with α and β we find

$$x_1 : p_{13} = x_2 : p_{23} = x_4 : p_{43},$$

$$y_1 : p_{14} = y_2 : p_{24} = y_3 : p_{34}.$$

After substitution, and elimination of λ, we find an equation of the form

$$p_{23}(a_1 p_{14} + a_2 p_{24} + a_3 p_{34})^{(n)} = p_{13}(b_1 p_{14} + b_2 p_{24} + b_3 p_{34})^{(n)},$$

by which the exponent between brackets reminds us that we must think here of a *symbolical* raising to a power.

If in $p_{k4} = x_k y_4 - x_4 y_k$ we put the coordinate x_4 equal to zero, we find for the intersection of the cone of the complex of Y on β the equation

$$(y_3 x_2 - y_2 x_3)(a_1 x_1 + a_2 x_2 + a_3 x_3)^{(n)} = (y_3 x_1 - y_1 x_3)(b_1 x_1 + b_2 x_2 + b_3 x_3)^{(n)},$$

or shorter

$$y_1 x_3 b_x^n - y_2 x_3 a_x^n + y_3 (x_2 a_x^n - x_1 b_x^n) = 0.$$

This proves anew, that the intersections of the cones of the complex form a net.

Mathematics. — "*On nets of algebraic plane curves*". By Prof. JAN DE VRIES.

If a net of curves of order n is represented by an equation in homogeneous coordinates

$$y_1 a_x^n + y_2 b_x^n + y_3 c_x^n = 0$$

to the curve indicated by a system of values $y_1 : y_2 : y_3$ is conjugated the point Y having y_1, y_2, y_3 as coordinates and reversely.

A homogeneous linear relation between the parameters y_k then indicates a right line as locus of Y, corresponding to a pencil comprised in the net.

To the Hessian, H, passing through the nodes of the curves belonging to the net, a curve (Y) corresponds of which the order is easy to determine. For, the pencil represented by an arbitrary right line l_Y has $3(n-1)^2$ nodes. So for the order n'' of (Y) we find $n'' = 3(n-1)^2$.

If one of the curves of a pencil has a node in one of the base-points, it is equivalent to two of the $3(n-1)^2$ curves with node belonging to the pencil. Then the image l_Y touches the curve (Y) and reversely.

Let us suppose that the net has b fixed points, then H passes twice through each of those base-points; so it has with the netcurve c^n_Y indicated by a definite point Y yet $(nn'-b)$ single points in common; here $n'=3(n-1)$ represents the order of H. The curve c^n having a node in D, determines with c^n_Y a pencil represented by a tangent of the curve (Y). From this ensues that the class of (Y) is indicated by $k''=3n(n-1)-2b$.

The genus g'' of this curve is also easy to find. As the points of (Y) are conjugated *one* to *one* to the points of H these curves have the same genus. So we have

$$g''=\tfrac{1}{2}(n'-1)(n'-2)-b=\tfrac{1}{2}(3n-4)(3n-5)-b.$$

We shall now seek the number of nodes and the number of cusps of (Y). These numbers δ'' and $\varkappa''$ satisfy the relations

$$2\delta''+3\varkappa''=n''(n''-1)-k'',$$

$$\delta''+\varkappa''=\tfrac{1}{2}(n''-1)(n''-2)-g''.$$

From this ensues after some reduction

$$\delta''=\tfrac{3}{2}(n-1)(n-2)(3n^2-3n-11)+b,$$

$$\varkappa''=12(n-1)(n-2).$$

The curve (Y) has nodes in the points Y_B which are images of the curves c^n_B possessing a node in a base-point of the net. For, to each right line through a point Y_B a pencil corresponds, in which c^n_B must be counted for two curves with node.

Each of the remaining nodes of (Y) is the image of a curve c^n, possessing two nodes.

So a net N^n contains $\tfrac{3}{2}(n-1)(n-2)(3n^2-3n-11)$ curves with two nodes.

To a cusp of (Y) will correspond a curve replacing in each pencil to which it belongs two curves with node. According to a well-known property that curve itself must have a cusp. For a definite pencil its cusp is one of the base-points; this pencil has for image the tangent in the corresponding cusp of (Y).

So a net N^n contains $12(n-1)(n-2)$ curves with a cusp.

The two properties proved here are generally indicated only for a net consisting of polar curves of a c^{n+1}. We have now found that they hold good for every net, independent of the appearance of fixed points B.

We can now easily determine the class z of the envelope Z of the nodal tangents of the net.

Through an arbitrary point P of a right line l pass z of these tangents. If we add the second tangent in the corresponding node to each of these tangents, these new set of z tangents intersects the right line l in z points P'. The coincidences of the correspondence (P,P') are of two kinds. They may originate in the first place from cuspidal tangents, in the second place from the points of intersection of l with the curve H; each of these latter points of intersection however is to be regarded as a double coincidence. Thus $2z = 12(n-1)(n-2) + 6(n-1) = 6(n-1)(2n-3)$.

The curve of ZEUTHEN *is of class* $3(n-1)(2n-3)$.

ERRATA.

Page 504, line 13, for members read member.
" 504, " 15, " not wanting read wanting.
" 509, " 24, " blewish read bluish.

(April 19, 1905).