Citation:

Versluys, W.A., The singularities of the focal curve of a plane general curve touching the line at infinity [å] times and passing [î] times through each of the imaginary circle points at infinity, in:
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where $\mathfrak{F}$ denotes an arbitrary function and u and v the functions of p and q, derived from the equations (4).

In the particular case that the function θ satisfies the two members of the equation (3) separately, we have to distinguish two cases according to the double sign in

$$\frac{\partial D}{\partial q} - \frac{\theta_{12} \pm \sqrt{\theta_{12}^2 - \theta_{11}\,\theta_{22}}}{\theta_{11}} \frac{\partial D}{\partial p} = 0.$$

The general integral can be written in both cases respectively

$$\left.\begin{aligned} p &= \frac{1}{u} f\,[u\,(v \mp g'')] + r''\,(u) \\ q &= v \\ \theta &= (2\,u^2\,g'' - 4\,u\,g' + 4\,g \mp u^2\,v)\,p - 2\int (u\,g''' \mp v) u\,p\,d\,u \end{aligned}\right\}, \quad (5)$$

where $g = g\,(u)$ has the same meaning as before, whilst f and r represent two new arbitrary functions of the arguments placed after these symbols.

In these cases the two intermediate integrals are

$$z \mp y\,(u\,g''' - g'') - x(u\,r''' + r'') = \mathfrak{F}\,(u),$$
$$y + xf'\,[u\,(v \mp g'')] = \mathfrak{F}\,[u\,(v \mp g')],$$

the values of u and v being expressed in p and q with the aid of the formulae (5).

The condition (3) appears, although in a different shape, already in the excellent dissertation of J. Vályi (Klausenburg 1880).

Mathematics. — *"The singularities of the focal curve of a plane general curve touching the line at infinity σ times and passing ε times through each of the imaginary circle points at infinity."* By Dr. W. A. Versluys. (Communicated by Prof. P. H. Schoute).

(Communicated in the meeting of January 30, 1904).

In "Verhandeling" 5 of the "Kon. Ak. v. W." at Amsterdam Vol. VIII, I have deduced some formulae expressing the singularities of the focal developable and of the focal curve in function of the singularities of a plane curve having no particular position.

In a similar way it is possible to deduce the following formulae expressing the singularities of the focal developable and of the focal curve of a plane curve touching the line at infinity σ times and passing ε times through each of the imaginary circle points at infinity.

Let the plane curve be of order μ, of class ν and let ι represent the number of its inflectional points. Then the singularities of the evolute or of the cuspidal curve of its focal developable are the following:

rank, $r = 2(\mu + \nu - 2\varepsilon - \sigma)$

class, $m = 2\nu$

number of stationary planes, $\alpha = 2\iota$

double osculating planes, $G = \nu^2 - \nu - \mu - 3\iota + 3\sigma^2 + 2\varepsilon - \sigma$

stationary tangents, $v = 0$

double points, $H = 3(\mu - \nu) + \iota$

double tangents, $\omega = 0$

order, $n = 2(3\mu + \iota - 6\varepsilon - 3\sigma)$

stationary points $\beta = 2(6\mu - 2\nu + 3\iota - 12\varepsilon - 6\sigma)$

stationary points not at infinity and not in the plane of the curve

$\beta' = 2(5\mu - 3\nu + 3\iota - 8\varepsilon - 3\sigma)$

order of the nodal curve $x = 2(\mu + \nu)^2 - 10\mu - 2\nu - 3\iota - 8\mu\varepsilon - 4\mu\sigma - 8\nu\varepsilon - 4\nu\sigma + 8\varepsilon^2 + 8\varepsilon\sigma + 2\sigma^2 + 20\varepsilon + 10\sigma.$

The chief singularities of the focal curve are:

order, $n = 2\mu^2 + 4\mu\nu + \nu^2 - 11\mu - \nu - 3\iota - 8\mu\varepsilon - 4\mu\sigma - 8\nu\varepsilon - 2\nu\sigma + 8\varepsilon^2 + 8\varepsilon\sigma + \sigma^2 + 20\varepsilon + 9\sigma$

rank, $r = 4\mu\nu + \nu^2 - 4\mu - 4\nu - 8\nu\varepsilon - 2\nu\sigma - 3\sigma^2 + 8\varepsilon + 5\sigma$

number of stationary tangents, $v = 0$

class, $m = 6\mu^2 + 6\mu\nu + 4\mu\iota + 2\nu\iota - 36\mu - 12\nu - 18\iota - 24\mu\varepsilon - 6\mu\sigma - 12\nu\varepsilon - 4\nu\sigma - 8\iota\varepsilon - 2\iota\sigma + 24\varepsilon^2 + 12\varepsilon\sigma - 8\sigma^2 + 60\varepsilon + 28\sigma$

number of stationary points $\beta = 2(3\mu + \iota)(2\mu + \nu) - 57\mu + 21\nu - 27\iota - 48\mu\varepsilon - 18\mu\sigma - 12\nu\varepsilon - 4\nu\sigma - 8\iota\varepsilon - 2\iota\sigma + 48\varepsilon^2 + 36\varepsilon\sigma + 4\sigma^2 + 36\varepsilon\sigma + 96\varepsilon + 40\sigma.$

Mathematics. — *"On the position of the three points which a twisted curve has in common with its osculating plane."* By Dr. W. A. VERSLUYS. (Communicated by Prof. P. H. SCHOUTE.)

(Communicated in the meeting of January 30, 1904).

§ 1. Let d be the section of the osculating plane V in a point P of the twisted curve C with the developable O of which C is the cuspidal curve; then the twisted curve C and the section d have in the point P only two points in common, that is they have in P a contact of the first order.