Violet are chalkopyrite and pentlandite.

Pale brown, lilac tinged are pyrite, and more or less smaltine, cobaltine and ilmenite. The colour of the last mentioned mineral approaches some more reddish brown tetraedrites.

Pale yellowish brown are stibnite and jamesonite, whereas hausmannite and manganite in colour approach the following group.

Reddish brown are boulangerite and clausthalite: bournonite is less red and as to coulour forms a transition from the two last to stephanite, which approaches yellowish black.

Yellowish black are galena (greenish tinge) enargite and chalcosine, further berzilianite, argentite and berthierite. Finally anthracite might be mentioned here.

Pale brownish grey are magnetite and polianite; further stannine and corynite, although the streak of these last mentioned minerals is of a rather pure grey colour.

Still purer is the grey of graphite and pyrrhotine.

The above mentioned colours were an immediate result of the fineness of the particles growing transparent in consequence of that fineness. Still an other effect is produced by rubbing down the streak of certain minerals. I will just passingly mention it here, later I shall treat it more fully.

The effect I mean is most apparent in minerals which contain copper and best in cuprite. In rubbing out the brownish red streak the colour grows more and more greenish; at last to dissolve into a bluish green. However when shutting out the air with a drup of glycerine, no change of colours takes place. This same final colour is obtained in azurite and malachite.

I need hardly point out, that all those colours may be a great help in determining the so called opake minerals.

Mathematics. — On "*The spacial anharmonic ratio of curves* ϱ^n *of order* n *in the space* S_n *with* n *dimensions*". By Prof. P. H. SCHOUTE.

1. If on the curve ϱ^n in S_n, forming the subject of this short treatise, we take arbitrarily $n-1$ points A_i, $(i = 1, 2, \ldots n-1)$, we also determine thereby a space S_{n-2} containing these points, and we can assign the points of the curve one by one to the spaces S_{n-1} through S_{n-2} containing them. This gives rise to a correspondence one by one between the points of the curve and the spaces S_{n-1} of the pencil of spaces with the basis S_{n-2}, which proves the

well-known theorem, that the genus of the curve ϱ^n is zero, and that we can just as well speak of the anharmonic ratio of four points of ϱ^n as of that of four points of a right line.

2. This simple consideration proves in general, that an invariable anharmonic ratio λ must be found by connecting any space S_{n-2} through $n-1$ variable points of the curve by spaces S_{n-1} with each of four fixed points A_1, A_2, A_3, A_4, which of course forms an extension of the well-known property of the conic ϱ^2 in the plane, that the quadruples of lines connecting a variable point P of the curve with four fixed points of the curve have an anharmonic ratio independent of P.

3. With this the generation of ϱ^n by means of n projectively related pencils of spaces S_{n-1} is closely connected. Moreover ensues from it that ϱ^n is determined by $n+3$ points. For, by dividing $n+3$ given points into two groups, one of n points and one of three, we can form by means of the n points of the first group n spaces S_{n-2} through $n-1$ points of the curve and after this determination of the n bases we can fix with the aid of the three points of the second group the projective correspondence between the n pencils of spaces S_{n-1}.

4. As is known the conic ϱ^2 in the plane can be considered as the locus of the points P which connected with four fixed points A_1, A_2, A_3, A_4 produce quadruples of lines of a definite anharmonic ratio λ, so that we can speak of the conic through A_1, A_2, A_3, A_4 containing the anharmonic ratio λ; by varying λ appears the pencil of conics filling the plane; of this pencil the points A_1, A_2, A_3, A_4 form the base. In like manner, if of a curve ϱ^n we give not $n+3$ but $n+2$ points $A_1, A_2, \ldots A_{n+2}$, we find instead of a single ϱ^n an $n-1$ fold infinite system of curves ϱ^n, filling S_n. And now arises the question whether it is not possible to indicate individually the curves of this $n-1$ fold infinite system by means of $n-1$ anharmonic ratios. This question must be answered in the affirmative. For, we have seen that on a given ϱ^n four given points represent a determined anharmonic ratio, and now out of the $n+2$ given points are to be formed by completing the triple $A_1\ A_2\ A_3$ with each of the remaining A_j to a quadruple $A_1\ A_2\ A_3\ A_j$, $(j=4, 5, \ldots n+2)$, exactly $n-1$ mutually independent anharmonic ratios $\lambda_j = (A_1\, A_2\, A_3\, A_j)$. From this follows for the present nothing but this that to a given curve ϱ^n through the $n+2$ given points belongs a definite set of anharmonic

ratios λ_j, whilst on the other hand the possibility is not excluded that inversely to a given set of anharmonic ratios λ_j belongs more than one curve ϱ^n passing through $n+2$ given points. It is however easily proved that all those curves ϱ^n belonging to a given set of anharmonic ratios — supposing there be more than *one* — are projected from A_{n+2} by the same conical space of order $n-1$. For if we project the figure considered, from the point A_{n+2} upon the space S_{n-1} determined by $A_2, A_3, \ldots A_{n+1}$, the curves ϱ^n through $A_1, A_2, \ldots A_{n+2}$ with the anharmonic ratios $\lambda_4, \lambda_5, \ldots \lambda_{n+2}$ in S_n are transformed into the curves ϱ^{n-1} through $A'_1, A_2, \ldots A_{n+1}$ with the anharmonic ratios $\lambda_4, \lambda_5, \ldots \lambda_{n+1}$ in S_{n-1}. And by repeating this reduction, passing from the applied space S_{n-1} to the space S_{n-2} determined by $A_2, A_3, \ldots A_n$ etc. till we arrive at the plane of projection $A_2\ A_3\ A_4$ where the original point A_1 may finally arrive in A''_1, two curves ϱ^n which pass through the $n+2$ given points, belong to the anharmonic ratios $\lambda_4, \lambda_5, \ldots \lambda_{n+2}$ and are projected from A_{n+2} by different conical spaces of order $n-1$, will finally be transformed into two different conics through A''_1, A_2, A_3, A_4, to which belongs the same anharmonic ratio λ_4. This being impossible the different curves ϱ^n through the $n+2$ given points which might belong to the given set of anharmonic ratios $\lambda_4, \lambda_5, \ldots \lambda_{n+2}$, must be projected from A_{n+2} by means of the same conical space. And what is true for the point A_{n+2} is applicable to all the remaining given points. So all curves ϱ^n which may belong to a given set of anharmonic ratios $\lambda_4, \lambda_5, \ldots \lambda_{n+2}$, being projected from all points A_i, $(i = 1, 2, \ldots n+2)$, by the same conical spaces, must coincide. So in general we find:

"We can determine in S_n a curve ϱ^n passing through any $n+2$ points A_i, $(i=1, 2, \ldots n+2)$, by indicating the $n-1$ anharmonic ratios $\lambda_j = (A_1\, A_2\, A_3\, A_j)$, $(j = 4, 5, \ldots n+2)$. And these anharmonic ratios assuming all possible values, the determined curve ϱ^n generates the $n-1$ fold infinite linear system with the base $A_1, A_2, \ldots {}_{n+2}$, filling the space S_n, which system is of course projectively related to the likewise $n-1$ fold infinite linear system of the anharmonic ratios $\lambda_4, \lambda_5, \ldots \lambda_{n+2}$, or in other words to the linear system of points in a space S_{n-1} of which the coordinates are given by means of these systems of values."

The preceding proof wants some completion. We might ask, a. o. with a view to the structure of the anharmonic ratios, where on one side A_1, A_2, A_3 and on the other side $A_4, A_5, \ldots A_{n+2}$ play different parts, whether we are allowed to extend to all points A_i what has been found true for A_{n+2}. Yet leaving alone whether it be necessary to know that all points A_i behave in this respect alike, it is easy to see at once, that the difference between the points of the two groups is in this respect but an apparent one. For by giving the $n-1$ anharmonic ratios $(A_1\,A_2\,A_3\,A_j)$, $(j=4, 5, \ldots n+2)$, all anharmonic ratios of quadruples of points A_i are determined. For, if on a right line l, corresponding point by point to ϱ^n, we assume arbitrarily the points belonging to A_1, A_2, A_3, the given anharmonic ratios determine the points belonging to $A_4, A_5, \ldots A_{n+2}$; so on l the n corresponding points and thus all the anharmonic ratios are known.

5. Probably it is recommendable to call the complex of the $6\frac{(n+2)(n+1)n(n-1)}{1\,.\,2\,.\,3\,.\,4.}$ anharmonic ratios, determined by $n-1$ mutually independent ones out of them, simply a *spacial* anharmonic ratio and to represent it by the symbol $\lambda_{(n-1)}$. We can then say, that the $n-1$ fold infinite linear system of curves ϱ^n with $n+2$ points A_i as base is projectively related to that spacial anharmonic ratio, in so far that a curve of the system corresponds to a definite spacial anharmonic ratio and reversely. By this the analogy of the obtained result with the special case $n=2$ becomes as great as possible.

6. The above led us to the following theorem of cyclic inversion, which we shall first indicate for the special case $n=3$ of our space. The net of the skew cubics ϱ^3 is then given by its five base points. If we put

$$(A_1A_2A_3A_4)=\lambda_0,\ \ (A_2A_3A_4A_5)=\lambda_1,\ \ (A_3A_4A_5A_1)=\lambda_2,$$
$$(A_4A_5A_1A_2)=\lambda_3,\ \ (A_5A_1A_2A_3)=\lambda_4,$$

where $(PQRS)$ always stands for

$$\frac{PR}{QR}:\frac{PS}{QS},$$

then the question rises by which recurrent relation the five quantities λ_i, $(i=0, 1, 2, 3, 4)$, are connected. By direct reckoning we find

$$\lambda_{m+2}=\frac{\lambda_{m+1}}{\lambda_m(\lambda_{m+1}-1)},$$

where $m, m+1, m+2$ are to be replaced by their remainders after division by five; and really by repeated substitution we arrive at $\lambda_{m+5}=\lambda_m$. Of course in the general case of an arbitrary n a result will be obtained of the form

$$\lambda_{m+n-1} = f(\lambda_m, \lambda_{m+1}, \ldots \lambda_{m+n-2}).$$

However, the definition of the form of the function f we leave to others.

7. We shall point out a few particularities which already appear when we restrict ourselves to space with three dimensions.

In the following way the net of the skew cubics ϱ^3 through five points is deduced from considerations on conics and cones.

If A, B, C, D, E are five points given arbitrarily in space, the locus of the vertices T of the cones $T(A,B,C,D,E)$ through these points, on which the edges $T(A, B, C, D)$ represent a definite anharmonic ratio λ, is the cone $E_\lambda(A,B,C,D)$ through A,B,C,D with vertex E, on which the edges passing through A, B, C, D determine the anharmonic ratio λ. And if λ assumes all possible values this cone generates the pencil with EA, EB, EC, ED as base edges, filling the space.

For, if P is an arbitrary point of $E_\lambda(A, B, C, D)$, then according to definition

$$P(AE, BE, CE, DE) = \lambda$$

and this is identical with

$$E(AP, BP, CP\ DP) = \lambda.$$

If A, B, C, D, E are again five points given arbitrarily in space, the locus of the vertices T of the cones $T(A,B,C,D,E)$ through these points, on which $T(A, B, C, D)$ and $T(A, B, C, E)$ represent respectively the anharmonic ratios λ and μ, is the skew cubic $\varrho^3_{\lambda,\mu}$ through A, B, C, D, E, forming with the line DE the complete intersection of the cones $E_\lambda(A, B, C, D)$ and $D_\mu(A, B, C, E)$. And if λ and μ assume all possible values, this curve generates the net with the base A, B, C, D, E; this net filling the space is projectively related to the points (λ, μ) of a point-field.

The anharmonic ratios $T(A, B, C, D)$ and $T(A, B, C, E)$ on the cone $T(A, B, C, D, E)$ being identical with the anharmonic ratios (A, B, C, D) and (A, B, C, E) on the curve $\varrho^3_{\lambda,\mu}$, this result is nothing more but at the same time nothing less than the special case $n=3$ of the general result obtained before. And from this we could have gone on to the case $n = 4$ in order to prove the anticipated general result by the conclusion from n to $n+1$. It occurred to us however that the deduction given above of the general result is shorter and clearer.

8. By assuming between λ and μ the bilinear relation

$$p\lambda\mu + q\lambda + r\mu + s = 0$$

we form a projective correspondence between the cones $E_\lambda(A,B,C,D)$ and $D_\mu(A, B, C, E)$. So we find.

"The locus of the curves $\varrho^3_{\lambda,\mu}$, for which λ and μ satisfy a given bilinear relation, is a surface F^4 of order four, on which the two triples of lines DA, DB, DC and EA, EB, EC are simple lines, the points A, B, C are double points and DE is a double line. All these different surfaces F^4 form a threefold infinite linear system, projectively related to the linear system of the rectangular hyperbolae, represented by the equation of correspondence, if λ and μ indicate the rectangular coordinates of a point in the plane."

"If in particular $p = 0$, then $\lambda = \infty$ and $\mu = \infty$ correspond to each other and likewise the pairs of planes $E(AD, BC)$ and $D(AE, BC)$. Then the surface F^4, passing through BC, breaks up into the plane ADE and a surface F^3, i. e. the locus of the curves $\varrho^3_{\lambda,\mu}$ is then an F^3 through the edges of the tetrahedron $BCDE$. All these surfaces pass through A and have B, C, D, E as double points; so they form a net of course projectively related to the net of the right lines $q\lambda + r\mu + s = 0$."

"If at the same time $q = 0$, we then find $\lambda = \infty$ and $r\mu + s = 0$, so that F^4 breaks up into the pair of planes $E(AD, BC)$ and a cone $D_\mu(A, B, C, E)$. By the addition of $q=0$ a new plane i. e. BCE has separated from F^4."

"The linear system of the surface F^4 contains a net of surfaces F^3 and two pencils of cones."

Also analytically we can easily find that the surfaces F^4 having DE as double line, A, B, C as double points and passing through the triples of lines $D(A, B, C)$, $E(A, B, C)$ form an at least threefold infinite series. Firstly we learn out of the equation.

$$z^2\varphi_2(x,y) + zt\psi_2(x,y) + t^2\chi_2(x,y) + z\xi_3(x,y) + t\eta_3(x,y) + \zeta_4(x,y) = 0$$

of a surface $f_4(x, y, z, t) = 0$ of order four with the right line $x = 0$, $y = 0$ as double line, that of the 35 coefficients of the complete equation only

$$3 + 3 + 3 + 4 + 4 + 5$$

or 22 are extant, so that the compound condition of having DE as a double line is equivalent to 13 simple ones. The condition of having A, B, C as double points and that of passing through six given lines count respectively for 12 and at most for 6 simple ones, so that at least 3 remain at our disposal. Only if each surface F^4 with DE as a double line and A, B, C as double points, which is brought through five of the six lines, contained by that already also the sixth — a peculiarity which appears as will be seen in the following series of surfaces — the number of conditions to be disposed of could become greater than 3; so this peculiarity does not appear here.

It is also easy to see that the surfaces F^3 through A with the double points B, C, D, E form a twofold infinite series. For, the surfaces $f_3(x, y, z, t) = 0$ with the vertices of the tetrahedron of coordinates as double points form a linear system

$$a\,y\,z\,t + b\,z\,t\,x + c\,t\,x\,y + d\,x\,y\,z = 0,$$

etc.

9. If we assume in space six given points A, B, C, D, E, F, we arrive in the following way at a generation of skew biquadratics:

"The locus of the common vertices T of the cones $T(A, B, C, D, E)$ and $T(A, B, C, D, F)$, on which the four edges $T(A, B, C, D)$ determine respectively the anharmonic ratios λ and μ, is the skew biquadratic $\varrho^4_{\lambda,\mu}$ through A, B, C, D forming the complete intersection of the cones $E_\lambda(A, B, C, D)$ and $F_\mu(A, B, C, D)$; of this curve E and F are two of the four vertices of cones containing it. And if λ and μ assume all possible values this ϱ^4 generates the net with the base A, B, C, D and the vertices E, F; this net filling the space is a. o. projectively related to the point-field (λ, μ)."

"The locus of the curves $\varrho^4_{\lambda,\mu}$, for which λ and μ satisfy a given bilinear relation, is a surface F^4 of order four, having the points A, B, C, D, E, F as double points and containing the quadruples of lines $E(A, B, C, D)$ and $F(A, B, C, D)$. All these surfaces F^4 form again a threefold infinite linear system, projectively related to the linear system of the rectangular hyperbolae represented by the equation of correspondence."

Some difficulty arises regarding the proof, that the surfaces F^4 found here really represent a threefold infinite series. For, the condition first of having six double points and secondly of passing through eight lines connecting four of these points with the remaining two is equivalent to 24 and apparently to 8 simple conditions more; from which would ensue, that only two simple conditions remain at our disposal. This difficulty can be removed only by the supposition, that each surface F^4 with the double points A, B, C, D, E, F

passing through seven of the eight lines (E, F) (A, B, C, D) also contains the eighth. In reality the surface of the series degenerated into four planes show that the series is threefold infinite. For of the nine degenerated surfaces

I	II	III	IV	V	VI	VII	VIII	IX
EAB	EAB	EAB	EAC	EAC	EAC	EAD	EAD	EAD
ECD	ECD	ECD	EBD	EBD	EBD	EBC	EBC	EBC
FAB	FAC	FAD	FAB	FAC	FAD	FAB	FAC	FAD
FCD	FBD	FBC	FCD	FBD	FBC	FCD	FBD	FBC

the individuals of each of the triples (I, II, III), (IV, V, VI), (VII, VIII, IX), (I, IV, VII), (II, V, VIII), (III, VI, IX) belong to a same pencil, as is shown by the identity

$$(x-y)(z-t)+(x-z)(t-y)+(x-t)(y-z)=0.$$

So I, II, IV, V are four degenerations no three out of which belong to a pencil. Moreover the fourth not belonging to the net determined by the others — for I, II, IV contain AB and CD and these lines do not lie on V — they form a linear system of threefold infinity.

10. For seven points A, B, C, D, E, F, G given arbitrarily in space we have farthermore:

"Four vertices T are to be found for which the common edges $T(A, B, C, D)$ determine respectively on the three cones $T(A,B,C,D,E)$, $T(A, B, C, D, F)$, $T(A, B, C, D, G)$ the anharmonic ratios λ, μ, ν. These four points form with A,B,C,D the eight points of intersection of the three cones $E_\lambda(A,B,C,D)$, $F_\mu(A,B,C,D)$, $G_\nu(A,B,C,D)$. And if λ, μ, ν assume all possible values, this quadruple of points generates a biquadratic involution of quadruples of points filling the space and projectively related to the points (λ, μ, ν) of space."

According to the general character of the involution a quadruple of points P, Q, R, S is determined by one of its points; if P is given the cones $E(A, B, C, D, P)$, $F(A, B, C, D, P)$, $G(A, B, C, D, P)$ are determined and likewise the three other new points of intersection. In fact, we have not to deal with *all* quadruples of points completing A, B, C, D to eight *associated* points, in which case we might arbitrarily assume seven out of the eight points, but only with those octuples A, B, C, D, P, Q, R, S, for which E, F, G are three vertices of quadratic cones containing them. Of course a great number of problems appear immediately. We can ask what Q, R, S generate together when P describes a right line or a plane, what the locus is of the quadruple of points under the condition that one of six connecting lines passes through a given point or intersects a given line, etc. In order not to be too prolix we shall discuss but two other loci.

Of these the first is connected with the trilinear equation

$$k\lambda\mu\nu + l\mu\nu + m\nu\lambda + n\lambda\mu + p\lambda + q\mu + r\nu + s = 0$$

between λ, μ, ν. We find:

"The locus of the quadruples of points of intersection of the three cones $E_\lambda(A, B, C, D)$, $F_\mu(A, B, C, D)$, $G_\nu(A, B, C, D)$, for which λ, μ, ν satisfy a given trilinear equation, is a surface of order six with A, B, C, D as threefold points, E, F, G as double points and the three quadruples of lines obtained by connecting each of three points E, F, G with the four points A, B, C, D as simple lines. All those surfaces F^6 form a sevenfold infinite linear system, projectively related to the in like way sevenfold infinite system of cubic surfaces represented by the equation of correspondence."

Here is again immediately shown, that the found surfaces F^6 form an at least sevenfold infinite series. For of the 83 simple conditions determining an F^6 the four threefold points take 40, the three double points 12 and the 12 right lines at most 24, so that at least 7 remain at our disposal. The system of the surfaces F^6 being really sevenfold infinite, from this ensues reversely that the determining quantities — threefold points, double points and simple lines — represent mutually independent data.

Secondly we look for the locus of the quadruples of points of intersection, for which λ, μ, ν are equal to one another. We find:

"The locus of the quadruples of points of intersection of the three cones $E_\lambda(A, B, C, D)$, $F_\mu(A, B, C, D)$, $G_\nu(A, B, C, D)$, for which λ, μ, ν are equal to one another, is a skew sextic not passing through A, B, C, D, E, F, G."

According to the above the locus of the intersection $\varrho^4_{\lambda,\mu}$ of $E_\lambda(A, B, C, D)$ and $F_\mu(A, B, C, D)$, for which $\lambda = \mu$, is a surface F^4 passing through the two quadruples of lines $E(A, B, C, D)$ and $F(A, B, C, D)$ with the double points A, B, C, D, E, F, which passes — the values 0, 1, ∞ of λ corresponding to equal values of μ — likewise through the edges of the tetrahedron $ABCD$.

In like manner the locus of the intersection of $E_\lambda(A, B, C, D)$ and

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$G_\nu(A,B,C,D)$, for which $\lambda = \nu$, is an F^4 with the double points A, B, C, D, E, G passing through the quadruples of lines $E(A,B,C,D)$ and $G(A,B,C,D)$ and the edges of the tetrahedron $ABCD$. Of the total intersection ϱ^{16} of these surfaces, having A, B, C, D, E as fourfold points, the ten right lines connecting the points A, B, C, D, E two by two, separate; so has been proved what was asserted.

Botanics. — "*Preservatives on the stigma against the Germination of Foreign Pollen.*" By Dr. W. BURCK. (Communicated by Professor HUGO DE VRIES.)

It is well known that the pollen of many plants gets destroyed as soon as it comes into contact with water. The both coats (exine and intine) are then seen to burst, while the contents stream out vigorously [1]).

Further it is known that frequently pollen is successfully brought into germination in sugar solutions at different degrees of concentration, or also in gelatin, agar-agar, gum, dextrine etc., or in mixtures of these substances with sugar [2]).

For number of pollen species, however, there has not yet been found, hitherto, a solution in which germination was observed (many *Compositae*, *Umbelliferae*, *Urticaceae*, *Malvaceae*, *Ericaceae*, and many others).

The idea that chemical substances occurring in the moisture of the stigma would here play a part, has been frequently expressed, among others by MOLISCH [3]), in 1892, who inferred it from the fact

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1) On the relation of pollen to water compare, among others, BENGT LIDFORSS, Zur Biologie des Pollens. Pringsheim's Jahrbucher Bd. XXIX, 1896, pag. 1—39.

HANSGIRG, Beiträge zur Biologie und Morphologie des Pollens. Sitzungsber. der K. Bohm. Gesellsch. 1897, XXIII.

BENGT LIDFORSS, Weitere Beitrage zur Biologie des Pollens. Pringsheim's Jahrb. Bd. XXXIII, 1899.

2) See, among others, VAN TIEGHEM, Recherches physiologiques sur la végétation libre du pollen et de l'ovule. Annales des sc. nat. Bot. 5e série, tom. XII, 1872.

L. KNY, Sitzungsber. d. botanischen Vereines d. Provinz Brandenburg XXIII, 1881.

E. STRASBURGER, Neuere Untersuchungen uber den Befruchtungs-Vorgang bei den Phanerogamen etc. Jena 1884.

E. STRASBURGER, Ueber fremdartige Bestaubung. Pringsheim's Jahrb. für w. Botanik Bd. XVIII, 1886.

H. MOLISCH, Zur Physiologie des Pollens, mit besonderer Rucksicht auf die chemotropischen Bewegungen der Pollenschlauche. Sitzungsber. der math. naturw. Classe der K. Akademie der Wissensch. Wien Bd. CII, Abth. I, 1893.

3) MOLISCH, l.c. pag. 429.