Citation:

H.A. Lorentz, The intensity of radiation and the motion of earth, in:
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But, for this system, by (22), joined to the 5th and 7th of the equations (4),

$$\mathfrak{D}' - \frac{1}{4\pi c^2}\mathfrak{E} = \mathfrak{M}\ ;$$

so that we find by using (21)

$$\mathfrak{M} = \frac{1}{\sigma}\mathfrak{E} - \frac{j}{\sigma^2}\,Rot\,\mathfrak{E} + \frac{j}{c^2\sigma^2}[\dot{\mathfrak{E}} \cdot \mathfrak{p}].$$

This is precisely the formula (7), if for k we take the value (3).

Physics. — H. A. LORENTZ. "*The intensity of radiation and the motion of the earth*".

(Communicated in the meeting of March 29, 1902).

Many years ago FIZEAU [1]) remarked that, if the aether does not follow the earth in its annual motion, the radiation, emitted by a terrestrial source of light or heat L, might possibly have unequal intensities in different directions. Let A be a point that is likewise fixed to the earth, and whose distance from L we shall denote by l. Then, if $L\,A$ have the direction of the earth's velocity v, a vibration produced by L will have to travel over a length

$$l \cdot \frac{c}{c - v}$$

(c velocity of light), before it reaches A. On the contrary, its course will be

$$l \cdot \frac{c}{c + v},$$

if $L\,A$ has the opposite direction. FIZEAU expected that the intensities received by A in the two cases would be inversely as the squares of these expressions, so that there would be a difference which one might hope to detect by means of suitable experiments with a thermo-electric battery.

From our present views regarding electric and optical phenomena in moving bodies it may be inferred that the experiment, proposed by FIZEAU would have a negative result, the amount of heat which is imparted to an absorbing body being independent of the earth's motion.

It will suffice to consider a simple case, omitting all terms depending

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[1]) Pogg. Ann., Bd. 92, p. 652, 1854.

on the square of v. Let there be a single radiating particle in the origin of coordinates, and let it have an electric moment

$$\mathfrak{m}_y = a \cos nt,$$

in the direction of OY. In order to find the dielectric displacement $\mathfrak{d}$ and the magnetic force $\mathfrak{H}$ in the surrounding field, we may start from the formulae, I have developed in § 33 of my „Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern". Let the velocity v of the earth be in the direction of OX, let r be the distance to O,

$$t' = t - \frac{v}{c^2}x \, , \quad \ldots \ldots \ldots \quad (1)$$

and

$$\psi = \frac{a}{r} \cos n\left(t' - \frac{r}{c}\right).$$

Then

$$\mathfrak{d}_x = \frac{1}{4\pi}\frac{\partial^2 \psi}{\partial x \, \partial y},$$

$$\mathfrak{d}_y = -\frac{1}{4\pi}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + \frac{v}{4\pi c^2}\frac{\partial^2 \psi}{\partial t' \, \partial x},$$

$$\mathfrak{d}_z = \frac{1}{4\pi}\frac{\partial^2 \psi}{\partial z \, \partial y},$$

$$\mathfrak{H}_x = -\frac{\partial^2 \psi}{\partial t' \, \partial z},$$

$$\mathfrak{H}_y = - \, v \frac{\partial^2 \psi}{\partial z \partial y},$$

$$\mathfrak{H}_z = \frac{\partial^2 \psi}{\partial t' \partial x} - v\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2}\right).$$

The auxiliary quantity ψ is to be regarded as a function of x, y, z, t', and it is only after the differentiations have been performed, that the value (1) must be substituted.

We may further confine ourselves to values of r, very much larger than the wave-length λ. In this case we have only to retain the terms whose denominator has the first power of r, all other terms being, with respect to these, of the order $\frac{\lambda}{r}$ or $\frac{\lambda^2}{r^2}$. For points situated on the positive axis of x, we find

$$\mathfrak{d}_x = 0, \; \mathfrak{d}_z = 0,$$

$$\mathfrak{d}_y = \frac{n^2}{4\pi c^2}\left(1 + \frac{v}{c}\right)\frac{a}{r}\cos n\left\{t - \left(1 + \frac{v}{c}\right)\frac{x}{c}\right\},$$

$$\mathfrak{H}_x = 0 \, , \; \mathfrak{H}_y = 0,$$

$$\mathfrak{H}_z = \frac{n^2}{c}\left(1 + \frac{v}{c}\right)\frac{a}{r}\cos n\left\{t - \left(1 + \frac{v}{c}\right)\frac{x}{c}\right\},$$

so *that*

$$\mathfrak{d}_y = \frac{1}{4\pi c}\mathfrak{H}_z .$$

The corresponding energy per unit of space is

$$2\pi c^2 \mathfrak{d}_y^2 + \frac{1}{8\pi}\mathfrak{H}_z^2 = \frac{1}{4\pi}\mathfrak{H}_z^2, \quad \ldots \ldots \quad (2)$$

and, according to POYNTING's theorem, there is a flow of energy along OX

$$c^2 \mathfrak{d}_y \mathfrak{H}_z = \frac{c}{4\pi}\mathfrak{H}_z^2,$$

this quantity being the amount of energy per unit of time and unit of area, which traverses an element of surface, perpendicular to OX, and *not* moving with the earth.

In what follows we have only to attend to the mean values of the energy and its flow, taken for a full period or for a lapse of time, embracing a large number of periods. The mean value of (2) is

$$U = \frac{n^4}{8\pi c^2}\left(1 + \frac{2v}{c}\right)\frac{a^2}{r^2}, \quad \ldots \ldots \quad (3)$$

and for that of the energy-current we may write

$$c\,U.$$

Since v may be negative as well as positive, the above formulae apply not only to the vibrations, sent out in the direction of the earth's motion, but equally to those which go forth in opposite direction. The factor $1 + \frac{v}{c}$ in the expressions for $\mathfrak{d}_y$ and $\mathfrak{H}_z$ is different in the two cases; it would however be rash, to conclude from this, without closer examination, that the difference will make itself felt in measurements on the heating of a body exposed to the rays.

Let there be, in any point of the positive axis of x, and placed perpendicular to it, a disk of infinitely small area ω, and composed of a perfectly black material, so that it reflects no part of the incident radiation. This disk will be supposed to be fixed to the earth, and we shall deduce the amount of heating from the law of conservation of energy, taking into account that the rays exert on the disk a certain normal pressure, the amount of which per unit area is given precisely by U [1]).

Imagine a right cylinder C, having ω for its base and turned towards the source of heat, and suppose the face of it, that is opposite

[1]) See e g. my "Versuch u. s. w.", §§ 16 and 17.

to ω — we shall call this ω' — to have a fixed position in space. Let θ be a lapse of time, consisting of a large number of periods, and consider, for this interval, the change of the amount of energy, contained within C. Let the cylinder be of so great a length h, that, if v should be negative, the disk ω cannot reach the plane ω', before the end of the time θ, and let h at the same time be so small in comparison with the distance r, that terms which are of the order $\frac{h}{r}$ with respect to the quantities we are considering may be neglected. Then we need not trouble ourselves about the difference between the values of U for ω and ω'; neither will it be necessary to attend to the flow of energy through the cylindrical surface of C.

If, for the time θ, e_1 is the amount of energy, by which the plane ω' is traversed, e_2 the increment of the energy, contained within the cylinder, and e_3 the work done by the pressure exerted on ω, the absorption is evidently given by

$$e = e_1 - e_2 - e_3.$$

Now:

$$e_1 = c\, U \omega \theta,$$

and, the volume of the cylinder being increased by $v \omega \theta$,

$$e_2 = v\, U \omega\, \theta.$$

Finally we have, since the displacement of the disk is $v\, \theta$,

$$e_3 = v\, U \omega\, \theta.$$

The result is therefore

$$e = (c - 2\, v)\, U \omega\, \theta,$$

or, by (3), if we continue to neglect terms in v^2,

$$e = \frac{n^4}{8\pi c} \frac{a^2}{r^2} \omega\, \theta\, ,$$

independent of the velocity of the earth.

Physics. — "*Ternary systems*". III. By Prof J. D. VAN DER WAALS. (Continued from page 560).

The quantity $(\varepsilon_{21})_v$ occurring in equation (1) as a factor of $\frac{dT}{T}$, is negative for normal substances. It represents (Cont. II, pag. 101 and following pages) the decrease of energy per molecule, when we have a finite quantity of the first phasis and an infinitely small quantity of the second phasis, and when we then make the substance fill the volume homogeneously, keeping volume and temperature constant; so we may also say that it represents the heat which in