chromosphere gases. Now we can hardly assume that these gases really do not emit any light; the question is only, in what cases and how far the intensity of the true chromospheric emission is comparable with the intensity of the abnormally refracted photosphere light.

Perhaps the photographs obtained by our expedition are accidentally so extremely fit to show the part played by anomalous dispersion in causing chromosphere light, that they induce one to overestimate the importance of the new principle.

It would therefore be very interesting if the plates of other expeditions were also studied from this point of view.

Mathematics. — "*Considerations in reference to a configuration of* SEGRE". By Prof. P. H. SCHOUTE (first part).

1. In a treatise published in 1888 "Sulle varietà cubiche dello spazio a quattro dimensioni, ecc" (*Memorie di Torino*) Dr. C. SEGRE proved the following remarkable theorem:

The locus of the right lines cutting any four planes assumed in the space S_4 is a curved space of order three containing besides these four planes eleven planes more; one of these eleven new planes is intersected by all the right lines cutting the four given planes. The fifteen planes pass six by six through one of ten points, which are double points of the cubic locus.

If we call the four given planes $\alpha, \beta, \gamma, \delta$ and if we denote by α' the plane through the three points of intersection $(\gamma\delta)$, $(\delta\beta)$, $(\beta\gamma)$, by β' the plane through the three points of intersection $(\delta\alpha)$, $(\alpha\gamma)$, $(\gamma\delta)$, etc., then the four points of intersection lie in one and the same space ε and the five planes form such a quintuple, that each right line cutting four of these planes, also cuts the fifth.

In a study also published in 1888 "Alcune considerazioni elementari sull' incidenza di rette e piani nello spazio a quattro dimensioni" (*Rendiconti del circolo matematico di Palermo*, vol. 2, pages 45—52) the same writer gives a rather simple geometrical proof of the second part of this theorem, and then ascends to the configuration mentioned in the first part by the indication that the ten points spoken of are the points of intersection of the five planes $\alpha, \beta, \gamma, \delta, \varepsilon$ two by two and the ten new planes are deduced out of the triplets of these five planes as α' out of $(\beta\gamma\delta)$, etc.

In the following pages we will submit the configuration of SEGRE to a simple analytical investigation. For this let us consider the dualistically opposite figure of fifteen lines and ten three-dimensional spaces.

2. If we begin with the second part of the theorem, then we have to deal with the figure consisting of eight lines

$$\begin{vmatrix} a_1, & a_2, & a_3, & a_4 \\ b_1, & b_2, & b_3, & b_4 \end{vmatrix}$$

corresponding in this respect with the wellknown double six of SCHLÄFLI, that each of these eight lines intersects only those three of the remaining ones, corresponding with them neither in letter nor in index. We suppose the four given right lines a_1, a_2, a_3, a_4 to be given in S_4 in such a way that among the six connecting spaces $(a_1 a_2), \ldots (a_3 a_4)$ there are not three having a plane in common. And b_1 is then again the line of intersection of the three spaces $(a_3 a_4)$, $(a_4 a_2)$, $(a_2 a_3)$, etc. To this figure which in a previous study we considered as the basis of a particular net of quadratic curved spaces we therefore gave the name of "double four" ("Ein besonderer Bündel von dreidimensionalen Räumen zweiter Ordnung im Raum von vier Dimensionen", *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 9, pages 103—114).

If we consider the spaces $(a_1 a_3)$ and $(a_2 a_4)$, it is immediately evident that each of these spaces contains four of the eight lines of the double four and that this is therefore broken up into the two skew quadrilaterals

$$(a_1 \ b_2 \ a_3 \ b_4),$$

$$(b_3 \ a_4 \ b_1 \ a_2),$$

of which the sides written here under each other meet each other in four points of the plane of intersection of the spaces $(a_1 a_3)$ and $(a_2 a_4)$. If to begin with we draw only the first of those skew quadrilaterals (fig. 1), then it is clear that the lines $P_1 P_2$ and $P_3 P_4$, connecting those points in which the pairs of sides (a_1, b_2) and (a_3, b_4)

are intersected by this plane, will meet each other on the line of intersection l of the planes $(a_1 b_2)$ and $(a_3 b_4)$. In like manner the line of intersection m (fig. 2) of the planes $(b_1 a_2)$ and $(b_3 a_4)$ passes through the point of intersection O of $P_1 P_2$ and $P_3 P_4$.

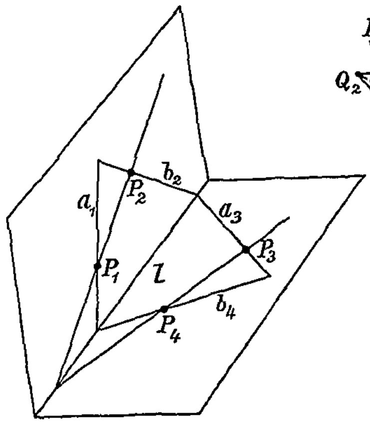

Fig. 1.

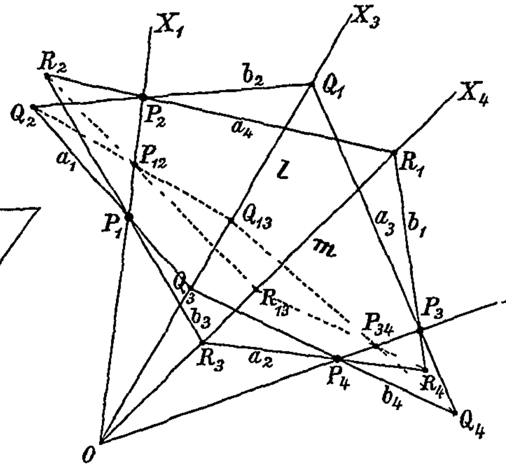

Fig 2.

If we now indicate the two skew quadrilaterals according to the vertices in the way pointed out in fig. 2 by $Q_1\, Q_2\, Q_3\, Q_4$ and $R_1\, R_2\, R_3\, R_4$ and if P_{12}, P_{34}, Q_{13}, R_{13} represent the points separating O harmonically from the pairs of points (P_1, P_2), (P_3, P_4), (Q_1, Q_3), (R_1, R_3), it is easy to see that

$$(Q_2\, P_{12}\, Q_{13}),\ (Q_4\, P_{34}\, Q_{13}),\ (R_2\, P_{12}\, R_{13}),\ (R_4\, P_{34}\, R_{13})$$

are four triplets of points on a right line,

$$(P_{12}\, P_{34}\, Q_2\, Q_4\, Q_{13}),\ (P_{12}\, P_{34}\, R_2\, R_4\, R_{13})$$

two quintuples of points in a plane and that

$$P_{12}\, P_{34}\, Q_2\, Q_4\, Q_{13}\, R_2\, R_4\, R_{13}$$

are eight points of a same space. We now choose the five-cell of which O is a vertex, the four lines $P_1 P_2$, $P_3 P_4$, $Q_1 Q_3$, $R_1 R_3$ are the edges passing through this point and the space just found is the side-space situated opposite O, as five-cell of coordinates; this has the five points O, P_{12}, P_{34}, Q_{13}, R_{13} as vertices. By assuming the point of intersection of the four spaces

$$(P_1\, P_{34}\, Q_{13}\, R_{13}),\ (P_4\, P_{12}\, Q_{13}\, R_{13}),\ (Q_1\, P_{12}\, P_{34}\, R_{13}),\ (R_1\, P_{12}\, P_{34}\, Q_{13})$$

as point of unity of the homogeneous system of coordinates and by following for the rest the notation of fig. 2 we obtain the following table of proportions of coordinates of the points the knowledge of which is sufficient for the determination of the equations of the lines of the double four:

$$\begin{array}{llll}
P_1 \ldots (\ \ 1, & 0, 0, 0, 1), & Q_1 \ldots (0, 0, & \ \ 1, \ \ 0, 1), \\
P_2 \ldots (-1, & 0, 0, 0, 1), & Q_3 \ldots (0, 0, & -1, \ \ 0, 1), \\
P_3 \ldots (\ \ 0, & -1, 0, 0, 1), & R_1 \ldots (0, 0, & \ \ 0, \ \ 1, 1), \\
P_4 \ldots (\ \ 0, & \ \ 1, 0, 0, 1), & R_3 \ldots (0, 0, & \ \ 0, -1, 1).
\end{array}$$

So the equations of the two quadruples of lines are

$$\left.\begin{array}{lll}
a_1 \ldots\ldots x_1 - x_3 = x_5 & , \ x_2 = 0 & , \ x_4 = 0 \\
a_2 \ldots\ldots x_2 - x_4 = x_5 & , \ x_1 = 0 & , \ x_3 = 0 \\
a_3 \ldots\ldots x_3 - x_2 = x_5 & , \ x_1 = 0 & , \ x_4 = 0 \\
a_4 \ldots\ldots x_4 - x_1 = x_5 & , \ x_2 = 0 & , \ x_3 = 0
\end{array}\right\},$$

$$\left.\begin{array}{lll}
b_1 \ldots\ldots x_4 - x_2 = x_5 & , \ x_1 = 0 & , \ x_3 = 0 \\
b_2 \ldots\ldots x_3 - x_1 = x_5 & , \ x_2 = 0 & , \ x_4 = 0 \\
b_3 \ldots\ldots x_1 - x_4 = x_5 & , \ x_2 = 0 & , \ x_3 = 0 \\
b_4 \ldots\ldots x_2 - x_3 = x_5 & , \ x_1 = 0 & , \ x_4 = 0
\end{array}\right\}.$$

So the equations of the spaces $(a_i\, b_i)$, $i = 1, 2, 3, 4$, through the opposite sides of the double four are

$$\left.\begin{array}{ll}
(a_1\, b_1) \ldots\ldots & -x_1 + x_2 + x_3 - x_4 + x_5 = 0 \\
(a_2\, b_2) \ldots\ldots & x_1 - x_2 - x_3 + x_4 + x_5 = 0 \\
(a_3\, b_3) \ldots\ldots & -x_1 + x_2 - x_3 + x_4 + x_5 = 0 \\
(a_4\, b_4) \ldots\ldots & x_1 - x_2 + x_3 - x_4 + x_5 = 0
\end{array}\right\}$$

and these four spaces have evidently the right line

$$x_1 = x_2 \quad , \quad x_3 = x_4 \quad , \quad x_5 = 0$$

in common.

It remains to be proved that the relation between the five lines, obtained by adding this new line a_5 to the four given lines a is mutual in such a sense, that all quadruples of which a_5 is a line lead back in the indicated way to the fifth line.

We prove this for the quadruple $a_1 a_2 a_3 a_5$, and for this complete this to the double four

$$\left| \begin{array}{cccc} a_1 , & a_2 , & a_3 , & a_5 \\ c_1 , & c_2 , & c_3 , & b_4 \end{array} \right|$$

and then verify that the four spaces $(a_1 c_1)$, $(a_2 c_2)$, $(a_3 c_3)$, $(a_5 b_4)$ have the line a_4 in common.

The lines c_1, c_2, c_3 as the lines of intersection of the triplets of spaces

$$\left. \begin{array}{ll} (a_3\ a_5) \ldots\ldots & -x_1 + x_2 - x_3 + x_4 + x_5 = 0 \\ (a_2\ a_5) \ldots\ldots & x_1 - x_2 - x_3 + x_4 + x_5 = 0 \\ (a_2\ a_3) \ldots\ldots & x_1 = 0 \end{array} \right\} \ldots\ldots c_1 ,$$

$$\left. \begin{array}{ll} (a_3\ a_5) \ldots\ldots & -x_1 + x_2 - x_3 + x_4 + x_5 = 0 \\ (a_1\ a_5) \ldots\ldots & -x_1 + x_2 + x_3 - x_4 + x_5 = 0 \\ (a_1\ a_3) \ldots\ldots & x_4 = 0 \end{array} \right\} \ldots\ldots c_2 ,$$

$$\left. \begin{array}{ll} (a_1\ a_5) \ldots\ldots & -x_1 + x_2 + x_3 - x_4 + x_5 = 0 \\ (a_2\ a_5) \ldots\ldots & x_1 - x_2 - x_3 + x_4 + x_5 = 0 \\ (a_1\ a_2) \ldots\ldots & -x_1 - x_2 + x_3 + x_4 + x_5 = 0 \end{array} \right\} \ldots\ldots c_3$$

are then represented by the equations

$$\left. \begin{array}{llll} c_1 \ldots\ldots & x_3 - x_4 = x_5 , & x_2 = 0 , & x_1 = 0 \\ c_2 \ldots\ldots & x_1 - x_2 = x_5 , & x_4 = 0 , & x_3 = 0 \\ c_3 \ldots\ldots & x_1 = x_3 , & x_2 = x_4 , & x_5 = 0 \end{array} \right\} ;$$

so the four spaces $(a_1\ c_1)$, $(a_2\ c_2)$, $(a_3\ c_3)$, $(a_5\ b_4)$ with the equations

$$\left.\begin{array}{lr} (a_1\ c_1) \ldots . & x_2 \qquad\qquad\qquad = 0 \\ (a_2\ c_2) \ldots . & x_3 \qquad\qquad = 0 \\ (a_3\ c_3) \ldots . & x_1 + x_2 - x_3 - x_4 + x_5 = 0 \\ (a_5\ b_4) \ldots . & x_1 - x_2 + x_3 - x_4 + x_5 = 0 \end{array}\right\}$$

really pass through the line a_4 with the equations

$$x_4 - x_1 = x_5\,, \quad x_2 = 0, \quad x_3 = 0.$$

If we complete in the same way the remaining quadruples to double fours out of the five lines a according to the notation

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 & a_3 & a_5 \\ c_1 & c_2 & c_3 & b_4 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 & a_4 & a_5 \\ d_1 & d_2 & c_3 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_3 & a_4 & a_5 \\ e_1 & d_2 & c_2 & b_2 \end{vmatrix}, \begin{vmatrix} a_2 & a_3 & a_4 & a_5 \\ e_1 & d_1 & c_1 & b_1 \end{vmatrix},$$

then it is also evident that the spaces

$$\begin{array}{lll} & (a_1\ d_1),\ (a_2\ d_2),\ (a_4\ c_3),\ (a_5\ b_3) & \text{pass through } a_3\,, \\ & (a_1\ e_1),\ (a_3\ d_2),\ (a_4\ c_2),\ (a_5\ b_2) & \text{through} \quad a_2 \\ \text{and} & (a_2\ e_1),\ (a_3\ d_1),\ (a_4\ c_1),\ (a_5\ b_1) & \quad ,, \qquad a_1. \end{array}$$

We then find for the right lines d_1, d_2, e_1 the equations

$$\left.\begin{array}{llll} d_1 \ldots . & x_2 - x_1 = x_5\,, & x_3 = 0\,, & x_4 = 0 \\ d_2 \ldots . & x_4 - x_3 = x_5\,, & x_2 = 0\,, & x_1 = 0 \\ e_1 \ldots . & x_1 = x_4\,, & x_2 = x_3\,, & x_5 = 0 \end{array}\right\},$$

by which the equations of the fifteen lines have been indicated.

By this we are now able to point out which of the fifteen lines intersect each other; it is evident that each of the fifteen lines cuts six of the remaining fourteen according to the following table:

a_1	$b_2\ b_3\ b_4\ c_2\ c_3\ d_2$	b_1	$a_2\ a_3\ a_4\ c_2\ c_3\ d_2$	c_2	$a_1\ a_3\ a_5\ b_1\ b_3\ d_1$
a_2	$b_1\ b_3\ b_4\ c_1\ c_3\ d_1$	b_2	$a_1\ a_3\ a_4\ c_1\ c_3\ d_1$	c_3	$a_1\ a_2\ a_5\ b_1\ b_2\ e_1$
a_3	$b_1\ b_2\ b_4\ c_1\ c_2\ e_1$	b_3	$a_1\ a_2\ a_4\ c_1\ c_2\ e_1$	d_1	$a_2\ a_4\ a_5\ b_2\ b_4\ c_2$
a_4	$b_1\ b_2\ b_3\ d_1\ d_2\ e_1$	b_4	$a_1\ a_2\ a_3\ d_1\ d_2\ e_1$	d_2	$a_1\ a_4\ a_5\ b_1\ b_4\ c_1$
a_5	$c_1\ c_2\ c_3\ d_1\ d_2\ e_1$	c_1	$a_2\ a_3\ a_5\ b_2\ b_3\ d_2$	e_1	$a_3\ a_4\ a_5\ b_3\ b_4\ c_3$

From this it is easy to deduce that the fifteen lines are situated six by six in ten spaces of which four pass through each line, according to the following table:

$a_1\ a_2$	$b_3\ b_4\ c_3\ e_1$	$a_3\ a_4$	$b_1\ b_2\ c_3\ e_1$
$a_1\ a_3$	$b_2\ b_4\ c_2\ d_1$	$a_1\ a_5$	$b_1\ c_2\ d_1\ d_2$
$a_1\ a_4$	$b_2\ b_3\ c_1\ d_2$	$a_2\ a_5$	$b_2\ c_1\ d_1\ d_2$
$a_2\ a_3$	$b_1\ b_4\ c_1\ d_2$	$a_3\ a_5$	$b_3\ c_1\ c_2\ e_1$
$a_2\ a_4$	$b_1\ b_3\ c_2\ d_1$	$a_4\ a_5$	$b_4\ d_1\ d_2\ e_1$

As follows from the first table each of these sextuples of right lines consists of two triplets of generatrices of the two systems of rays lying on the same quadratic surface.

With this the configuration of SEGRE is found back analytically.

We now point to another particularity which will soon be of service. Out of the first table follows that the fifteen triplets

$$a_1\,b_2\,c_3\,,\quad a_2\,b_1\,c_3\,,\quad a_3\,b_1\,c_2\,,\quad a_4\,b_1\,d_2\,,\quad a_5\,c_1\,d_2\,,$$

$$a_1\,b_3\,c_2\,,\quad a_2\,b_3\,c_1\,,\quad a_3\,b_2\,c_1\,,\quad a_4\,b_2\,d_1\,,\quad a_5\,c_2\,d_1\,,$$

$$a_1\,b_4\,d_2\,,\quad a_2\,b_4\,d_1\,,\quad a_3\,b_4\,e_1\,,\quad a_4\,b_3\,e_1\,,\quad a_5\,c_3\,e_1$$

consist of three lines intersecting each other two by two. The question whether the three lines of a triplet lie in a same plane, pass through a same point or show both particularities of situation is analytically easy to answer. We immediately find that the lines of a triplet always pass through a same point but never lie in a same plane. So we find fifteen new points connected with the configuration and in accordance with this fifteen "spaces through three lines."

If the 15 "points in three lines" and the 45 "planes through two lines" are considered as parts of the configuration, we then find that each of the points lies in three lines, in fifteen planes and in seven spaces, each of the lines lies in six planes and in three spaces and each of the planes lies in three spaces, whilst reversely each of the lines passes through three points, each of the planes passes through five points and through two lines, each of the spaces passes through seven points, through three lines and through nine planes. This all is given summarily in the symbol

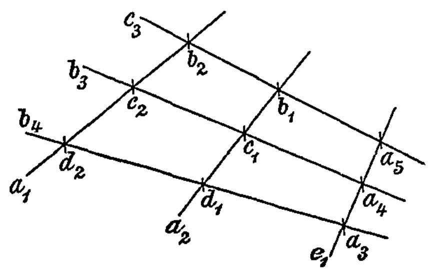

Fig. 3.

$$Cf.\ (15_{3,15,7} \mid {}_{3,}15_{,6,3} \mid {}_{5,2,}45_{,3} \mid {}_{7,3,9,}15),$$

by which we represent the configuration extended in this way.

For clearness' sake we assemble in fig. 3 the elements of the configuration lying in the space $(a_1\, a_2)$; the nine lines not lying in it are represented by their points of intersection.

3. The fifteen found lines are written as follows in the form of a determinant which with respect to a diagonal, here the diagonal of the missing elements, is symmetric

a_1	a_2	a_3	a_4	a_5	
b_1	b_2	b_3	b_4		a_5
c_1	c_2	c_3		b_4	a_4
d_1	d_2		c_3	b_3	a_3
e_1		d_2	c_2	b_2	a_2
	e_1	d_1	c_1	b_1	a_1

and the following laws are proved:

1°. If we state that the five lines among which the relation of the five lines a exists, are conjugate to each other, then each row or each column of the determinant contains five conjugate lines.

2°. Each of these six quintuples of conjugate lines leads back to the fifteen lines, if we search for the lines of intersection of the ten spaces through the lines of the quintuples two by two.

3°. Every two rows or two columns of the determinant furnish a double four after omission of the two elements of which the corresponding ones are wanting.

The proof for the first law is immediately given. If we connect with the first row one of the others, say the fourth, it follows from the definition of the last merely, that

$$\begin{vmatrix} d_1 & , & d_2 & , & c_3 & , & b_3 \\ a_1 & , & a_2 & , & a_4 & , & a_5 \end{vmatrix}$$

is a double four and that the four spaces $(d_1\, a_1)$, $(d_2\, a_2)$, $(c_3\, a_4)$, $(b_3\, a_5)$ intersect each other according to a right line, which must be a_3, the five lines a being conjugate; so the five lines of the fourth row are also conjugate.

The second law is immediately proved out of the second of the two tables. And the third follows out of the first table, when in connection with the above-named determinant we reduce this to the observation that two of the fifteen lines intersect each other when they belong nowhere in the determinant to the same row or the same column. The word "nowhere" inserted here refers to the circumstance, that each line appears twice. So taken all together the system of the fifteen lines contains fifteen double fours; each of these we can call in the configuration opposite to the right line, which is the section of the four spaces passing through the opposite elements of the double four.

The analytical representation of the fifteen right lines as well as the notation of the determinant leaves still something to be desired. In one as well as the other the circumstance that four of the fifteen lines a are brought to the foreground harms the regularity. Firstly we shall now try to improve the notation of the determinant.

Starting from the five lines a the other ten lines are found as common transversals of the triplets to be formed out of those five lines. This leads to the idea of representing all lines by a, where the original lines a retain their index for the present, each of the remaining lines however is to be indicated by an a with two indices derived from those two of the original quintuple which does not cut it. Then the ten lines

$$b_1\ ,\ b_2\ ,\ b_3\ ,\ b_4\ ,\ c_1\ ,\ c_2\ ,\ c_3\ ,\ d_1\ ,\ d_2\ ,\ e_1$$

become

$$a_{15}\ ,\ a_{25}\ ,\ a_{35}\ ,\ a_{45}\ ,\ a_{14}\ ,\ a_{24}\ ,\ a_{34}\ ,\ a_{13}\ ,\ a_{23}\ ,\ a_{12}\,.$$

If moreover we add to each of the lines of the original quintuple the index 6 and if we allow $a_{i,k}$ to be written $a_{k,i}$, then the previous determinant passes after a quarter of rotation into the entirely regular form:

	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
a_{21}		a_{23}	a_{24}	a_{25}	a_{26}
a_{31}	a_{32}		a_{34}	a_{35}	a_{36}
a_{41}	a_{42}	a_{43}		a_{45}	a_{46}
a_{51}	a_{52}	a_{53}	a_{54}		a_{56}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	

In this entirely regular notation two different lines a cross or intersect each other according to their having an index in common or not. In the first place we of course think here of the fifteen lines of the three-dimensional space which are left when from the 27 right lines of a cubic surface a double six is set aside; for, these lines behave in regard to the crossing and cutting in quite the same way with corresponding notation. However, we must not lose sight of the fact, that in the configuration three lines cutting each other two by two pass through a point without lying in a plane, whilst with an arbitrary surface of order three they lie in a plane without passing through a point. If however we polarize the surface of order three to a surface of class three, for instance with respect to a sphere lying in the same space, then the 27 right lines, bearers of points and of double tangent planes, pass into 27 right lines, bearers of tangent planes and double points, whilst each double six of the surface of order three transforms itself into a double six of the surface of class three. In the second place — and with more right — we see therefore in the notation with two indices applied above to the lines of the configuration the notation of the fifteen lines of three-dimensional space forming with a double six the 27 right lines of a surface of class three.

If we project the fifteen right lines of the configuration out of any point P, neither with two crossing lines of the fifteen in the same space nor with two cutting lines of the fifteen in the same plane, on any space S_3 not containing this point P, we then have in S_3 fifteen lines a'_{kl}, which are really bearers of tangent planes of a single surface of class three. By polarization of a wellknown proof we find namely, that the lines

$$\begin{vmatrix} a'_{12} & a'_{34} & a'_{56} \\ a'_{45} & a'_{61} & a'_{23} \\ a'_{36} & a'_{25} & a'_{14} \end{vmatrix}$$

as the lines connecting two triplets of points representing degenerated surfaces of class three form the developable surface degenerated into nine pencils of planes enveloping a tangential pencil of surfaces of class three. And now the surface belonging to that tangential pencil touching at the same time the plane passing through a'_{13} and a'_{24} has with each of the six pencils of planes round the remaining lines

$$a'_{13} \;,\; a'_{15} \;,\; a'_{24} \;,\; a'_{26} \;,\; a'_{35} \;,\; a'_{46}$$

four planes in common, so that this surface has all the fifteen lines a'_{kl} as bearers of tangent planes.

But this same surface of class three is moreover connected in a simple manner with the sextuples of the conjugate lines which in future we shall briefly indicate by the sign (v_i), where i stands for the common index of the lines. For, this surface contains besides the fifteen lines a' still twelve lines forming a double six. And if we indicate this double six in connection with these lines a' in the manner customary with the surface of order or class three by

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$$\begin{vmatrix} b_1 , & b_2 , & b_3 , & b_4 , & b_5 , & b_6 \\ c_1 , & c_2 , & c_3 , & c_4 , & c_5 , & c_6 \end{vmatrix}$$

then each pair of opposite elements b_i, c_i meets the lines of the conjugate quintuple (v_i), from which ensues that the pairs of planes (Pb_i), (Pc_i) do so likewise. So we have proved the following theorem:

Through any point P two planes β_i, γ_i pass cutting the lines of the conjugate quintuple (v_i). The six pairs of planes β_i, γ_i, $(i = 1, 2, \ldots 6)$, and the fifteen planes connecting P with the lines of the configuration are cut by an arbitrary space not passing through P according to the 27 right lines of a same surface of class three.

Of the lines cutting four arbitrary planes $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ given in S_4 two lie in an arbitrary space S'_3; for this space cuts the four given planes according to four crossing lines, which admit of two common transversals. By dualistic reversion it ensues from this, that through an arbitrary point P two planes pass cutting any four given lines a_1, a_2, a_3, a_4. And then the above mentioned theorem teaches us that the connection among the five conjugate lines a_i is also expressed by the circumstance, that each plane cutting four of the five lines also cuts the fifth. This characteristic property forms the starting point of Dr. SEGRE's considerations.

In the second part of this communication will be indicated how the analytical representation of the fifteen planes can be simplified; so I will have occasion to show the relation of my results to two studies of Dr. G. CASTELNUOVO, of the existence of which I was not aware in the moment this first part was passing through the press.

Chemistry. — Professor LOBRY DE BRUYN presents a dissertation from Dr. N. SCHOORL and a communication on: "*Urea derivatives* (*carbamides*) *of sugars*". II.

In continuation of the first communication (see report of 29 Dec. 1900) the following has been taken from the said dissertation.

The method of determining the *molecular weight* by means of the increase in the boiling point gave unsatisfactory results with glucose-ureïde because, as was shown afterwards, this is decomposed by