echelon spectroscope concerning the difference in wave-length between the components of the outer components of the sextet of the blue (4358) line of mercury. The following table is an extract ($\delta\lambda_3$ in A.U.)

H	$\delta\lambda_3$
5.000	.....
12.100	.....
12.900	0.052
20.000	0.098?
21.300	0.09
23.400	0.098

For a value of the field between 12.100 and 12.900 the splitting up of the lines becomes sufficient to make them appear as separate lines *on a photograph* (upon which the measurements were taken). Two lines can of course be *seen* separated at a considerably smaller distance.

Thus now $q = \frac{0{,}052}{4358} = 11{,}9 . 10^{-6}$ and q_e considerably smaller. For the echelons of these observers we have $t = 7{,}5$, $n = 15$.

With these data I calculate $q_t = 5{,}3 . 10^{-6}$.

Thus it appears from the data given in this paper that it is possible to manufacture echelons, performing nearly as well as they are theoretically capable.

Mathematics. — "*Considerations in reference to a configuration of* SEGRE". By Prof. P. H. SCHOUTE. (Second part).

5. We have already remarked that the form of the equations of the fifteen lines obtained in the first part of this communication was not yet a quite regular one. If we shorten $x_1 - x_3 = x_5$ into $(1 - 3)$, $x_1 = x_2$ into 12 and if everywhere we omit the equations $x_1 = 0$, $x_2 = 0, \ldots x_5 = 0$, then the following table gives the obtained result in the form of the determinant repeatedly used

(1—3)	(2—4)	(3—2)	(4—1)	(12, 34)	
(4—2)	(3—1)	(1—4)	(2—3)		(12, 34)
(3—4)	(1—2)	(13, 24)		(2—3)	(4—1)
(2—1)	(4—3)		(13, 24)	(1—4)	(3—2)
(14, 23)		(4—3)	(1—2)	(3—1)	(2—4)
	(14, 23)	(2—1)	(3—4)	(4—2)	(1—3)

This table shows that so far there is regularity in the irregularity, that each conjugate quintuple as to this irregularity corresponds with the quintuple of the lines a_i.

Before we pass to an entirely regular form of the equations of the fifteen lines, we determine, also to show the fitness of the system of equations, the locus of the planes cutting the four lines a given originally. For this we search for the conditions, under which the space

$$p_x \equiv p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4 + p_5 x_5 = 0$$

contains such a plane. The number of planes cutting four lines given arbitrarily in S_4 being twofold infinite, this investigation must lead us to a homogeneous equation $f(p) = 0$ in the five spacial coordinates p_i, the tangential equation of the curved space enveloped by the spaces $p_x = 0$.

The coordinates of the points of intersection of the space $p_x = 0$ with the four lines a_1, a_2, a_3, a_4 are the elements of the four rows of the matrix

$$\left|\begin{matrix} p_3 - p_5, & 0, & -(p_1 + p_5), & 0, & p_1 + p_3 \\ 0, & p_4 - p_5, & 0, & -(p_2 + p_5), & p_2 + p_4 \\ 0, & -(p_3 + p_5), & p_2 - p_5, & 0, & p_2 + p_3 \\ -(p_4 + p_5), & 0, & 0, & p_1 - p_5, & p_1 + p_4 \end{matrix}\right| ;$$

by putting this matrix equal to naught, we let the space $p_x = 0$ satisfy the given condition. Here, however, an obstacle seems to present itself. For the five equations obtained by putting the determinants comprised in the matrix equal to naught, furnish in general two respectively independent relations, which cannot be the case here. However, as is immediately evident after development, each of those five determinants consists really of the form

$$(p_1 + p_2 + p_3 + p_4) p_5^2 + p_2 p_3 p_4 + p_1 p_3 p_4 + p_1 p_2 p_4 + p_1 p_2 p_3$$

every time multiplied by another linear form, and we find the wanted equation of the enveloped surface by putting this common factor equal to naught.

The same obstacle seems to appear when we make use of the following method to determine the equation of the enveloped space. If $p_x = 0$, $q_x = 0$ represent an arbitrary plane, it intersects the four lines a_1, a_2, a_3, a_4 under the conditions

$$\begin{vmatrix} p_1 & p_3 & p_5 \\ q_1 & q_3 & q_5 \\ -1 & 1 & 1 \end{vmatrix} = 0, \quad \begin{vmatrix} p_2 & p_4 & p_5 \\ q_2 & q_4 & q_5 \\ -1 & 1 & 1 \end{vmatrix} = 0, \quad \begin{vmatrix} p_3 & p_2 & p_5 \\ q_3 & q_2 & q_5 \\ -1 & 1 & 1 \end{vmatrix} = 0, \quad \begin{vmatrix} p_4 & p_1 & p_5 \\ q_4 & q_1 & q_5 \\ -1 & 1 & 1 \end{vmatrix} = 0$$

and by eliminating q_1, q_2, q_3 we arrive at the equation

$$\begin{vmatrix} p_3 - p_5, & 0, & -(p_1 + p_5), & (p_1 + p_3) q_5 \\ 0, & p_4 - p_5, & 0, & -(p_2 + p_5) q_4 + (p_2 + p_4) q_5 \\ 0, & -(p_3 + p_5), & p_2 - p_5, & (p_2 + p_3) q_5 \\ -(p_4 + p_5), & 0, & 0, & (p_1 - p_5) q_4 + (p_1 + p_4) q_5 \end{vmatrix} = 0,$$

which also furnishes two equations, as it must hold good for all values of the quotient $\frac{q_4}{q_5}$. We recognise in these two equations immediately those which are obtained by omitting from the matrix found above respectively the last column and the last but one.

By the way we notice that the second method can prove in a simple way how each plane intersecting a_1, a_2, a_3, a_4 also cuts a_5. If we introduce for the determinant $p_k q_l - q_l p_k$ the notation (kl) the four conditions can be written in the form

$$\left.\begin{array}{l} (53) + (51) + (13) = 0 \\ (54) + (52) + (24) = 0 \\ (25) + (35) + (23) = 0 \\ (15) + (45) + (14) = 0 \end{array}\right\}.$$

So addition gives

$$(13) + (14) + (23) + (24) = 0,$$

which is the condition that the plane cuts the line a_5. For substituting

$$x_1 = x_2 \quad , \quad x_3 = x_4 \quad , \quad x_5 = 0$$

into $p_x = 0$ and $q_x = 0$ we find

$$(p_1 + p_2)\,x_2 + (p_3 + p_4)\,x_4 = 0,$$

$$(q_1 + q_2)\,x_2 + (q_3 + q_4)\,x_4 = 0;$$

by eliminating the quotient $\frac{x_2}{x_4}$ we get

$$\begin{vmatrix} p_1 + p_2 & , & p_3 + p_4 \\ q_1 + q_2 & , & q_3 + q_4 \end{vmatrix} = 0,$$

which can be immediately developed into

$$(13) + (14) + (23) + (24) = 0.$$

6. Now that we have found the equation of the enveloped space the full investigation of it may be omitted. We shall confine ourselves to some ready observations.

In the first place it is evident that the ten "spaces through six lines" with the spacial coordinates

$(a_1 a_2)$	(−1,	−1,	1,	1,	1)
$(a_1 a_3)$	(0,	0,	0,	1,	0)
$(a_1 a_4)$	(0,	1,	0,	0,	0)
$(a_1 a_5)$	(−1,	1,	1,	−1,	1)
$(a_2 a_3)$	(1,	0,	0,	0,	0)
$(a_2 a_4)$	(0,	0,	1,	0,	0)
$(a_2 a_5)$	(1,	−1,	−1,	1,	1)
$(a_3 a_4)$	(1,	1,	−1,	−1,	1)
$(a_3 a_5)$	(−1,	1,	−1,	1,	1)
$(a_4 a_5)$	(1,	−1,	1,	−1,	1)

are double spaces of the enveloped space, so that this must be of order four and cannot admit of an eleventh double space, because a space of order two cannot be of class three.

Moreover it is evident, that the fifteen "points in three lines" are points for which the tangent planes of the curved space, passing through it, envelope a conic space degenerated into three nets of planes (all planes through a line). So the three lines a_5, c_3, e_1 cut each other in the point

$$x_1 = x_2 = x_3 = x_4, \quad x_5 = 0$$

with the equation

$$p_1 + p_2 + p_3 + p_4 = 0$$

and the combination of this with the equation of the cubic envelope causes the latter to be transformed by elimination of p_4 into

$$(p_2 + p_3)(p_3 + p_1)(p_1 + p_2) = 0,$$

which in connection with the first furnishes the planes

$$\left.\begin{matrix} p_2 + p_3 = 0 \\ p_1 + p_4 = 0 \end{matrix}\right\}, \quad \left.\begin{matrix} p_3 + p_1 = 0 \\ p_2 + p_4 = 0 \end{matrix}\right\}, \quad \left.\begin{matrix} p_1 + p_2 = 0 \\ p_3 + p_4 = 0 \end{matrix}\right\}$$

with the axes e_1, c_3, a_5.

But the following must be pointed out particularly: the surface of the third class out of n⁰. 4 is connected in a simple way with the enveloped space. In the cone by which this surface is projected from the point P taken there, we have namely before us the envelope of all the tangent planes of the cubic space, passing through this point P. This will be clear if we resume in the following form the dualistically opposite results forming an extension of the theorem of SEGRE mentioned in n⁰. 1:

If we take quite arbitrarily in S_4 four planes α_{13}, α_{14}, α_{15}, α_{16}, and if we determine the planes α_{23}, α_{24} α_{25}, α_{26}, forming with the former one a double four

$$\begin{vmatrix} \alpha_{13} , & \alpha_{14} , & \alpha_{15} , & \alpha_{16} \\ \alpha_{23} , & \alpha_{24} , & \alpha_{25} , & \alpha_{26} \end{vmatrix}$$

— where two planes have a point or a line in common according to their symbols having a common index or not —, then the four points of intersection of the opposite elements of the double four — placed here under each other — lie in a same plane α_{12}.

If we add this plane α_{12} to the assumed planes, we obtain a quintuple of "conjugate planes" with the remarkable property, that each of those planes plays the same part in reference to the double four, of which the four remaining planes form one of the two quadruples, as α_{12} in reference to the above mentioned double four.

If we complete all quadruples to be formed out of this quintuple to double fours, we find fifteen planes in all, which can be characterised by symbols $\alpha_{k,l}$ in such a way, that two planes have a line or point in common according to their symbols having a common index or not. We then find that ordering of those symbols in form of determinants — as the corresponding $a_{k,l}$ in n⁰. 4 — gives six rows or six columns of conjugate quintuples (φ_i), $i=1,2,...6$. Each line intersecting four planes of a conjugate quintuple also cuts the fifth.

Each triplet of planes $(\alpha_{12}, \alpha_{34}, \alpha_{56})$, cutting each other two by two in a line, lie in a same space indicated by $S_{12,34,56}$; in this space they pass through a same point $P_{12,34,56}$. There are fifteen of such spaces and points.

Each sextuple of points as $(\alpha_{12}, \alpha_{13}, \alpha_{23}, \alpha_{45}, \alpha_{46}, \alpha_{56})$, built up of two triplets $(\alpha_{12}, \alpha_{13}, \alpha_{23})$ and $(\alpha_{45}, \alpha_{46}, \alpha_{56})$, with the property that each plane of one triplet cuts each plane of the other in a line, pass through a same point $P_{123,456}$. There are ten such points.

A threedimensional space S_3 taken arbitrarily cuts each of the six conjugate quintuples (φ_i) in five lines, which admit of two common transversals (b_i, c_i); these six pairs of lines (b_i, c_i) are opposite elements of a double six of a surface F^3 of order three of which the 27 right lines consist of these twelve lines and the fifteen lines of intersection of S_3 with the planes $\alpha^{k,l}$.

The locus of the lines intersecting four planes belonging to a same conjugate quintuple — and so also the fifth — is always the same curved space $S^{3,4}$ of order three and class four through the fifteen planes $\alpha_{k,l}$, whichever of the six quintuples (φ_i) are taken; so this space $S^{3,4}$ contains six different twofold infinite systems of right lines. It has the ten points $P_{123,456}$ as double points, the quadratic conic spaces of contact of which contain the sextuples of planes passing through those points; it is cut into three planes by each of the fifteen spaces $S_{12,34,56}$. Its section with the above introduced arbitrary space S_3 must contain the double six of the pairs of lines (b_i, c_i) as well as the fifteen lines of intersection of S_3 with the planes $\alpha_{k,l}$ and so it must coincide with the surface F^3 found there, of which the points lying outside these 27 lines are points of intersection of S_3 with lines of the locus $S^{3,4}$ not situated in S_3.

7. If we apply to the equations

$$\left.\begin{array}{lll} x_1 - x_3 = x_5 \ , & x_2 = 0 \ , & x_4 = 0 \\ x_2 - x_4 = x_5 \ , & x_1 = 0 \ , & x_3 = 0 \\ x_3 - x_2 = x_5 \ , & x_1 = 0 \ , & x_4 = 0 \\ x_4 - x_1 = x_5 \ , & x_2 = 0 \ , & x_3 = 0 \\ x_1 = x_2 \ , & x_3 = x_4 \ , & x_5 = 0 \end{array}\right\}$$

of the lines $a_1, a_2, \ldots a_5$ the transformation

$$\left.\begin{aligned} -x_1+x_2+x_3+x_4-x_5&=y_1\\ x_1-x_2+x_3+x_4-x_5&=y_2\\ x_1+x_2-x_3+x_4-x_5&=y_3\\ x^1+x_2+x_3-x_4-x_5&=y_4\\ -2x_5&=y_5 \end{aligned}\right\},$$

which it is possible to write in the reversed form

$$\left.\begin{aligned} 4x_1&=-y_1+y_2+y_3+y_4-y_5\\ 4x_2&=y_1-y_2+y_3+y_4-y_5\\ 4x_3&=y_1+y_2-y_3+y_4-y_5\\ 4x_4&=y_1+y_2+y_3-y_4-y_5\\ 2x_5&=-y_5 \end{aligned}\right\},$$

these equations pass into

$$\left.\begin{aligned} y_1=y_5\;,\quad y_2=y_4\;,\quad y_3=0\\ y_2=y_5\;,\quad y_1=y_3\;,\quad y_4=0\\ y_3=y_5\;,\quad y_1=y_4\;,\quad y_2=0\\ y_4=y_5\;,\quad y_2=y_3\;,\quad y_1=0\\ y_1=y_2\;,\quad y_3=y_4\;,\quad y_5=0 \end{aligned}\right\}.$$

Moreover the equations of all the fifteen lines of the configuration present themselves in the collective formula

$$y_p=y_q\,,\quad y_r=y_s\,,\quad y_t=0\,,$$

where p, q, r, s, t indicate one of the permutations of the five indices $1, 2, \ldots 5$.

The regular representation of the lines obtained in this way allows of a very simple geometrical realization. If we suppose for simplicity's sake that the homogeneous system of coordinates y_i with respect to the five-cell of the spaces $y_i=0$ is a system of normal distance coordinates, then it is evident that the fifteen lines of the configuration are the lines connecting the mid-points of the pairs of edges of that five-cell crossing each other. So, if we suppose (fig. 4)

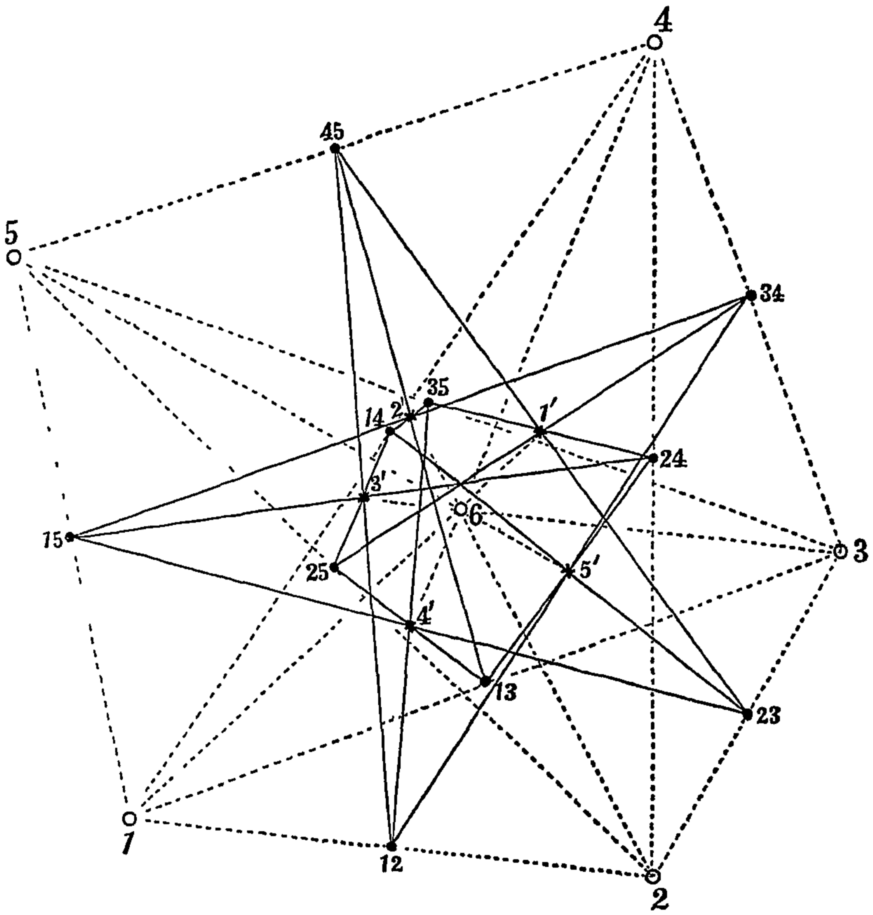

Fig. 4.

five points 1, 2, 3, 4, 5 not lying in a three-dimensional space to be the bearers of equal masses and if we determine the barycentres 12, 13, . . . 45 of the ten pairs of these masses, then the lines (12, 34), . . . , (23, 45) connecting two of these points belonging to four different masses will form the fifteen lines of a configuration of SEGRE. These lines pass through one of the barycentres 1', 2', 3', 4', 5' of four of the five masses; moreover the five lines (1, 1'), (2, 2'), . . , (5, 5') pass through the barycentre 6 of the five masses. Out of this figure we easily find the remaining elements of the configuration

Cf. (15, 3, 15, 7 | 3, 15, 6, 3 | 5, 2, 45, 3 | 7, 3, 9, 15).

For we recognize in the ten barycentres 12, 13, . . , 45 of two masses and the five barycentres 1', 2', . . , 5' of four of the five masses the fifteen points, in the thirty planes (12, 34, 35, 4', 5') and the fifteen planes (1', 24, 35, 23, 45) the forty-five planes, in the ten spaces (12, 34, 35, 45, 3', 4', 5') and the five limiting spaces of the five-cell the fifteen spaces of the configuration. And if we assume in the points 1, 2, 3, 4, 5 entirely arbitrary different masses instead of equal ones, then the special case represented in fig. 4 passes into the general one. If we then also call the barycentre of those unequal masses the point 6, we arrive at the following simple representation of the configuration:

If in space S_4 six points 1, 2, . . , 6 are taken in such a way that no five of these points lie in a three-dimensional space, we can find fifteen three-dimensional spaces $R_3^{(1,2)} = (3456)$, $R_3^{(1,3)} = (2456)$, . . , $R_3^{(5,6)} = (1234)$ each one of which contains four of the six points. Of these spaces $S_3^{(i,k)}$ any three, the indices i, k of which complete each other to 1, 2, . . , 6, pass through a same line furnishing in all fifteen lines. And these same lines forming with each other the chief part of a configuration of SEGRE are also found if each of the lines l_{12}, l_{13} . . , l_{56} connecting the six points two by two is cut by the opposite space $R_3^{(12)}$, $R_3^{(13)}$, . . , $R_3^{(56)}$ by which operation we obtain fifteen points 12, 13, . . , 56 characterized by the property that any three points the indices of which complete each other to 1, 2, . . , 6 are lying on a right line. Every five lines containing together the fifteen points 12, 13, . . , 56 form a quintuple of conjugate lines.

8. The simple representation we have now given of SEGRE's configuration is closely related to results published already in 1888 by Dr. G. CASTELNUOVO in his treatise „Sulle congruenze del terzo ordine dello spazio a quattro dimensioni" (*Atti del R. Istituto Veneto*, serie 6, vol. 6). If namely we assume the five points 1, 2, 3, 4, 5 as vertices $p_i = 0$, $i = 1, 2, 3, 4, 5$, of the five-cell of coordinates and the point 6 as point of unity $p_6 \equiv (p_1 + p_2 + p_3 + p_4 + p_5) = 0$, then the pair of equations

$$p_1 + p_2 = 0, \quad p_3 + p_4 = 0,$$

into which the system

$$x_1 = x_2, \quad x_3 = x_4, \quad x_5 = 0$$

passes by means of the relation

$$p_x \equiv p_1 x + p_2 x_2 + p_3 x_2 + p_4 x_4 + p_5 x_5 = 0$$

between the coordinates x_i of points and the coordinates p_i, of spaces, can be completed by

$$p_5 + p_6 = 0$$

and from this is evident, that each of the fifteen lines of SEGRE's configuration is represented by a triplet of equations of the form

$$p_1 + p_2 = 0, \quad p_3 + p_4 = 0, \quad p_5 + p_6 = 0$$

and that the lines are situated six by six in ten spaces with the coordinates

$$(1, 1, 1, -1, -1, -1), \ (1, 1, -1, 1, -1, -1), \ldots (-1, -1, -1, 1, 1, 1).$$

So the six lines

(14, 25, 36), (15, 24, 36), (16, 23, 45)

(14, 26, 35), (15, 23, 46), (16, 24, 35)

are situated in the first of those ten spaces, etc.

In the quoted treatise CASTELNUOVO has represented the fifteen planes of the dualistically opposite figure by the system of the fifteen corresponding triplets of equations

$$x_1 + x_2 = 0, \quad x_3 + x_4 = 0, \quad x_5 + x_6 = 0.$$

Though the connection between the above-mentioned treatise and these considerations have already hereby been indicated, yet I desire to acknowledge that not until the existence of that treatise had been brought to my notice by SEGRE did I succeed in deducing from my considerations founded upon the double four the above-given simple representation.

Out of the fifteen triplets of equations

$$p_1 + p_2 = 0, \quad p_3 + p_4 = 0, \quad p_5 + p_6 = 0$$

it directly ensues that the curved space of class three enveloped by the pencils of planes with one of the lines of the configuration as axis must have the equation

$$p_1^3 + p_2^3 + p_3^3 + p_4^3 + p_5^3 + p_6^3 = 0,$$

the first member disappearing for each of those triplets.

So the six points $p_i = 0$ are for this curved space of class three what the pentaeder of SYLVESTER is for the surface of order three.

9. In a second treatise published in 1891 and entitled: „Ricerche di geometria delle rette nello spazio à quattro dimensioni" (*Atti del. R. Istituto Veneto*, series 7, vol. 2) CASTELNUOVO has represented the curved space

$$\varrho^3 \equiv x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 = 0$$

on our space S_3 in such a way, that the spacial sections of $\varrho^3 = 0$ correspond with quadratic surfaces passing through five fixed points. In that case the fifteen planes correspond to the five vertices and to the ten faces of a complete quintangle in S_3, whilst the ten double points of $\varrho^3 = 0$ correspond to the ten edges of this quintangle. Instead of continuing these researches we put the question in how far the configuration of fifteen lines is unique in its kind.

Of course it is not difficult to point out in the poly-dimensional spaces configurations having characteristics in common with the configuration of SEGRE. So we find one in each group of $n + 2$ points taken arbitrarily in S_n when $n + 2$ is not a prime number. Let us take as an example nine arbitrary points 1, 2, 3, . . , 9 in S_7 and let us represent the point of intersection of the plane (1, 2, 3) with the space S_5 through the six remaining points by the symbol P_{123}; then each three points P_{123}, P_{456}, P_{789}, whose indices complete each other to 1, 2, 3. . . , 9, will be situated on a right line. It may be, that the easiest way of proving this is by the aid of a system of parallel forces in equilibrium applied to the points 1, 2, 3, . . , 9. As is known the parallel forces applied to the points 1, 2, 3, . . , 8 may be chosen in such a way that the resultant acts on point 9; if we add to these eight forces a force applying in point 9 equal and opposite to this resultant, then such a system of forces in equilibrium has been obtained. It is now evident that point P_{123} is the point of application ot the resultant of the three forces working at the forces 1, 2, 3 as well as that of the resultant of the six remaining ones. For, these points of application must coincide, on account of the equilibrium, in a point situated in the plane (1, 2, 3) as well as in the space S_5 through the six other points. If we reduce the nine forces to three by compounding those operating in 1, 2, 3 and those operating in 4, 5, 6 and those operating in 7, 8, 9, we obtain those parallel forces applied in P_{123}, P_{456}, P_{789} and these three forces can only then be in equilibrium when the three points of application lie on a right line. So the figure of the nine points S_7 leads to $(9)_3 = 84$ points P_{123}, situated three by three on $\frac{1}{6}\ (9)_3 . (6)_3 = 280$ lines, whilst reversely ten of those 280 lines pass through each of those 84 points. So we can deduce out of 12 arbitrary points in S_{10}, performing the decomposition of 12 into two factors in different ways, 66 points P_{12}, 220 points P_{123} or 495 points P_{1234} and remark that the points P_{12} are situated six by six in 10395 spaces S_4, the points P_{123} four by four in 15400 planes and the points P_{1234} three by three in 13305600 lines, etc.

Although the configuration of SEGRE is undoubtedly a part of the general group indicated here, it is certainly distinguished from most of them and probably from all of them by the property that it is determined by a quadruple of crossing lines taken arbitrarily and these lines fix in a narrower sense a fifth of the fifteen lines, which group of conjugate lines then bear together the fifteen points of the configuration. Indeed, in S_4 the system of six points as well as that of the four lines is dependent on 24 parameters and so these figures agree in number of constants, according to an expression of SCHUBERT. If on this point we examine the configuration deduced from the nine points of S_7, it is even evident by merely consulting the numbers of constants 63 and 12 of the nine points and of a line in S_7, that it is impossible to determine the 280 right lines (123, 456, 789) by some of them crossing each other;

for 63 is not divisible by 12. In the second example of the twelve points in S_{10} this is likewise proved for the configuration of the 13305600 lines from the fact, that 18 is not a factor of 120. However, the numbers of constants of plane and four-dimensional space in S_{10}, namely 24 and 30, being factors of 120, considerations of another kind only can teach us that the 15400 planes and 10395 spaces S_4 cannot be determined by some of them crossing each other. So a characteristic difference between these two examples in S_{10} and the configuration of SEGRE would already appear if it was proved that five planes in a narrower sense do not lead to 55 planes through the 220 points, and four spaces S_4 in a narrower sense not to eleven spaces S_4 through the 66 points. And should this prove to be the case in one of the two, there still remains the difference that in S_4 five lines are found intersecting six arbitrary planes and that these lines are related in such a way that each line cutting four of the five lines also cuts the fifth; whilst according to a general formula of SCHUBERT (*Mitt. der math. Gesellschaft in Hamburg*, vol. 2, pag. 87, 1883) in S_{10} are found not only 55 but 116848170 planes and not only 11 but 689289872070 spaces S_4, which have respectively a point in common with each of the 24 arbitrary spaces S_7 and with each of the 30 arbitrary spaces S_5.

Finally we must stated that H. W. RICHMOND has made the figure of the six arbitrary points in the space S_4, called by him "hexastigm", the subject of two papers (*Quarterly Journal of Math.*, vol. 31, pag. 125—160, 1899 and *Math. Ann.*, vol. 53, pag. 161—176, 1900). In these important studies the configuration of SEGRE and its simplest analytical representation is brought into close connection with G. VERONESE's theorems about the PASCAL hexagram; but, comparatively spoken, the curved space of SEGRE is only cursorily mentioned.

Chemistry. — Professor LOBRY DE BRUYN presents a communication from Dr. J. J. BLANKSMA: "*On the influence of different atoms and atomic groups on the conversion of aromatic sulphides into sulphones.*"

It is known that organic sulphides may be converted first into sulphoxides and then into sulphones by means of oxidising agents, nitric acid for instance. From the following observations it appears to what extent atoms or groups which occupy an ortho-position in regard to the sulphur atom, render the latter unoxidisable.