Physics. — J. C. Schalkwijk: "*Precise Isothermals. II. Accuracy of the measurement of pressure by means of the open manometer of* Kamerlingh Onnes." (Communication N°. 70 from the Physical Laboratory at Leiden, by Prof. H. Kamerlingh Onnes).

§ 1. As the accuracy, which could be attained in the measurement of pressure, was of the greatest importance for attaining that high degree of accuracy which is required for my determinations of isothermals (Comp. Comm. N°. 67), I have made the measurements with the open manometer mentioned the subject of a separate investigation. Although there is little reason to doubt the exactness of the observation, after the necessary corrections described in Comm. N°. 44 have been applied yet some reason for doubt might exist in the fact that the correction for the depression of the mercury menisci, and also those resulting from the friction of the mercury in the capillary tubes are not sufficiently known. The former were borrowed from the tables of Mendelejeff and Gutkowsky [1]) and the latter were calculated by means of Poiseuille's law; especially the latter give rise to many difficulties, as I could calculate only the mean bore of the manometer capillaries, whereas in Poiseuille's [2]) equation the mean of the square of the bore is required. It is however a favourable circumstance that the variation of the level of the mercury in each of the manometer tubes hardly ever exceeds the amount of 1 c.m. an hour, and as in the narrowest of the capillaries this would give a difference of pressure of 0.02 c.m per tube, with our degree of accuracy it may be neglected in comparison with a pressure of 304 c.m. per tube.

§ 2. The lengths of the tubular measuring rods (comp. Communication N°. 44) which are of chief importance for the measurement of the pressure, were determined in the following way: the

1) As the reading of the level and the top of the meniscus would double the time of observation the depression is already accounted for in the reading, by estimating and adding for each tube the required correction to the height.

2) In order to test the use of the formula for tubes of the bore used (of 0.055 c.m. radius on an average), I have calculated from the velocity of the mercury through a similar tube the value of the co-efficient of friction η and found 0.00001725, differing only a little from the value 0.00001633 found by Warburg (0.01609 in the C.G.S. system).

first rod L_0, graduated over its whole length, and hanging between the limbs of the U-shaped manometer-tube A, was compared with the Standardmeter N^0. 1 of the Physical Laboratory of Leiden, which according to its correction is at 7° C. shorter by 0,005 mm. than the Mètre des Archives. From this we calculate its length at 0° C. 999.91 mm. The length between the marks 0 and 304 (corresponding to 4 atm.) of this measuring rod was measured in two ways and found to be 303.976 c.m. and 303 971 cm., which two values are in sufficient agreement. In order to determine the lengths of the other measuring rods B_I—B_{VII} they were suspended together with the measured rod at short distances from each other, and at 425 c.m. distance from the theodolites to be used for the reading of the mercury levels, the same distance at which they are used. The readings were made with telescopes rotating round perfectly vertical axes; during rotation I could not observe on the very sensitive levels a larger variation than $^1/_4$ mark, corresponding to $^1/_{28}$ mm. on the measuring tube. In four measurements the mean difference of the readings was less than 0.1 mm.

The following corrections have to be applied in the measurements:

A. The correction for the depression, mentioned above.

B. The correction for the friction of the mercury, also discussed above.

C. The corrections for the inclination of the telescope of the theodolite.

But in the most unfavourable case, namely in the outer tube from 56—60 atm., this correction is only $^1/_{160}$ of the vertical distance between the mercury meniscus and the level of the theodolite, and as in the measurements this distance was hardly ever more than 3 c.m. the correction may be neglected, especially for the other tubes where it is much less.

D. The corrections arising from the temperature of the mercury.

The temperature is measured by eight thermometers distributed over different parts of the apparatus. If the mean temperature is t, tables can be calculated for this correction from the formula $\Delta = L\{1-(a-k)\,t\}$, in which a is the co-efficient of cubic expansion of mercury and k the co-efficient of linear expansion of brass. (The measuring rods are of brass).

E. The corrections for the weight of the air which is compressed between the mercury of the successive manometer tubes. By means of the known values of the pressure belonging to each column, the specific weight of the air and the mean temperature, tables can also be calculated for these corrections.

F. The corrections of the thermometers necessary for the corrections mentioned in D and E. They were determined by suspending all the thermometers in a water-bath at a constant temperature, together with a standard thermometer of which the scale had been tested at the Physikalisch-Technische Reichsanstalt, and of which the zero error had been determined every two months in melting ice.

G. The correction for the compression of the mercury; this is $\Delta = {}^1/_2 \beta P^2$, if the coefficient of compressibility is β. As according to AMAGAT β is only 0.00000392 for mercury if P is expressed in atmospheres, this correction need only be considered above 40 atm.

H. The correction for the lengthening of the measuring rods during the suspension.

The rod L_0 was measured in a horizontal position, whereas the rods are used, suspended at about $^1/_3$ of their height from the top by a Cardanus-collar. Hence the upper part will be compressed and the lower part will be lengthened; if l is the distance to the point of suspension we can easily calculate $\Delta = {}^1/_2 \frac{S}{E} l^2$, for the displacement of the lower end. For brass $S = 8.45$; $E = 1037{,}000{,}000$ while $l' =$ about 215 cm.; so we calculate $\Delta = 0.0002$ cm. which therefore may be entirely neglected.

§ 3. In order to judge of the influence of the uncertainty in the corrections A and B, the open manometer (comp. Communication n°. 50 § 2) was divided into two parts (see Plate Comm. n°. 44). For this tube A was entirely cut off by closing cock K_1 and by loosing the steel capillary of tube B_1 from the T-piece T_1, and the steel capillary of tube B_8 from the T-piece T_8, while this opening of T_8 was closed tightly by a nut with leather packing. If all the other cocks are open and pressure is very slowly admitted by means of cock X, we shall see the mercury rise in the capillary both in tube B_1 and B_8. If in B_1 the mercury has reached the mark X_1 the cock K_2 is closed, after which the mercury is forced up in B_2, etc. At the same time attention must be paid to the mercury in B_8 and the cock K_9 is closed when the mercury in B_8 has reached the required height, which now however will be higher than the mark X_8 of the figure and which must be determined experimentally beforehand. In this way we can continue until the mercury has reached the lower end of the upper reservoir in tube B_7 and therefore also in B_{14}; then the cock K_8 is closed while the cock X is left open until the mercury in tube B_{14} has reached the upper part of the reservoir; then also the cock X is closed [1]). In this way it was possible to raise the pressure in the second system of tubes more than $^1/_4$ atm. higher than in the first system. At the reopening of the cock K_8 the communication between the two systems was reestablished and thus while the pressure rose for the first system it fell for the second. The influence both of the friction and of the depressions was opposite for the two systems, for in the first system the menisci were rounded at the upper end and flat at the lower end, whereas in the second they were rounded at the lower and flat at the higher end. As during the reading which lasted each time 30 minutes we could expect a variation of the menisci the observations have each time been made symmetrically, and so first in system I at the higher end from the left to the right, then in system II at the higher end from the left to the right, then in system II at the lower end from the right to the left and lastly in system I at the lower end from the right to the left.

§ 4. *Results.* In order to give a survey of the value of the corrections to be applied and the calculation of the pressure this has been carried out for one measurement. For the further determinations which will be communicated in the table of comparisons (p. 28) all the pressures are similarly calculated [2]).

In order to obtain the uncorrected height at the mean time viz. at 4.36 we need only add to the sum of the lengths of the measuring rods (entered in the table for both system as $\Sigma^I L$ and $\Sigma^{II} L$), the sum of the upper readings (for both systems $\Sigma_1{}^I B$ and $\Sigma_1{}^{II} B$) taking into account that the measuring rod L serves as well for the manometer tube B_1 as for B_2 etc. and to subtract from this sum the sum of the lower readings (for both systems $\Sigma_2{}^I B$ and $\Sigma_2{}^{II} B$). In determining the mean temperature given as $T_1{}^I$ and $T_1{}^{II}$ etc. we must pay attention to the positions of the thermometers; they were arranged in the following way: t_1, t_2 and t_3 were suspended at the upper end, in the middle and at the lower end of tube B_1; t_4 and t_5 were suspended at $^1/_4$ and $^3/_4$ of the tube B_7 and t_6, t_7 and t_8 again

1) In order to exclude unnecessary possibilities of leakage the differential manometer tube C was always loosened from the T-piece T_{16} and the crosspiece N, which were both again closed.

2) In this equation the height of the barometer can be left out of consideration, because it equally influences both systems.

Readings on June 28 1900.

Thermometer	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	$T_1^{I} = 17°.68$
Reading.	18°	17°.9	17°.7	17°.7	17° 75	17°.8	17°.7	17°.45	$T_1^{II} = 17°.61$
Corrected.	17°.87	17°.77	17°.52	17°.6	17°.67	17°.7	17°.62	17°.37	
time				4.26					
Manometer tube B. top left to right.	I.	II.	III.	IV.	V.	VI.	VII.		
	3.62	4.70	0.77	0.68	3.50	4.27	6.14		$\Sigma_1{}^{I}B = 23.68$
Manometer tube B. top left to right.	VIII	IX.	X.	XI.	XII.	XIII.	XIV		
	4.38	5.66	5.58	6.81	7 27	6.84	5.83		$\Sigma_1{}^{II}B = 42.37$
time				4.36					
Manometer tube B. bottom right to left.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.		
	5.87	8.44	9.73	9.75	9 25	8.14	10.17		$\Sigma_2{}^{II}B = 61.35$
Manometer tube B. bottom right to left	I.	II.	III.	IV.	V.	VI.	VII.		
	7.18	7.49	4.32	4.59	9.97	4.83	3.80		$\Sigma_2{}^{I}B = 42.13$
time				4.46					
Thermometer	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	$T_2^{I} = 17°.63$
Reading.	17°.9	17°.8	17°.8	17°.65	17°.7	17°.8	17°.7	17°.5	$T_2^{II} = 17°.59$
Corrected.	17°.77	17°.67	17°.62	17°.55	17°.62	17°.7	17°.62	17°.42	
Measuring rod L.	I.	II.	III.	IV.	V.	VI	VII.		$\Sigma^{I}L = 2123.36$
Length.	302.18	303.45^5	304.19^5	303.70	303.68^5	302.74^5	303.52^5		$\Sigma^{II}L = 2123.61$

at the higher end, in the middle and at the lower end of tube B_{14}. Hence the mean temperature will be given pretty accurately for the first system by $\frac{1}{8}(t_1 + 2t_2 + t_3 + 2t_4 + 2t_5)$ and for the second system by $\frac{1}{8}(2t_4 + 2t_5 + t_6 + 2t_7 + t_8)$. The following table is obtained:

System.	ΣL	$\Sigma_1 B$	$\Sigma_2 B$	Height mercury.	Corr. leakage.	Mean temp.	Corr. for temp.	Corr for weight air.	Corrected mercury height.
I.	2123.36	23.68	42.13	2104.91	0.00	17°.66	— 6.05	— 2.38	2096 48
II.	2123.61	42.37	61 35	2104.63	0.00	17°.60	— 6.02	— 2.38	2096.23

All the lengths are expressed in cms.

The heights of the mercury found on different days are combined in the following table, which also gives the difference of those calculated heights in the two systems.

Table of comparisons.

Date.	Time.	System I.	System II.	Difference.
28 June	4.36	2096 48 cm	2096.23 cm.	+ 0 25 cm.
» »	5.14	2095.33 »	2095.45 »	— 0.12 »
29 »	3.50	2107.03 »	2107.15 »	— 0.12 »
» »	4.30	2106.13 »	2106.27 »	— 0.14 »
» »	5.07	2104 25 »	2104.41 »	— 0.16 »
30 »	2.25	2117.55 »	2118.01 »	— 0.46 »
» »	3.02	2117.42 »	2117.73 »	— 0.31 »
» »	3.42	2116.01 »	2116.23 »	— 0.22 »
» »	4.13	2115.23 »	2115.51 »	— 0.28 »
» »	4.49	2113.81 »	2113.75 »	+ 0.06 »
5 July	4.00	2118.76 »	2118.53 »	+ 0.23 »
» »	4.25	2117.67 »	2117.96 »	— 0.29 »
» »	5.00	2116.59 »	2116.64 »	— 0.05 »

For the *mean difference* at one measurement we find therefore 0.24 c.m., this amounts to $\frac{1}{8800}$ of the pressure measured. But there appears to be *a systematic error* in the observations; for the reading in the second system is on an average 0.13 c.m. higher than in the first system. Perhaps it may be ascribed to the fact that all the tubes of the second system are much narrower than those of the first, so that if the height of the high menisci at the lower end is estimated only a little too low, the depression becomes too small and hence the mercury height read too large.

This systematic error amounts only to $\frac{1}{16000}$ of the pressure measured and moreover will only have any influence at pressures above 32 atm. (for then only the tubes of the second system are used). By reading the real height of the menisci perhaps even this slight deviation might be prevented.

Physics. — J. C. SCHALKWIJK: "*Precise Isothermals.* III. *A water-jacket of constant ordinary temperature.*" (Communication N°. 70, (continued) from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES).

§ 1. *The necessity of having at disposal a current of water at a constant temperature.* In consideration of the small heat conductivity of gases and the great thickness of the walls of piezometer tubes, used in my experiments (comp. Communication N°. 50 of prof. H. KAMERLINGH ONNES, June 24th 1899) it is desirable to take care that the temperature of the surrounding water cannot vary more than some hundredths of a degree per hour, and that it can be accurately adjusted at the desired value and be kept almost constant during 5 hours. This offers however many difficulties; for the bath must be more than 80 c. m. high, and we must be able to read accurately the marks over the whole length of the tube, which excludes coating the bath with a badly conducting substance as a protection. Hence there will be a continual large loss of heat from the surface of the bath. The distance of the piezometertube from the glass wall must be very small with a view to the refraction of the rays emerging from the water, as we must be able to read very accurately the difference in the height of the mercury in the piezometer tube C_1 (comp. the plate Proceedings June 24th 1899) and in the measuring glass P [1]; therefore 6 c. m. was taken as

[1]) A means used by Dr. N. QUINT (comp. his thesis for the doctorate 1900, p. 15 and fig. 5) cannot be used in our case. He placed the observation-tube in a rectangular basin containing about 40 liters of water and kept up a constant temperature by means of two liquid-resistances (saturated ammonium chloride) carrying an alternating current, an exterior krüppin resistance to be regulated by hand and rotating bladed wheel. Moreover in accurate determinations of isothermals, the measurements proper occupy each time about three hours and as therefore many readings have to be made, the regulation of the krüppin-resistance would cause an undesirable interruption of the series of observations.