(X_1 X_2 \ldots X_p)$ remains in this latter space on the quadratic space represented by (1), then the relation holds good

$$tg\, a = \frac{1}{OR}.$$

If $b_1, b_2, \ldots b_p$ are the half axes of this new quadratic space, we shall find

$$tg\, \alpha_1 = \frac{1}{b}, tg\, \alpha_2 = \frac{1}{b_2}, \ldots, tg\alpha_p = \frac{1}{b_p}.$$

Now (1) passes into the symbolic form

$$(A_1\, x_1 + A_2\, x_2 + \ldots + A_p\, x_p)^{(2)} = 1$$

by the substitutions

$$\sum_{i=1}^{n-p} a^2{}_{k,i} = A_{k,k} \quad , \quad \sum_{i=1}^{n-p} a_{k,i}\, a_{l,i} = A_{k,l};$$

so the well know secular equation

$$\begin{vmatrix} A_{11} - \lambda & A_{21} & \ldots & A_{p,1} \\ A_{12} & A_{22} - \lambda & \ldots & A_{p,2} \\ \ldots & \ldots & \ldots & \ldots \\ A_{1,p} & A_{2,p} & & A_{p,p} - \lambda \end{vmatrix} = 0$$

furnishes by its roots $\lambda_1, \lambda_2, \ldots \lambda_p$ the coefficients of the equation of that quadratic space reduced on the axes.

From the relations

$$\lambda_1 = \frac{1}{b_1^{\,2}}, \lambda_2 = \frac{1}{b_2^{\,2}}, \ldots \lambda_p = \frac{1}{b_p^{\,2}}$$

ensues immediately that the demanded equation is arrived at by replacing in the above mentioned determinant λ by $tg^2\, a$.

Mathematics. — *"The locus of the principal axes of a pencil of quadratic surfaces".* By Prof. J. CARDINAAL.

1. The envelope of the axes of a pencil of conics was investigated among others by M. TREBITSCHER[1]). He found that the axes of the above mentioned conics envelop a curve of class three touching the right line at infinity of the plane in two points conjugate to the directions of the axes of the two parabolae of the pencil with respect to the

[1]) Ueber Beziehungen zwischen Kegelschnittbüscheln und rationalen Curven dritter Classe, Sitzungsber. der Kaiserl. Akademie der Wissenschaften, Bnd. LXXXI, p. 1080.