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The curve extending upwards from P should therefore be bent in such a way, that its initial direction was the same as that of the curve of the three-phase-pressure.

The tangent plane to the (p, T, x_f) surface being normal to the plane of the figure, because it contains a line which in P is normal to the figure, every curve on that surface, passing through P, will have its projection in the section of the tangent plane with the plane of the figure; and so both the curves extending upwards from P and those extending downwards, will have their projections in this same section. This follows also from the values of p. 241 (preceding communication). We have for the three-phase-pressure:

$$T\frac{dp}{dT}=\frac{\dfrac{w_{s_1}}{x_s-x_1}-\dfrac{w_{21}}{x_2-x_1}}{\dfrac{v_{s_1}}{x_s-x_1}-\dfrac{v_{21}}{x_2-x_1}}.$$

In the immediate neighbourhood of a plaitpoint $\dfrac{w_{21}}{x_2-x_1}$ and $\dfrac{v_{21}}{x_2-x_1}$ is equal to zero, (Cont. II, p. 125); and we find $\dfrac{dp}{dT}=\left(\dfrac{\partial p}{\partial T}\right)_x$.

One more remark to conclude with. Now that we have concluded to the existence of the tops of the curves $V_{sf}=0$ and $W_{sf}=0$, we shall also have to accept the conclusion, that the complications in the course of the (p, x) and the (p, T) sections of the surface of fluid phases coexisting with solid ones, remain restricted to the neighbourhood of the critical phases. It is therefore uncertain, whether in a section for given x, if the latter is e.g. chosen halfway between x_e and x_a, the two vertical tangents still occur. As soon as they have coincided, the section has no longer any special point, and so the retrograde solidification has also disappeared.

Mathematics. — "*Centric decomposition of polytopes.*" By Prof. P. H. Schoute.

In the following lines it will be shown how a regular polytope can be decomposed according to its vertices or to its limiting spaces of the greatest number of dimensions into a system of congruent regular polytopes with a common centre. For this $P^{(r)}_{m,n}$ shall represent a regular polytope, limited by m spaces S_{n-1} in S_n, with a length r of the edges; and moreover we shall omit as much as possible the number n of the dimensions and always each of the predicates "regular", "congruent" and "concentric".

In our space the theorems hold good:

I^a. "The eight vertices of a cube $P_6^{(1)}$ can be arranged into two quadruples of vertices not connected by edges, or of non-adjacent vertices, the vertices of two tetraeders $P_4^{(\sqrt{2})}$."

I^b. "The eight faces of an octaeder $P_8^{(1)}$ can be arranged into two quadruples of planes of which no two planes pass through one and the same edge, i. e. of quadruples of non-adjacent planes, the faces of two tetraeders $P_4^{(2)}$."

II^a. "The twenty vertices of a $P_{12}^{(1)}$ form taken *twice* the vertices of five $P_6^{[\frac{1}{2}(1+\sqrt{5})]}$ and taken *once* in two different ways the vertices of five $P_4^{[\frac{1}{2}(1+\sqrt{5})\sqrt{2}]}$."

II^b. "The twenty faces of a $P_{20}^{(1)}$ form taken *twice* the faces of five $P_8^{[\frac{1}{4}(3+\sqrt{5})\sqrt{2}]}$ and taken *once* in two ways the faces of five $P_4^{[\frac{1}{2}(3+\sqrt{5})\sqrt{2}]}$"

The length of the edges indicated for the components follows immediately from the observation that for the decomposition according to the vertices the radius of the circumscribed sphere, for the decomposition according to the faces the radius of the inscribed sphere remains unchanged.

For S_4 we have the following theorems:

III. "The sixteen vertices of a $P_8^{(1)}$ can be arranged into two octuples of non-adjacent vertices, the vertices of two sixteen-cells $P_{16}^{(\sqrt{2})}$. In like way $P_{16}^{(1)}$ gives according to the limiting spaces two $P_8^{(\frac{1}{2}\sqrt{2})}$."

IV. "The twenty-four vertices of a $P_{24}^{(1)}$ form the three octuples of vertices of three $P_{16}^{(\sqrt{2})}$. In like way $P_{24}^{(1)}$ gives according to the limiting spaces three $P_8^{(\sqrt{2})}$."

V. "The one hundred and twenty vertices of a $P_{600}^{(1)}$ form in five different ways the vertices of five $P_{24}^{[\frac{1}{2}(1+\sqrt{5})]}$. In like way $P_{120}^{(1)}$ gives according to the limiting spaces in five ways five $P_{24}^{[\frac{1}{4}(7+3\sqrt{5})\sqrt{2}]}$."

VI. "The six hundred vertices of a $P_{120}^{(1)}$ form in two different ways the vertices of five $P_{600}^{[\frac{1}{2}(1+\sqrt{5})\sqrt{2}]}$. In like way $P_{600}^{(1)}$ gives in two ways the limiting spaces of five $P_{120}^{[\frac{1}{2}(\sqrt{5}-1)\sqrt{2}]}$."

With the aid of these theorems it is easy to arrive at the remaining possible centric decompositions of the four-dimensional polytopes.

In spaces with a greater number of dimensions it is known that but three regular polytopes are to be found, i. e. in S_n the simplex P_{n+1}, the polytope of measure P_{2n} and the polytope P_{2^n} reciprocally related to the preceding. With respect to these there is an extension for theorem I and theorem III only, namely I for $n = 2^p - 1$ and III for $n = 2^p$. These extensions run as follows:

VII. "In space S_{2^p-1} the 2^{2^p-1} vertices of a $P_{2(2^p-1)}^{(1)}$ form the vertices of 2^{2^p-p-1} simplexes $P_{2^p}^{[\sqrt{2^{p-1}}]}$. In like way a $P_{2^{2^p-1}}^{(1)}$ gives according to the limiting spaces of 2^p—2 dimensions 2^{2^p-p-1} simplexes $P_{2^p}^{[\sqrt{2^p}]}$."

VIII. "In space S_{2^p} the 2^{2^p} vertices of a $P_{2^p+1}^{(1)}$ from the vertices of 2^{2^p-p-1} $P_{2^{2^p}}^{[\sqrt{2^{p-1}}]}$. In like way a $P_{2^{2^p}}^{(1)}$ gives according to the limiting spaces of 2^p—1' dimensions 2^{2^p-p-1} $P_{2^p \mp 1}^{[\sqrt{2^{1-p}}]}$."

In the meeting of June 27, 1903 Prof. J. M. van Bemmelen communicated a paper: "*On absorptioncompounds which may change into chemical compounds or solution.*"

(This paper will not be published in these Proceedings.)

(December 23, 1903).