Finally we come to the last question, of great signification for the physiological bacteriology of the intestinal canal: Have the micro-organisms of the intestinal canal of the rabbit to play a part in the digestion?

If attention is paid to the following facts:

1°. The very small number of living bacteria with respect to the number of grams of intestinal contents;

2°. The very small number of living bacteria with regard to the number of dead ones, in particular perceptible from the high sterility-indices of the whole intestinal canal, and from the slight number of living bacteria found on 1 million of dead organisms, and

3°. That at no single place there is a multiplication, on the contrary, that nearly in the whole intestinal canal there is a mortality on large scale of living bacteria, we are obliged to deny the bacteria playing any part in the digestion in the intestinal canal of the rabbit.

Mathematics. — „*On the motion of variable systems*" by Prof. CARDINAAL.

1. With considerations relative to the theory of motion, we generally start from the principle that two phases of the system are congruent. If the two systems considered in this way are situated in a plane the pole of the motion is the only real point of coincidence of the two systems; if they are situated in space the principal axis of the motion is their line of coincidence. If we suppose the second system to have approached the first at infinitesimal distance, the rays connecting the homologous points are directions of velocities and one of the principal problems of motion consists of the construction of the directions of these velocities. Special constructions exist for this, the second system not being suitable for use.

2. In the plane the construction of the direction of velocities is a simple matter, the polar rays being normals to the orbits and the velocities touching them. In space the construction becomes already more elaborate; however, we can notice that the directions of velocities are the rays of a tetraedral complex; to this complex belongs moreover a focal system, the properties of which enable us to find the points belonging to these rays. This paper now purposes to investigate this same subject for systems of points in space changing projectively during their motion. The investigation is independent

of the length of the velocities; however it is necessary to investigate more closely the above-mentioned tetraedral complex.

3. When a system remaining congruent to itself is displaced, two opposite edges of the tetraeder of coincidence (principal tetraeder) are real: the principal axis l and the line l_∞ at infinity common to the planes normal to l. Now there must be on l as well as on l_∞ two real or imaginary vertices; if we call the first pair P, Q and the second R, S, then P, Q are the double points of two congruent ranges of points on l and therefore united in *one* point at infinity; R, S are the double points of two congruent systems in the plane λ_∞ at infinity, therefore two cyclic points of any plane normal to l.

From this ensues:

Of the four vertices of the principal tetraeder of the complex two coincide in λ_∞, the two other being the cyclic points of a plane normal to l; of the faces of this tetraeder two likewise coincide in λ_∞.

Suppose the direction of a velocity v_a is given and we wish to construct the point A possessing this direction of velocity; then we have to construct the line d of shortest distance from l and v_a; this cuts v_a in A. If we bring a plane α through d normal to v_a then this plane is normal in A to the locus of A; the rays through A in α are the normals to the orbits and at the same time rays of a focal system having l and l_∞ as conjugate polars. Thus the focal system is connected with the tetraedral complex.

4. Let us now suppose, that the system in motion changes projectively. If we imagine two positions of the system, then the points of coincidence are the vertices of the principal tetraeder $PQRS$, which tetraeder we suppose for our further consideration to be constructed and for the present entirely real. The tetraedral complex is determined by the principal tetraeder and the line connecting one pair of homologous points. This ray, however, is not only the line connecting two homologous points; the same complex appears, when it is regarded as the bearer of ∞^2 pairs of homologous points.

If now this ray is the direction of velocity v_a of a point A of this system, it is evident that for the determination of this point further conditions must be introduced, namely such as permit the construction of the point A as well as of the plane α.

5. In the first place a pair of opposite edges of the tetraeder must be conjugate polars of a focal system; PQ and RS to be taken for these. This condition, however, is not yet sufficient, as in the

preceding problem d was not only normal to l but also to v_a. The plane ld constructed according to the supposition made there and a plane through l parallel to v_a are normal to each other and are thus conjugate in respect to the planes through l and the cyclic points in the plane normal to l; by applying this last named principle to the case of projectively altering systems, we obtain the following construction for the point A, whose direction of velocity is v_a.

Suppose the given direction of velocity v_a cuts the plane PRS in the point L; construct in this plane the ray PL' harmonically conjugate to PL with respect to PR and PS, which cuts RS in L'; bring through L' a ray cutting PQ and v_a; then this ray will cut v_a in the point A possessing the given direction of velocity.

For the determination of the focal system the construction of the ray $L'A$ is not sufficient, as we know of the focal plane α belonging to A only that it passes through $L'A$. To determine α entirely we have to notice that with the motion of congruent systems α cuts the plane λ_∞ in the polar of the point of intersection L of v_a and λ_∞ in respect to the imaginary circle in λ_∞. This circle passes through the imaginary vertices in λ_∞ of the principal tetraeder and the point of intersection of l and λ_∞ is the centre of it. By applying these principles to the case of systems changing projectively, we obtain the following construction.

We assume a conic K^2, touching PR and PS in R and S; we construct the polar p of L through L' in reference to K^2 and we bring the plane α through A and p; now α is the focal plane of A. So with the motion of systems projectively varying the complex of rays and the focal system are connected with each other.

6. The edges PQ and RS determine with v_a a hyperboloid H^2, on which the polar of v_a in reference to the focal system is also situated. This cuts the plane PRS besides in RS also in PL; so from this ensues that the polar of v_a relatively to the focal system cuts the plane PRS in a point of PL. It is then easy to see, that PL is the polar of L' relatively to K^2.

7. Not until the inverse problems are solved, are the constructions complete, thus (a) when for each point the direction of velocity and the focal plane are constructed, (b) when for each plane the focus and the direction of velocity of this focus are constructed. We suppose in these constructions the complex of rays to be determined and K^2 moreover constructed.

a. Given point A. Draw through A the line cutting PQ and

RS, the latter in L'; construct PL harmonically conjugate to PL' relatively to PR and PS; according to the preceding L must be determined on PL. When the complex of rays is determined the ray through A must cut not only PL, but also a ray in the plane QRS belonging to the pencil of rays with centre Q, projective to the pencil P/RSL; this ray QL'' corresponds to PL. So we have to determine this ray QL'' according to the known projectivity of the pencils with the centres P and Q and to construct the line through A cutting PL and QL''; by this the plane α is at the same time known.

b. Given α; the connection of the point of intersection of α and PQ with the point of intersection L' of α and RS produces a focal ray. We determine farthermore the pole L of the line of intersection of α and PRS relatively to K^2, construct the ray QL'' corresponding to the ray PL in the two projective pencils P/RSL, Q/RSL'' and bring through L a right line, cutting QL'' and the constructed focal ray. This right line is the direction v_a and its point of intersection with the focal ray is A.

8. The preceding considerations point to a connection existing between the investigations of A. SCHOENFLIES "Geometrie der Bewegung" pages 79—129 and those of L. BURMESTER "Kinematisch geometrische Untersuchung der gesetzmässig veränderlichen Systeme", Zeitschrift für Mathematik und Physik, vol. 20, pag. 395—405. The former treats very completely of the constructions ensuing from the focal system and the tetraedral complex belonging to it, when the system remains congruent to itself during its motion; the latter assumes the projective variability of the moving systems, but does not make use of the focal system.

It would not be difficult to give a more general form to most constructions appearing in the former consideration; this would however give rise to unnecessary repetitions; so it will be sufficient if this is shown in a single example.

9. To do so we take the construction corresponding to that of the characteristic of invariable systems; so the question is to determine according to the foregoing principles in the plane α the right line a containing the points the directions of velocities of which lie in α. For this a must be the line of intersection of two homologous planes of the two systems at infinitesimal distance of each other; so if we think on v_a the point A to be determined and the point A' at an infinitesimal distance of it, and the planes α and α' to be

constructed, then a proves to be the polar of v_a relatively to the focal system.

According to (6) a passes through a point of PL, moreover a lies in α, so it has a point in common with the line of intersection of the planes α and PRS which is the polar P of L relatively to K^2; from this ensues:

The polar a of v_a relatively to the focal system cuts the plane PRS in the pole of LL' relatively to K^2 and lies on a hyperboloid of which PQ, RS and v_a are three generators.

10. Up till now we have supposed the four vertices of the principal tetraeder to be real. The constructions, however, can still be performed if we assume that the two vertices RS are conjugate imaginary. The two edges PQ and RS namely remain real, also the planes PRS and QRS, the imaginary edges PR and RS are represented as imaginary double rays of an elliptic involution of rays with the centre P; the construction of the conic K^2 and of the polar p remains however possible and therefore also the remaining constructions of the complex of rays and of the focal system.

If, however, the four vertices of the principal tetraeder are imaginary, then the construction can no longer be performed, because according to the preceding it ought to take place in a plane (PRS), which becomes imaginary itself. As the constructions treated of here will also be considered from another point of view, this case shall for the present remain unnoticed.

11. In the theory of the motion of an invariable system we imagine cylinders of revolution to be described round the principal axis. If one of these cylinders is constructed, the velocities touching these cross the principal axis under the same angle, so that they are tangents to helices of definite inclination. Let us now find out the analogon of these cylinders in the motion of projectively changing systems and let us to do so return to the formerly (5) constructed rays PL and PL' which are harmonically conjugate with respect to PR and PS.

We suppose furthermore a quadratic cone C^2 to be constructed, the vertex of which is P which touches the planes PQR and PQS of the tetraeder according to the edges PR and PS; then the planes PQL and PQL' are conjugate polar planes of C^2. If we now bring a tangent plane to C^2 through PL, this touches the cone according to a generator lying in the plane PQL'; from this ensues:

The right line d through L', cutting PQ and v_a, also cuts the generator according to which the tangent plane through PL touches C^2. If C^2 is constructed in such a way that the plane through PL and v_a touches it, d cuts v_a in the point of contact with C^2. We can thus make a geometric image of all directions of velocities namely in the following way:

Given a ray v_a of the complex, cutting plane PRS in L; construct a cone C^2, having P as vertex, touching the planes PQR and PQS according to PR and PS and touching moreover v_a. Construct the harmonic ray PL'; then the rays through L' cutting PQ, also cut the complex rays through L in the points of contact with C^2. If we construct all the rays of the complex, a pencil of cones is formed; to each tangent plane PLv_a belongs a cone and a pencil of complex rays through a point of PL.

So whilst to each tangent plane a cone belongs, two tangent planes PLv_a, PLv'_a belong to each cone; the ray d through L' cutting PQ and v_a, also cuts v'_a in a point of contact with C^2.

The planes dPQ, dRS, dv_a, dv'_a form a harmonic pencil.

12. Finally a few general observations may be in their place at the conclusion of this communication.

a It is clear, that if we consider the four vertices P,Q,R,S as the points of coincidence of two projective systems, each of these points plays the same part; by regarding, as was done in the beginning, the edges PQ and RS as conjugate polars of a focal system a limiting condition has been introduced.

And the introduction of this condition is allowed as the principal tetraeder and one direction of a velocity do not determine, the position of the homologous points of two projective systems though they determine the complex of rays. By the second assumption, that of the conic K^2, the focal system is determined. As it is possible to choose in three different ways a pair of edges as conjugate polars and moreover the point of intersection L can be assumed in two different planes, the point A can be determined in twelve different ways on a direction of velocity v_a.

b. The number of solutions for the determination of the point A on the direction of velocity v_a diminishes, when two of the vertices, say R, S are imaginary. So PQ and RS form the only possible pair of opposite edges. The point of intersection L can be determined in two faces (PRS and QRS). If now also the points P and Q coincide as is the case for the motion of invariable systems, only one solution is possible.

c. The entire preceding consideration is independent of the length of the velocities. It is also possible to find constructions for which use is made of that length. This will be done in a following communication.