Citation:

Everdingen Jr, E. van, On the HALL-effect and the resistance of crystals of bismuth within and without the magnetic field, in:
KNAW, Proceedings, 3, 1900-1901, Amsterdam, 1901, pp. 407-421

In other mammals this bottom-plate of the sphenoïdal sinus has been called by SEYDEL lamina terminalis or "untere Schlussplatte"

Though in Echidna it is well developed and easily visible in a paraseptal section through the macerated skull, its occurrence in this animal hitherto seems to have escaped notice, for not only is it absent in the figure ZUCKERKANDL has given in 1887 in his „Geruchsorgan der Säugethiere", but it is equally omitted in the more recent illustration of GEGENBAUR's new Handbook of Vertebrate comparative anatomy (1898).

The structure of the maxillo-turbinal is the same in both Monotremes; it corresponds to the „verästigte" (ramified) type of HARWOOD-WIEDEMANN, the only difference between the two forms being that in Ornithorhynchus it is somewhat larger and more complicated.

ZUCKERKANDL's statement, that there exists a difference in this respect between Echidna and Ornithorhynchus, the first having a doubly-coiled („doppeltgewundenes"), the latter a folded („gefaltenes") maxillo-turbinal is erroneous, and it is all the more desirable that this mistake should be elucidated, as it has found its way unaltered into GEGENBAUR's new handbook. Yet, as far as regards Ornithorhynchus, the veracity of the statement had already been challenged by SYMINGTON [1]), and for Echidna, by W. N. PARKER (l.c.) who, though agreeing with SYMINGTON, yet came to the conclusion, that Echidna's „maxillary turbinal apparently belongs to the folded („gefaltene") and not to the doubly-coiled („doppeltgewundene") variety."

Transverse sections through the organ, in the preserved as well as in the macerated state, leave no doubt that there exists a complete agreement between Ornithorhynchus and Echidna, both showing a well-marked branching type.

Physics. — Dr. E. VAN EVERDINGEN JR., "*On the* HALL-*effect and the resistance of crystals of bismuth within and without the magnetic field*" (Communication N°. 61 (continued) from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES).

4. *Complete results for the* HALL-*coefficient.* It was mentioned in § 2 of the first part of this Communication [2]) that the relation

[1]) SYMINGTON, J. On the nose, the organ of Jacobson and the dumb-bell-shaped-bone in the Ornithorhynchus. Proc. Zool. Soc. London 1891, pag. 575.

[2]) Versl. d. Verg. Kon. Ak. v. Wet. 29 Sept. 1900, p. 277. Comm. Phys. Lab. Leiden, N°. 61.

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found before between the HALL-coefficient and the position of the principal axis with respect to the lines of magnetic force was confirmed in these recent experiments. The bar N°. 1, with its longest dimension parallel to the principal axis, and N°. 2, 3 and 5, with their longest dimension and two sides perpendicular to the principal axis (for the position of these bars compare fig. 1 in the first part of this paper) were each tested in four positions. In these the longest dimension (also direction of current) was always horizontal and perpendicular to the lines of force of the horizontal electromagnet, while each of the four sides consecutively took the upper horizontal position. Hence with N°. 1 the principal axis was always perpendicular to the lines of force (position ⊥), with 2, 3 and 5 alternately perpendicular to and parallel to the lines of force (position //). For the sake of simplicity the differences between the results in the four positions of N°. 1 and in the two positions ⊥ or // of the other bars will not be mentioned, and only mean values will be given. Very likely these differences are caused by small irregularities in crystallisation and small deviations from the exact position in the experiments, and they are not to be compared to the differences between positions // and ⊥.

All observations have been reduced to the same magnetic fields [1]) and to the same temperature (15° C.).

HALL-coefficient *R*.

N°.	Magnetic field			
	4600		2600	
	⊥	//	⊥	//
1	— 8 0	—	—10 2	—
2	—10 6	—0 2	—12 6	—0.7
3	— 8.8	0 0	—11.1	—0.4
5	— 8.2	+0.6	—10 6	—0.1

The small value of the coefficient in the position // and the reversal of sign with N°. 5 were first pointed out in § 2 of the first part of this paper.

[1]) The numbeis given in § 2 for the magnetic field appeared afterwards a little too high. For this reason and on account of the correction for temperature the numbers for 2, 3 and 5 differ slightly from those given before.

From the numbers in the columns headed ⊥ it appears that the experiments afford no reason for making a distinction between the positions in which the *current* is parallel (as with 1) and those in which the *current* is perpendicular to the principal axis (as with 2, 3 and 5). This seems to indicate that, as was admitted before, only the angle between the principal axis and the lines of force determines the value of the HALL-coefficient.

In order to find the form of this relation the bars N°. 4 and 6, in which the principal axis makes an angle of 60° with two of the sides and is parallel to the other sides, were also tested in two positions.

In isotropic substances the HALL-effect for currents in an arbitrary plane V is determined by the product of "the" HALL-coefficient into the component of the magnetic force perpendicular to that plane. This may however also be regarded as the product of the whole magnetic force into a specific HALL-*coefficient for the plane* V. This coefficient would be obtained by multiplying the coefficient for a normal magnetic force by the cosine of the angle between the actual direction of magnetic force and the normal to the plane V. We shall apply this principle to the HALL-effect in a crystal of bismuth, and for this purpose resolve the magnetic force into the direction of the principal axis and the transverse direction. Let us assume that we found for currents in a plane ⊥ to the principal axis and the magnetic force a HALL-coefficient R_1, for currents in a plane // to the principal axis and ⊥ to the magnetic force a coefficient R_2. The simplest supposition in the case of a magnetic force M in a direction inclined at an angle α to the principal axis is then, that the HALL-effect in the plane ⊥ to M now consists of two parts, one caused by the component $M \cos \alpha$, // to the principal axis, one caused by the component $M \sin \alpha$, ⊥ to the principal axis. The HALL-coefficient R in this case is then given by

$$R = R_1 \cos^2 \alpha + R_2 \sin^2 \alpha.$$

In this deduction for simplicity R_1 and R_2 are taken as constants and not as functions of the magnetic force, as is the case with bismuth. As we aim only at an approximation this will not be open to objection; we remark only by the way that with an isotropic substance where R was a function of M, this method might lead to wrong results.

As appears from the table the value of R_1 for crystalline bismuth is very small as compared with R_2, so that we may omit the term with R_1 (which moreover is rather uncertain) except for very small values of α. Then R becomes equal to $R_2 \sin^2 \alpha$.

We give here the values, observed with the bars N°. 4 and 6 in a magnetic field 4600 for R_2 and R_1 and the calculated values $R_2 \sin^2 \alpha$, where α is 30°.

N°.	R_2	R	$R_2 \sin^2 \alpha$
4	10.3	2.5	2.6
6	12.2	3.1	3.1

The agreement between the observed and calculated R is as good as one could wish, so that the simple supposition leading to the formula for R is confirmed. The values for R_2 do not differ too much from those found with the bars 1, 2, 3 and 5.

If the equation for R is written in the following form:

$$1 = \frac{\left(\frac{1}{\sqrt{R}}\right)^2 \cos^2 \alpha}{\left(\frac{1}{\sqrt{R_1}}\right)^2} + \frac{\left(\frac{1}{\sqrt{R}}\right)^2 \sin^2 \alpha}{\left(\frac{1}{\sqrt{R_2}}\right)^2}$$

it appears that R may be obtained by the construction of an ellipsoid of revolution with $\sqrt{R_1}$ as axis of revolution parallel to the principal axis, and $\sqrt{R_2}$ as perpendicular axis. The radius vector in the direction of the magnetic force gives the value of $\frac{1}{\sqrt{R}}$ for the plane perpendicular to the magnetic force.

Also with a view to the results, mentioned below, obtained for the resistance in the magnetic field it appeared useless to connect the HALL-coefficient with the magnetisation (MAXWELL's vector $\mathfrak{J}$), as has been done before [1]).

5. *Resistance of the bismuth crystals.*

The first object of these measurements was to test whether in regularly crystallised bismuth an increase of resistance would occur when the current flows in the direction of the lines of magnetic force. For irregular (cast) bismuth-plates this question had been

[1]) Versl. der Verg. 21 April 1897, p. 501; 26 Juni 1897, p. 69. Comm. N°. 37 p. 18; Comm. N°. 40, p. 3.

answered in the affirmative by the experiments of GOLDHAMMER [1]) and others. The result of the investigation with three bars of bismuth from MERCK and mentioned in Communication N^0. 37, likewise gave an answer in the affirmative. The increase of resistance, though small, was comparable to that found in positions // when the current was perpendicular to the lines of force.

It was now considered desirable to carry out a set of measurements so complete that for an arbitrary relative position of principal axis and magnetic force the resistance in any direction would be known. The bar of which the greatest dimension was parallel to the principal axis, MERCK N^0. 3, was however hardly longer than the distance between the "resistance-electrodes", so that for this research other material was required. I found this in the crystal of bismuth put at my disposal by Mr. PERROT and shall now publish only the results obtained with that.

In these experiments we must take into account the relative positions of three directions: principal axis, magnetic force and current. In the figures 2*a*, *b* and *c* (Pl. I) the principal axis is always represented by a single arrow, the magnetic force by a double arrow, while the direction of the current, always coinciding with the longest dimension of the bars, is indicated by radii vectores *Oa*, *Ob*, *Oc* etc.

The experiments in the magnetic field may be divided into three groups:

I. Magnetic force ⊥ to principal axis.

II. " " // " " "

III. " " and " " at an angle of 60°.

For group I and II, and for the resistance without magnetic field it was very probable that the resistance in any direction with respect to the principal axis would be found by the aid of an ellipsoid with its axis coinciding with axes of symmetry of the crystal. For, these axes will remain axes of symmetry, so that the relation between electromotive force and current density can be expressed by equations like:

$$X = r_1 u \qquad Y = r_2 v \qquad Z = r_3 w.$$

1) Wied. Ann. 31, p. 360, 1887.

When a current I flows in a direction determined by the angles α, β and γ with the axes Oa, Ob and Oc,

$$u = I\cos\alpha, \qquad v = I\cos\beta, \qquad w = I\cos\gamma,$$

while the potential gradient E in the direction (α, β, γ) which measures the resistance, is given by

$$E = X\cos\alpha + Y\cos\beta + Z\cos\gamma$$

hence

$$E = I(r_1\cos^2\alpha + r_2\cos^2\beta + r_3\cos^2\gamma) = rI$$

and

$$r = r_1\cos^2\alpha + r_2\cos^2\beta + r_3\cos^2\gamma \quad \ldots\ldots \quad (*)$$

This written in the form

$$1 = \frac{\left(\frac{1}{\sqrt{r}}\right)^2\cos^2\alpha}{\left(\frac{1}{\sqrt{r_1}}\right)^2} + \frac{\left(\frac{1}{\sqrt{r}}\right)^2\cos^2\beta}{\left(\frac{1}{\sqrt{r_2}}\right)^2} + \frac{\left(\frac{1}{\sqrt{r}}\right)^2\cos^2\gamma}{\left(\frac{1}{\sqrt{r_3}}\right)^2}$$

indicates that r may be found by the construction of an ellipsoid with the square roots of the conductivities in three principal directions as axes.

The measurements indicate, that very likely also the resistances in group III can be found by means of such an ellipsoid.

We will treat now successively of:

1st. the resistances without the magnetic field;

2nd. the resistances along the axes in the three groups in the magnetic field;

3rd. the resistances in other directions, compared with values calculated from the results of 2nd by means of the above formula (*).

6. *The resistances without the magnetic field.*

With each of the six bars the resistance was measured at least four times, i.e. with the resistance electrodes at least once on each of the four sides (after the method described in Communication N°. 48) and mean values were calculated from the results.

The results are given here; r is expressed in the unit 10^{-5} C.G.S., the conductivity λ in the unit 10^{-6} C.G.S., $\sqrt{\lambda}$ in the unit 10^{-3} C.G.S.

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I

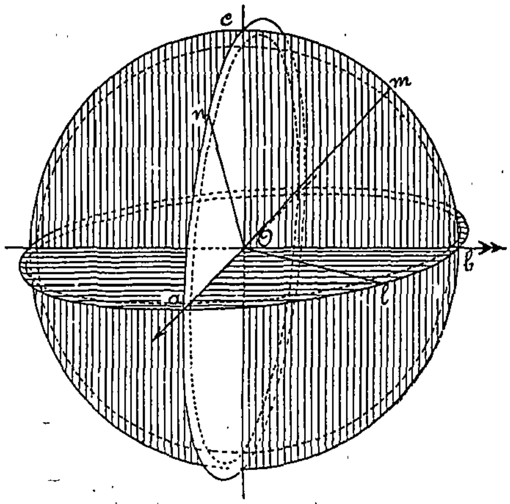

Fig. 2a.

II

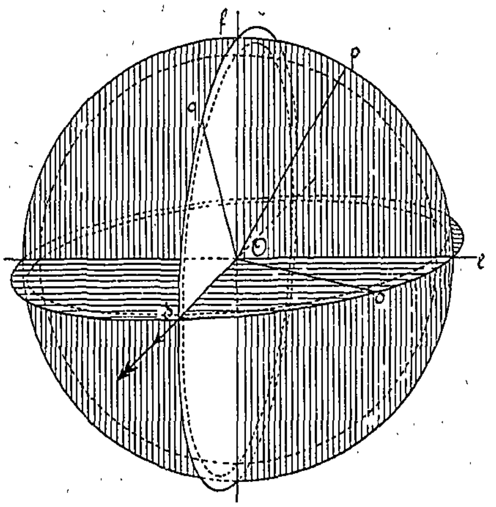

Fig. 2b.

III

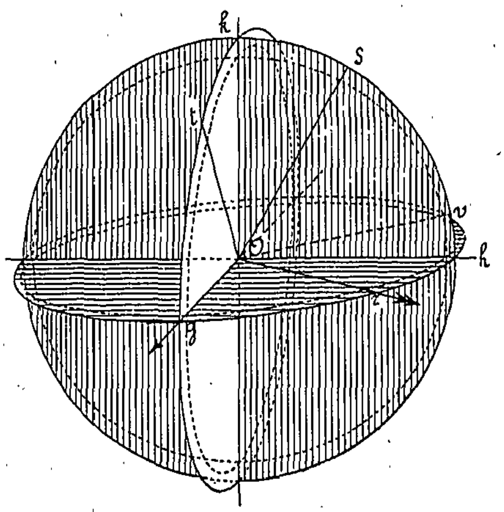

Fig. 2c.

N⁰	1	2	3	5	4	6
r	3.48	2 29	2 32	2 07	2 59	2 85
λ	2 87	4 37	4.31	4.83	3.86	3.51
$\sqrt{\lambda}$	1 70	2 09	2 08	2.20	1.96	1.87

It appears that the resistance of N⁰. 1, that is the resistance in the direction of the principal axis, is considerably larger than that in the transverse directions with 2, 3 and 5. As irregularities in crystallisation can only diminish the ratio of these resistances we are lead to assume that the ratio of the resistances of N⁰. 1 and 5, 3.48 : 2.07 or 1.68 : 1 approaches nearest to the ratio for a perfect crystal. (Also according to the results for HALL-effect 5 was the most regular bar). For the whole prism PERROT found as ratio of thermo-electric forces $\frac{//}{\perp}$ 2.00 as a mean, hence a ratio of the same order of magnitude.

The mutual differences between 2, 3 and 5 are relatively small. Hence we may assume that these differences would vanish in a perfect crystal, so that the ellipsoid of conduction without the magnetic field would be an ellipsoid of revolution. As axes for this we take the values of $\sqrt{\lambda}$ obtained with 1 and 5, that is 1.70 and 2.20. With these values in the figures 2*a*, 2*b* and 2*c* the lined circles and ellipses have been drawn, all dimensions parallel to the principal axis (←) being reduced in the ratio 2 : 1.

For the direction of N⁰. 4 and 6 a value of the resistance may now be calculated. We have $\alpha = 60°$, $\beta = 30°$ and $\gamma = 90°$ or $\alpha = 60°$, $\beta = 90°$ and $\gamma = 30°$, hence for both r is found from

$$r = 3.48 \cos^2 60° + 2.07 \cos^2 30° = 2.42$$

This value is smaller than both the observed values. If conversely from the numbers 2.59 and 2.85 α is calculated, then for N⁰. 4 53° is found, for N⁰. 6 42°, instead of 60°. It is not certain however that the differences are only caused by deviations from regular crystallisation. For, PERROT found for the density of his four best prisms numbers from 9,809 to 9,887, when the bismuth was always from the same source and had always been subjected to the same treatment; even in one and the same casting different densities were found. Hence it is possible that in the prisms too the density, and with it the resistance, varies in different points, as has been suggested by PERROT himself [1]). Accordingly the results with N°. 6 are only partially less satisfactory than those found with 4.

From the remainder of the crystalline piece another or seventh bar was cut, corresponding in original position as much as possible with N°.6. The resistance of this appeared to be 2.74, only slightly differing from N°. 6. Nevertheless in the further experiments this bar usually gave better results than N°. 6.

7. *The resistances along the axes in the magnetic field.* Without the magnetic field in the plane perpendicular to the principal axis all sets of two lines at right angles may be assumed to be axes. In the magnetic field a difference is possible between the direction which lies at the same time in the plane through principal axis and magnetic force and the perpendicular direction. We choose the original directions of the bars 2 and 3 as axes. In order that these directions shall remain axes also in the magnetic field, it is only necessary to suppose that the crystal revolves about the principal axis untill these directions coincide with the planes of symmetry determined by the magnetic force; nothing is thereby altered in the properties of the crystal as described with respect to the principal axis and the magnetic force. In the experiments the bars 2, 3 and 5 can then be used indiscriminately, for instance for measurements in the positions *Ob* and *Oc*, provided that care is taken to obtain a correct adjustment of the relative positions of principal axis, magnetic force and direction of current.

We first give only percentage increases of resistance, always in a field of 4600 C. G. S., and at 15° C.

Group I. Fig. 2a. Magnetic force ⊥ principal axis.

N°.	Position		
	Oa	*Ob*	*Oc*
1	13.0		
2		5 1	9.9
3		5.0	8 4
5		4 5	8.0

[1]) Arch. d. Sc. phys. et nat. (4) 6. p. 255. Septembre 1898.

Each of the numbers under *Oa* and *Ob* is a mean of four, each of those under *Oc* of two corresponding positions, which usually showed only small differences.

For the construction of the new ellipsoid (dotted) the values for 1 and 5 were used. These give for the new axis the values:

$$(Oa)\ g_1^{90} = 1.60 \qquad (Ob)\ g_2^{90} = 2.16 \qquad (Oc)\ g_3^{90} = 2.12.$$

Hence the ellipsoid of conduction has now *three unequal* axes. In the plane perpendicular to the magnetic force the resistances are not proportionally increased. The simple hypothesis, formulated before [1]) and reconcilable with the former imperfect material, which assumed a proportional increase of resistance in this plane, must now be abandoned. However for the explication of the dissymmetry of the HALL-effect in bismuth, which was originally the object of this research, and for the description of the increase of resistance in the magnetic field this is a simplification. According to the researches of LEBRET and of myself the unequal increase of resistance in two perpendicular directions causes the dissymmetry. It has now become superfluous to take the direction of the magnetisation ($\mathfrak{J}$) into account in order to explain this inequality. As will appear from what follows, in each case where the principal axis is not perpendicular to the plate a disproportional increase of resistance, and with that dissymmetry, will be found.

In the figure the differences between the new axes and the old ones are drawn on a twice magnified scale in order not to render the drawing indistinct.

Group II. Fig. 2b. Magnetic force // principal axis.

N⁰.	Position	
	Od	*Oe* or *Of*
1	2.5	
2		5.0
3		4.4
5		2.9

1) Versl. d. Verg. 21 April 1897, d. 501. Comm. N⁰. 37. p. 18.

The number under *Od* is a mean of four observations, the other numbers of two observations.

As there is no theoretical difference between N°. 2, 3 and 5 or the positions *Oe* and *Of* the latter are united in one column. So the ellipsoid remains one of revolution, while the whole variation is much smaller than in the preceding case. With the values for 1 and 5 the new axes become 1.68 and 2.17. With the value for 1 and the mean for 2 and 3, 1 68 and 2.15. For the figure we chose as new axes

$$(Od)\ g_1^0 = 1.68 \qquad (Oe,\ Of)\ g_2^0 = g_3^0 = 2.16.$$

In order to keep the drawing distinct it was here necessary to draw the variations to the scale of four.

Group III. Fig. 2c. *Magnetic force and principal axis at an angle of* 60°.

N°.	Position		
	Og	*Oh*	*Ok*
1	11.2		
2		4 1	9.1
5		4 0	7.6

Here also the three axes of the ellipsoid are unequal. With the values for 1 and 5 as a basis, the new axes become

$$(Og)\ g_1^{60} = 1.61 \qquad (Oh)\ g_2^{60} = 2.16 \qquad (Ok)\ g_3^{60} = 2.12,$$

hence only slightly differing from those in group I.

In the figure the differences are drawn on a double scale.

8. *Resistances along other directions in the magnetic field.* With regard to the differences between the results for corresponding bars even without the magnetic field, mentioned in § 6, it would not be allowable to directly compare resistances observed in the experiments of this § with calculated resistances, as in most cases the calculation will be based upon experiments with *other* bars. More is to be expected from a comparison between observed and calculated *increases*

of resistance in the magnetic field and we will make the comparison in this manner. We should not however expect more than an approximate agreement.

For the calculation the following method was used: for each direction of experiment the resistance was calculated by means of the formula:

$$r = r_1 \cos^2 \alpha + r_2 \cos^2 \beta + r_3 \cos^2 \gamma,$$

in which for r_1, r_2 and r_3 the values applying without and in the magnetic field were consecutively substituted. From the two results a percentage increase of resistance for the direction α, β, γ was deduced, and this was compared with the percentage increase directly observed.

As an example of calculation:

Magnetic force ⊥ principal axis. Direction *On* (fig. 2*a*).

$$\alpha = 60^\circ, \quad \beta = 90^\circ, \quad \gamma = 30^\circ.$$

$$r_1 = 3.48 \quad r_2 = r_3 = 2.07$$

$$r_1^{90} = 3.93 \quad r_2^{90} = 2.16 \quad r_3^{90} = 2.24$$

$$r = r_1 \cos^2 60^\circ + r_3 \cos^2 30^\circ = 2.42$$

$$r^{90} = r_1^{90} \cos^2 60^\circ + r_3^{90} \cos^2 30^\circ = 2.66$$

Percentage increase of resistance $\frac{0.24}{2.42} = 9.9\ \%$.

Here follow the results for the three groups; the indices of the r's correspond to those of the ϱ's.

$r_1 = 3.48 \quad r_2 = r_3 = 2.07 \quad r_1^{90} = 3.93 \quad r_2^{90} = 2\ 16 \quad r_3^{90} = 2.24$

Direction	α, β, γ	N°.	Perc. increase of resistance.			
			observed	calculated	along the axes of the corresponding ellipse greatest	smallest
Ol	60°, 30°, 90°	4	10.2	7.5	13.0	4.5
»		6	9.2	»		
»		7	6 6	»		
Om	90°, 45°, 45°	5	5.5	6.3	8 0	4.5
On	60°, 90, °30°	4	8.7	9 9	13.0	8.0
»		6	10.2	»		
»		7	9.4	»		

The most important deviations occur with the direction *Ol* where the increase of resistance for the axes is most different and accordingly a deviation of the direction of the axis has the largest influence. In each case the observed increase of resistance lies between the values of the last two columns.

Group II. Fig. 2b. Magnetic force // principal axis.

As there exists here no theoretical difference between the directions *Oe*, *Of* and *Op* and also between the bars 2, 3 and 5, for experimental purposes only the aequivalent directions *Oo* and *Oq* are left.

$r_1 = 3.48$ $r_2 = r_3 = 2.07$ $r^0_1 = 3.57$ $r^0_2 = r^0_3 = 2.15$

Direction	α, β, γ	N⁰.	Perc increase of resistance.			
			observed	calculated	along the axes of the corresponding ellipse	
					greatest	smallest
Oo or *Oq*	60°, 30°, 90°	4	3.5	3.5	3.8	2.5
		6	5.1	//		
		7	4.0	//		

Group III. Fig. 2c. Magnetic force at an angle of 60° *with the principal axis.*

As mentioned before in § 7 in this case a doubt might arise whether the resistances will allow of a deduction from an ellipsoid and whether casu quo the axes will still be in the same directions as in both the former cases. An experiment which throws some light directly upon this question is the comparison of the increase of resistance in the directions *Or* and *Ov*. For the ellipsoid these are aequivalent; but for one of them the current is parallel to the magnetic force, for the other one the current flows at an angle of 60° to the magnetic force. The result of this experiment was:

	Or	*Ov*
with N⁰. 4	9.3	9.3
» 6	6.8	7.9
» 7	6.8	6.0

Hence with N°. 4 the agreement is perfect; with 6 and 7 the deviations are in opposite directions. Therefore this result may be considered as confirming the supposition of an ellipsoid.

The results of the further experiments were:

$r_1 = 3.48$ $r_2 = r_3 = 2.07$ $\overset{60}{r_1} = 3.87$ $\overset{60}{r_2} = 2.15$ $\overset{60}{r_3} = 2.23$

Direction	α, β, γ	N°.	Perc. increase of resistance			
			observed	calculated	along the axes of the corresponding ellipse	
					greatest	smallest
Or or *Ov*	60°, 30°, 90°	4	7.7	6.6	11.2	4.0
"		6	6.9	"		
"		7	6.6	"		
Os	90°, 60°, 30°	5	6.7	6.8	7.6	4.0
Ot	60°, 90°, 30°	4	7.3	9.1	11.2	7.7
		7	8.5			

The deviations in this case are certainly not greater than in the other groups, so that they may be considered as not contradictory to the supposition that in this case also the resistances in all directions can be found by means of a conduction-ellipsoid on the axes of symmetry.

9. This result would at once be explained if we were allowed to assume that, in the case of a magnetic force inclined with respect to the principal axis, the increase of resistance for each axis would be found as the sum of two increases, one caused by the component of the magnetic force parallel to, the second by the component perpendicular to the principal axis.

In order to test this hypothesis by means of the experiments it was necessary to know the function connecting the increase of resistance with the magnetic force in this bismuth. For this purpose I could use the formula deduced before [1])

$$\Delta r = \frac{C_2 M^2}{1 + C_1 \sqrt{M^2}}.$$

[1]) Versl. d. Verg. 25 Maart '99, p. 485. Comm. N°. 48, p. 4.

As in most positions the increase was somewhat small for a reliable determination of the constants in this formula, I assumed that C_1 would have sensibly the same value for the various positions and axes, and only made some experiments for the direction *Oa*, in magnetic fields 2300, 3750 and 5800. These furnished for the constants the values

$$C_1 = 0.19 \qquad C_2 = 1.29.$$

In the experiments of group *III* the component of the magnetic force // principal axis was $4600 \cos 60° = 2300$, the component ⊥ principal axis $4600 \sin 60° = 3980$. Accordingly the increases of group *I* will have to be multiplied by

$$\frac{3.98^2}{4.60^2} \times \frac{1 + 4.60 \times 0.19}{1 + 3.98 \times 0.19} = 0.800 \text{ or } \frac{4}{5}$$

and those of group *II* by

$$\frac{2.3^2}{4.6^2} \times \frac{1 + 4.6 \times 0.19}{1 + 2.3 \times 0.19} = 0.326 \text{ or about } \frac{1}{3}.$$

So we find, using the values for N°. 1 and 5

Direction *Og*	$\frac{4}{5} \cdot 13.0 + \frac{1}{3} \cdot 2.5 = 11.2$,	observed	11.2
» *Oh*	$\frac{4}{5} \cdot 4.5 + \frac{1}{3} \cdot 2.9 = 4.6$	»	4.0
» *Ok*	$\frac{4}{5} \cdot 8.0 + \frac{1}{3} \cdot 2.9 = 7.6$	»	7.4

The agreement here may be considered very good, it is however favoured by the fact that in this case the *same two* bars could be used for calculation and experiment. Hence the observations do not afford any reason to doubt the validity of the principle of superposition in this case.

10. The results of this research may be summed up as follows: In crystalline bismuth the HALL-coefficient is large for a magnetic force ⊥ principal axis, very small for a magnetic force // principal axis (same order of magnitude as in other metals), while the coefficient for a magnetic force in any direction can be deduced from those in both principal cases with the aid of an ellipsoid of revolution.

Without a magnetic field the resistances in crystalline bismuth can be found for all directions by means of a conduction ellipsoid *of revolution* on the principal axis. (Axes in the ratio of 5 : 3).

In a magnetic field // principal axis there is an ellipsoid of revolution with comparatively slightly varied axes.

In a magnetic field ⊥ principal axis there is an ellipsoid with three more varied unequal axes.

In an arbitrary magnetic field there is an ellipsoid with three unequal axes which can be obtained by superposition of the variations of the axes in the principal cases.

The resistances in two directions at right angles in a plate of bismuth will generally increase *unequally* in the magnetic field, which explains the dissymmetry of the HALL-effect.

Physics. — J. C. SCHALKWIJK: "*Precise isothermals. I. Measurements and calculations on the corrections of the mercury meniscus with standard manometers*" (Communication N⁰ 67 from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES).

1. For the accurate investigation of isothermals of gases by means of piezometer tubes, into which mercury is forced, it is desirable to work with pretty large quantities of gas and to take care that the surface of the space it occupies is as small as possible with regard to its volume. For a given range of pressures we therefore desire to read the mercury meniscus in a tube the section of which is as large as is compatible with the accuracy of the adjustment and with the pressures which the piezometers have to resist. The correction for the capillary pressure to be applied to the pressure observed can only be applied with sufficient certainty when the piezometer tube is sufficiently large.

For such tubes, the volume of the meniscus may not in general be supposed to be equal to that of a spherical segment as it may allowably be considered in verry narrow tubes. This is the less permissible as the desired accuracy in the determination of the enclosed volume of gas is greater.

To attain in the measurements with the standard gasmanometers described in Communication N⁰ 50 of the Physical Laboratory at Leiden, the high degree of accuracy for which they are designed, an investigation of the volume of the meniscus which shuts off the gas is indispensable. For, these piezometers are made to accurately determine together with the standard open manometers, described in