We calculate:

for R=0,2832cM.; $\frac{p}{R^2}$=0,25 hence p=0,0201 and δ=0,0708; V=0,00265cc.
» »= » »; » =0,15 » »=0,0120 » »=0,0425; »=0,00158 »
» »=0,382 »; » =0,2 » »=0,0292 » »=0,0765; »=0,00725 »
» »= » »; » =0,1 » »=0,0146 » »=0,0383; »=0,00362 »

For still smaller values of δ we may use the development in series, in which $\left(\frac{dh}{dr}\right)^2$ is wholly neglected as compared with unity. We get

$$h = \mathrm{d} \sum \frac{1}{(n\,!)^2} \left(\frac{s}{4\,H}\right)^n r^{2n}.$$

and for the volume of the meniscus

$$V = \pi\, p\, R^2 \left\{ 1 - \frac{\sum \frac{1}{(n!)^2\,(n+1)} \left(\frac{s}{4\,H} R^2\right)^{n-1}}{\sum \frac{1}{(n!)^2} \left(\frac{s}{4\,H} R^2\right)^{n-1}} \right\}$$

(To be continued.)

Physics. — H. A. LORENTZ. — "*The Theory of Radiation and the Second Law of Thermodynamics*".

§ 1. In his celebrated theoretical researches on the emission and absorption of rays of heat and light, KIRCHHOFF was led to introduce a certain function of wave-length and temperature which is independent of the particular properties of the body considered. This function, whose mathematical form later investigators have tried to determine, represents the ratio, at a definite temperature and for a definite wave-length, between the emission E and the absorptive power A of a body, both taken in the sense assigned to them by KIRCHHOFF; indeed, by his law, this ratio is the same for all bodies, being always equal to the emission of what KIRCHHOFF calls a perfectly black body.

§ 2. The function in question has yet another physical meaning. If a space which contains nothing but aether is enclosed by perfectly black walls of the temperature T, it will be traversed in all directions by rays, and the aether will thus be the seat of a certain amount of energy. We may consider this energy as made up of a large number of parts, each of them belonging to the rays of a particular wave-length, and, for a given state, this repartition of the energy over the radiations of different periods can only be effected in a single way. Hence, if for unit space, we write

$$f(T, \lambda)\, d\, \lambda$$

for the energy, as far as it corresponds to the rays of wave-lengths between λ and $\lambda + d\,\lambda$, and

$$\mu = \int_0^\infty f(T, \lambda)\, d\, \lambda$$

for the whole energy, the function $f(T, \lambda)$ will be wholly determinate.

Now, this function is intimately connected with the emission of the black walls, and from KIRCHHOFF's law it follows that the state of the aether which it defines may also be the result of the radiation of a body that is not black.

To begin with, the walls of the enclosure may be made on the inside perfectly reflecting, instead of perfectly black. If, then, a certain part R_1 of the enclosed space be occupied by a black body M of the temperature T, and the remaining part R_2 by aether, it is easily seen that the state characterized by $f(T, \lambda)$, if once existing in R_2, will not be disturbed by the presence of M, but will be in equilibrium with the internal motions of the ponderable matter. It will even be the only state having this property, and must therefore of necessity be produced by the body, provided the geometrical conditions are such that, after a certain number of reflections by the walls, every ray in the space R_2 must ultimately strike the body M.

KIRCHHOFF's law further proves that the equilibrium will continue to exist, if the black body is replaced by any other body M of the temperature T, whatsoever be its physical and chemical state and its properties. What is more, such a body will also *give rise* to the same state of radiation as the black body did before, at least if the above geometrical condition is again fulfilled, and if, besides, the body has some absorptive power, be it ever so feeble, and consequently some emissivity, for every wave-length that is represented in the radiation of the black body. This may safely be assumed.

The function $f(T, \lambda)$ is thus seen to have a second universal physical meaning. The state of the aether to which it relates may for the sake of brevity be called the state corresponding to the temperature T.

§ 3. Since KIRCHHOFF's time great advances have been made in the investigation of the form of the function. By a most ingenious reasoning, founded partly on thermodynamic principles and partly on the electromagnetic theory of light, BOLTZMANN [1]) has shown that the total energy per unit of volume must be proportional to the fourth power of the absolute temperature, so that, if this is henceforth designed by T,

$$\int_0^\infty f(T,\lambda)\, d\lambda = CT^4, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C is a universal constant, whose numerical value will of course depend on the choice of the units.

A result that has been obtained by W. WIEN [2]) is likewise very remarkable. He found that $f(T, \lambda)$ is of the form

$$f(T,\lambda) = T^5 \varphi(T\lambda) = \frac{1}{\lambda^5} \psi(T\lambda), \quad . \quad . \quad . \quad . \quad (2)$$

$\varphi(T\lambda)$ or $\psi(T\lambda)$ being a function of the product $T\lambda$. Evidently BOLTZMANN's result is contained in the latter law.

WIEN [3]) and PLANCK [4]) have also endeavoured to discover the form of the function φ, but we need not here speak of these researches.

§ 4. The experiments of PASCHEN, and those of LUMMER and PRINGSHEIM have furnished a very satisfactory verification of the laws, expressed by (1) and (2), and have thus confirmed the fundamental supposition that the second law of thermodynamics holds in this domain of physics, as well as the validity of the reasoning by which the two formulae have been established. In fact, I don't see that any but perhaps some far fetched objection could be raised against the theories of BOLTZMANN and WIEN. In my opinion, we cannot but recognize all that has been said as legitimate deductions

1) BOLTZMANN, Wied. Ann, Bd. 22, p. 291; 1884.
2) WIEN, Wied. Ann., Bd. 52, p. 132; 1894.
3) WIEN, Wied. Ann. Bd. 58, p. 662; 1896.
4) PLANCK, Drude's Ann. Bd. 1, p. 116; 1900. Verhandl. der deutschen Physik. Ges. Jahrg. 2, p. p. 202, 237; 1900.

from CARNOT's principle, but in so doing we are forced to a remarkable and, at first sight, somewhat startling conclusion.

The state of the aether which corresponds to a given temperature is characterized not only by the amount of energy per unit of volume, but also by at least one definite linear dimension. We may for instance fix our attention on the wave-length for which $f(T, \lambda)$ has its maximum-value, and which I shall call λ_m, or we may calculate a certain mean wave-length by means of the formula

$$\bar{\lambda} = \frac{\int_0^\infty \lambda f(T, \lambda)\, d\lambda}{\int_0^\infty f(T, \lambda)\, d\lambda}.$$

Now, the form of the function may very well be such that the ratio between λ_m, $\bar{\lambda}$ and what other lengths [1]) it might be deemed convenient to introduce, is expressed by definite numbers, but we have to explain for what reason one of these, for instance λ_m, has precisely the length that has been found for it by observation. In considering this question we shall have to take into account that, by WIEN's law, λ_m is inversely proportional to the absolute temperature.

We have good reasons for believing that, in so far as the aether is concerned, the phenomena may be exhaustively described by means of the well known equations of the electromagnetic field. If this be true, it cannot be the properties of the aether which determine the amount of energy and the preponderating wave-length, the velocity V of light being the only constant quantity which these equations contain. Hence, within the enclosure considered in § 2, the value of the energy per unit volume and that of λ_m must be forced upon the aether by the ponderable body M. But then there must exist between different bodies a certain likeness, expressible by the equality

[1]) We might for instance, without decomposing the vibrations in the aether by means of FOURIER's theorem, define a length l by the formula

$$l^2 = \frac{[\alpha^2]}{\left[\left(\frac{\partial \alpha}{\partial x}\right)^2 + \left(\frac{\partial \alpha}{\partial y}\right)^2 + \left(\frac{\partial \alpha}{\partial z}\right)^2\right]},$$

in which α is one of the components of the dielectric displacement or the magnetic force, whereas the brackets serve to indicate the mean values, taken for a space whose dimensions are large in comparison with the wave-length, or with l itself.

of numerical quantities; else it would be inconceivable that two bodies call forth exactly the same values of μ and λ_m. Without some conformity, of one kind or another, in the structure of all substances, the consequences of the second law and this law itself cannot be understood If it did not exist, we could not even expect that a piece of copper and a mass of water for instance, after having been brought by contact into states in which they are in thermal equilibrium, would, under all circumstances, remain in these states, when exposed to their mutual radiation.

§ 5. It is by no means surprising that the validity of the rules of thermodynamics should require a certain similarity in the structure of different bodies, for in reality these rules do not teach us something about a single body, but always about two or more bodies and about the way in which these act on one another. The proposition that two bodies which, when brought into contact with a third one, do not interchange any heat with it, will also be in thermal equilibrium with each other, is clearly of this nature, and it is easily seen that our remark applies likewise to the law, that the absolute temperature is an integrating divisor of the differential expression for the quantity of heat, required for an infinitesimal change of state.

Let us suppose that an experimental investigation of the states of equilibrium of which a body (or a system of bodies) M_1, when considered by itself, is capable, has led to distinguish these states by the values of certain parameters α_1, β_1, $\gamma_1 \ldots$ Then, an infinitely small change of state may be defined by the simultaneous increments $d\alpha_1$, $d\beta_1$, $d\gamma_1, \ldots$ If, in every case, we measure the amount of heat dQ_1 that has to be supplied to the body, say by determining the equivalent mechanical energy, we may establish an equation of the form

$$dQ_1 = A_1\, d\alpha_1 + B_1\, d\beta_1 + C_1\, d\gamma_1 + \ldots\ldots, \quad . \quad . \quad . \quad . \quad (3)$$

in which the coefficients $A_1, B_1, C_1, \ldots$ are known functions of $\alpha_1, \beta_1, \gamma_1, \ldots$ The integrating divisors

$$\Delta_1', \ \Delta_1'', \ \Delta_1''', \ldots, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

of which the expression (3) admits, and which we may imagine to be determined by an ideal mathematician, will also be functions of the parameters.

Next, let M_2 be a second body or system of bodies. Operating with this, as we have done with the first one, we shall be led to the introduction of certain parameters $\alpha_2, \beta_2, \gamma_2, \ldots$, to an expression, corresponding to (3), say

$$dQ_2 = A_2\, d\alpha_2 + B_2\, d\beta_2 + C_2\, d\gamma_2 + \ldots,$$

and to its integrating divisors

$$\Delta_2', \Delta_2'', \Delta_2''', \ldots, \quad \ldots \ldots \quad (5)$$

These will be functions of $\alpha_2, \beta_2, \gamma_2, \ldots$ Now, the proposition that the temperature is an integrating divisor, ascribes a particular signification to one of the functions (4) and one of the functions (5), the inequality or equality of these functions, calculated each for a determined state of the body, having to decide as to whether the bodies, taken in these states, and placed near each other will exchange heat or not. However, in calculating the functions (4), we have not even thought of the body M_2, and in forming the functions (5), we have not had in view the system M_1. Therefore, the two functions could not be involved in what happens in the mutual action of the two bodies, if these had nothing at all in common.

§ 6. In our ordinary molecular theories, which leave out of account the phenomena in the aether, the question is very simple. So far as we know, the total want of order in the molecular motions, precisely the state of things which justifies the introduction of the calculus of probabilities, is, in these theories, a sufficient ground for the general validity of CARNOT's principle. This irregularity in the motion of the ultimate particles seems to be the only common feature of different bodies that is required. It has been found sufficient to prove the proposition that the mean kinetic energy of a molecule is the same for all gases of the same temperature, a result, which is of the highest importance in the theory of molecular motion, and is likely to be so too in that of radiation. Indeed, it is to be expected that in studying the state of the aether, corresponding to the temperature T, we shall meet again with the same definite amount of energy, with which a molecule of a gas, of that temperature, is, in the mean, endowed, and which must also play a part in the internal motions of a liquid or solid body.

I shall denote by ω this mean kinetic energy of a gaseous molecule at the temperature T.

§ 7. We shall now return to the question what similarity in the structure of all ponderable matter must lie at the bottom of the thermodynamic theory of radiation. Evidently, a perfectly satisfying answer could only be furnished by an elaborate theory of the mecanism of emission and absorption, such as has not yet been worked out, though PLANCK [1]) and VAN DER WAALS JR. [2]) have published interesting researches in this direction. We may however attack the problem in a way that does not require a knowledge of peculiarities. By comparing two systems, both composed of ponderable matter and aether, and which are, in a wide sense of the word, „similar", i. e. such, that, for every kind of geometrical or physical quantity involved, there is a fixed ratio between its corresponding values in the two systems, I shall try to show that, in all probability, the likeness in question consists in the equality of the small charged particles or *electrons*, in whose motions modern theories seek the origin of the vibrations in the aether. We shall begin by supposing that, in passing from one system to the other, the dimensions, masses and molecular forces may be arbitrarily modified; then we shall find that the charges of the electrons must remain unaltered, if the second system, as compared with the original one, is to satisfy BOLTZMANN's and WIEN's laws.

The consideration of similar systems has already proved of great value in molecular theory. It has enabled KAMERLINGH ONNES to give a theoretical demonstration of VAN DER WAALS's law of corresponding states; moreover, the experimental confirmation of this law has taught us that a large number of really existing bodies may, to a certain approximation, be regarded as similar.

Of course, if the theory is also to embrace the phenomena going on in the aether, we have less liberty in choosing the systems to be compared. Since the properties of the aether cannot be changed, the velocity of light is not in our power, and the similarity implies that all other velocities must likewise be left unaltered.

§ 8. Let the first of the two systems be the one that has been considered in § 2: a ponderable body M, and, next to it, a certain space, filled with aether, both enclosed by walls that are perfectly reflecting on the inside.

Let the ponderable body be built up of a large number of small particles, each of which has a certain volume, so that the density

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[1]) PLANCK, Drude's Ann. Bd. 1, p. 69, 1900.

[2]) VAN DER WAALS JR., Statistische behandeling der stralingsverschijnselen. Dissertation. Amsterdam. 1900.

of ponderable matter is finite everywhere. To these particles we shall ascribe an irregular "molecular" motion and the power of acting on one another with certain "molecular" forces.

We shall further suppose them — or some of them — to be electrically charged, and, for convenience' sake, we shall consider each charge to be distributed over a small space, with finite volume-density ϱ. This density may be treated as a continuous function, which sinks gradually into 0 at the surface of the electrons. Of course, if some of the particles have no charge, we have only to put for these $\varrho = 0$.

Finally, we shall take for granted that the aether pervades the space occupied by the particles, and that a dielectric displacement $\mathfrak{d}$ and a magnetic force $\mathfrak{H}$ may exist as well inside as outside a particle.

Then, if $\mathfrak{d}_x, \mathfrak{d}_y, \mathfrak{d}_z, \mathfrak{H}_x, \mathfrak{H}_y, \mathfrak{H}_z$ are the components of $\mathfrak{d}$ and $\mathfrak{H}$, and $\mathfrak{v}_x, \mathfrak{v}_y, \mathfrak{v}_z$ those of the velocity, we have the following equations [1]:

$$\left.\begin{aligned} \frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} &= 4\pi\left(\varrho\, \mathfrak{v}_x + \frac{\partial \mathfrak{d}_x}{\partial t}\right), \\ \frac{\partial \mathfrak{H}_x}{\partial z} - \frac{\partial \mathfrak{H}_z}{\partial x} &= 4\pi\left(\varrho\, \mathfrak{v}_y + \frac{\partial \mathfrak{d}_y}{\partial t}\right), \\ \frac{\partial \mathfrak{H}_y}{\partial x} - \frac{\partial \mathfrak{H}_x}{\partial y} &= 4\pi\left(\varrho\, \mathfrak{v}_z + \frac{\partial \mathfrak{d}_z}{\partial t}\right), \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (6)$$

$$\frac{\partial \mathfrak{d}_x}{\partial x} + \frac{\partial \mathfrak{d}_y}{\partial y} + \frac{\partial \mathfrak{d}_z}{\partial z} = \varrho, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

$$\left.\begin{aligned} 4\pi V^2\left(\frac{\partial \mathfrak{d}_y}{\partial z} - \frac{\partial \mathfrak{d}_z}{\partial y}\right) &= \frac{\partial \mathfrak{H}_x}{\partial t}, \\ 4\pi V^2\left(\frac{\partial \mathfrak{d}_z}{\partial x} - \frac{\partial \mathfrak{d}_x}{\partial z}\right) &= \frac{\partial \mathfrak{H}_y}{\partial t}, \\ 4\pi V^2\left(\frac{\partial \mathfrak{d}_x}{\partial y} - \frac{\partial \mathfrak{d}_y}{\partial x}\right) &= \frac{\partial \mathfrak{H}_z}{\partial t}, \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (8)$$

$$\frac{\partial \mathfrak{H}_x}{\partial x} + \frac{\partial \mathfrak{H}_y}{\partial y} + \frac{\partial \mathfrak{H}_z}{\partial z} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

[1] See f. i. LORENTZ, Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern. 1895.

These, with $\varrho = 0$ everywhere outside the electrons, and if we add proper conditions at the reflecting walls, serve to determine the state of the aether, as soon as we know the motions of the electrons.

The energy of the aether per unit volume is given by

$$2\pi\ V^2 \mathfrak{d}^2 + \frac{1}{8\pi} \mathfrak{H}^2, \quad \ldots \ldots \ldots \quad (10)$$

and the components of the force, exerted by the aether on the electrons, will be for unit charge

$$\left.\begin{array}{l} 4\pi\ V^2 \mathfrak{d}_x + \mathfrak{v}_y \mathfrak{H}_z - \mathfrak{v}_z \mathfrak{H}_y, \\ 4\pi\ V^2 \mathfrak{d}_y + \mathfrak{v}_z \mathfrak{H}_x - \mathfrak{v}_x \mathfrak{H}_z, \\ 4\pi\ V^2 \mathfrak{d}_z + \mathfrak{v}_x \mathfrak{H}_y - \mathfrak{v}_y \mathfrak{H}_x. \end{array}\right\} \quad \ldots \ldots \quad (11)$$

Besides these forces, there may be (molecular) forces of another kind, acting on the electrons.

§ 9. We have next to compare this really existing system S with a second system S', which perhaps will be only an imaginary one. Its enclosure is to be geometrically similar to that of S, the linear dimensions being a times what they are in the first system. By corresponding points in the spaces within the two enclosures, we shall mean points that are similarly situated, and to every instant in the interval of time, during which we consider the phenomena in S, we shall coordinate an instant for the second case, in such a way that the interval between any two moments in S' is a times the interval between the corresponding moments in S.

Let it further be assumed that, if at a particular instant ponderable matter or an electric charge is found at some point of one of the two systems, this will likewise be the case at the corresponding time and the corresponding point of the other system. As a consequence, the distribution of matter and of electric charge will be, at corresponding times, geometrically similar in the two cases, the dimensions of the particles in S' and their mutual distances bearing the ratio a to the corresponding quantities in S.

What has been said suffices to determine the internal motions in S', as soon as one knows those in S; the velocities will be the same in the two systems, because we have supposed the ratio of corresponding times to be equal to that of corresponding lengths. Of course, the motions in S and S' will present just the same degree of irregularity.

Now, our description of the state of the second system will become complete, if we indicate, for each of the physical quantities involved, the number by which we must multiply its value in S, in order to obtain its value in S' at corresponding points and times.

Let this factor be b for the density of ponderable matter, c for the density of electric charge, and $a\,c$ for the dielectric displacement and the magnetic force. Then, since the phenomena in the system S, which exists in reality, agree with the equations (6)—(9), those in S' will likewise satisfy these relations. Nor will the conditions imposed by the nature of the walls be violated. We may also remark that the formulae which are obtained for the two systems, if the motions are analyzed by means of FOURIER's theorem, will differ from each other only by the constant factors a and c. The ratio between corresponding wave-lengths, e. g. between the values of λ_m, will of course be a.

As to the motions we have attributed to the electrons in S', these will only be possible, if a, b and c satisfy a certain condition. The ratio of the accelerations being $\frac{1}{a}$, and that of the masses of corresponding elements of volume (or of corresponding particles) $a^3 b$, the forces acting on such elements must be in S' $a^2 b$ times what they are in S. Now, whereas the „molecular" forces may be supposed to be regulated according to this rule, the action of the aether on the electrons in S' has already been fixed by what has been said. The components (11) of the force on unit charge are, in S', $a\,c$ times what they are in S, and for the charges of corresponding elements of volume the ratio is $a^3 c$. The factor for the forces exerted by the aether on such elements will therefore be $a^4 c^2$, and we must have the relation

$$a^2 b = a^4 c^2,$$

or

$$b = a^2 c^2. \qquad (12)$$

This being the only condition, we may imagine a large variety of systems S', similar to S, and which must be deemed possible as far as our equations of motion are concerned. The coefficients a and c having been chosen, and b calculated by (12), we should find, by (10),

$$a^2 c^2 \qquad (13)$$

for the ratio of the kinetic energies per unit volume, and

$$a^3 b,$$

or, in virtue of (12),

$$a^5 c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$ [1]

for the ratio of the kinetic energies of a molecule or an electron.

The latter number will at the same time be the factor by which we have to multiply the temperature T of S in order to obtain that of S'. Indeed, in the formulae (1) and (2), we may suppose T to be measured by observations in which radiation does not come into play, say by means of a thermometer; we may therefore apply the result of molecular theory that T is proportional to the mean kinetic energy of a particle.

§ 10. If we had only to satisfy the equations of motion, a and c might be arbitrarily chosen. We could then take

$$c = a^{-\frac{5}{2}}$$

and $b = a^{-3}$. By this the value of (14) would become 1 and that of (13)

$$a^{-3},$$

which might have any magnitude we like. In this way we should have got two systems S and S' of equal temperatures, but with different amounts of energy in the same space. This being in contradiction with the results, deduced from CARNOT's principle, the choice of a and c must be appropriately limited.

If the two systems we have compared with each other are to agree with BOLTZMANN's law, (13) must be equal to the fourth power of (14). From this we conclude

$$a^3 c = 1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

that is to say, the charges of corresponding elements of volume,

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[1]) A moving charged particle produces in the surrounding aether an electromagnetic energy, which, for small velocities v, may be reckoned proportional to v^2. It may therefore be represented by $\frac{1}{2} k v^2$. The factor k plays the part of a mass, and may be called the electromagnetic or apparent mass, in order to distinguish it from the (true) mass in the ordinary sense of the word. Now, k is found to be proportional to the square of the charge, and inversely proportional to the dimensions of the particle. The condition (12) therefore means that the ratio between the true and the electromagnetic masses is the same in S and S'. There would be no necessity to introduce a condition of this kind, if there were no true mass at all; neither, if some of the particles had no charge, and the remaining ones no true mass.

We may also express the relation (12) by saying, that the ratio between the electromagnetic and the ordinary kinetic energy has to be the same in the two systems.

and also those of corresponding electrons must be the same in S and S'.

If (15) is satisfied, the two systems will accord with WIEN's law, as well as with that of BOLTZMANN. In the first place, the ratio of the temperatures, for which we found the number (14), now reduces to

$$\frac{1}{a}.$$

As the values of λ_m are to each other as 1 to a, they are inversely proportional to the temperatures of the two systems.

We may remark in the second place that the repartition of the energy over the rays of different wave-lengths will be similar in the two systems. Consider for instance the rays in S whose wave-lengths lie between λ and $\lambda + d\lambda$; by WIEN's law, the energy in unity of volume, depending on them, is

$$T^5 \varphi(T\lambda)\, d\lambda. \quad \quad (16)$$

The corresponding rays in the second system have their wave-lengths between λ' and $\lambda' + d\lambda'$, if

$$\lambda' = a\lambda, \qquad d\lambda' = a\, d\lambda,$$

and, in order to calculate the energy in unit space which is due to these rays, we have only to multiply (16) by the factor (13), which becomes $\frac{1}{a^4}$, in virtue of (15). Now, one gets the same expression

$$\frac{1}{a^4} T^5 \varphi(T\lambda)\, d\lambda,$$

if, in (16), one replaces λ by λ', $d\lambda$ by $d\lambda'$, and the temperature T of S by the temperature $T' = \frac{T}{a}$ of S'. It appears from this that the distribution of energy over the different rays in S' is exactly what it ought to be by WIEN's law at the temperature of the system.

§ 11. What precedes calls forth some further remarks. It might be argued that two bodies existing in nature will hardly ever be similar in the sense we have given to the word, and that therefore, if S corresponds to a real system, this will not be the case with S'. But this seems to be no objection. Suppose, we have formed an image of a class of phenomena, with a view to certain laws that have been derived from observation or from general principles. If then, we wish to know, which of the features in our picture are essential and which not, i. e., which of them are necessary for the agreement with the laws in question, we have only to seek in how far these latter will still hold after different modifications of the image; it will not at all be necessary that every image which agrees in its essential characteristics with the one we have first formed corresponds to a natural object.

We have many grounds for expecting that a theory of radiation can be developed on the lines drawn in § 8. In such a theory we shall have to distinguish between the hypotheses concerning the uncharged particles, the ordinary molecular motions and forces, and those which relate to the electrons, their dimensions, masses and charges and the non-electrical forces which, conjointly with the electromagnetic ones, determine their motion. Now, it seems natural to admit that in a theory of radiation the hypotheses which relate to the electrons form the essential part of the explanation, and that all the rest may be freely modified within the limits indicated by the ordinary molecular theories.

If we had a right, likewise to change at will the dimensions of the electrons, their true masses and the forces to which they are subject, the considerations of § 10 would only leave room for the conclusion, that a definite magnitude of the electric charges must be reckoned among the essential features of our picture. One might however be of opinion that these dimensions, masses or forces contain already elements that are necessary parts of the theory. For instance, the electrons could have a fixed, constant diameter, the same in all ponderable matter. If this were the case, our factor a could not be different from unity, and the formulae (12) and (15) would give $b = 1$, $c = 1$. The system S' would be identical with S, and it would be impossible to learn anything from it. Again, the ratio between the densities of ponderable matter and of electric charge might be a universal constant. This would require $b = c$, and by (12) and (15) $a = b = c = 1$. The way in which we have treated the molecular forces acting on the electrons is also liable to objection. If a definite intensity of these forces were a requirement in the theory, it would be impossible so to regulate them, that they are in S' $a^4 c^2$ times as great as in S.

These remarks do not, however, invalidate the general conclusion, that the electrons in two ponderable bodies cannot be wholly different. We may even remark that, if it were found necessary to ascribe equal dimensions to the electrons of different bodies, it would be not unnatural to suppose them equal in all other respects. This latter hypothesis would likewise recommend itself as the simplest possible, in case we ought to assume a constant ratio between the masses and the charges, and a fixed relation between the above mentioned forces in different bodies would in its turn point with some probability to an equality of the electrons.

Of course I do not mean to say that all electrons in nature must be of one and the same kind. Anyways, there must be both positive and negative particles, and we may imagine any number of kinds of electrons we please. The conformity between different substances should in this case be attributed to the existence of each of those kinds, with their definite charge, in every body.

We must leave these questions for future research. The theory will also have to explain why the phenomena always depend on the temperature in the way expressed by the equations (1) and (2). It is true, we have compared cases in which the temperatures were not the same, but in those cases we had to do with different bodies, whose molecular weights were such, that the velocities of the particles were equal at the two temperatures compared. It will be necessary also to compare the same body at different temperatures, and this cannot be done by barely comparing similar systems.

§ 12. The question remains, on what quantities that are involved in the constitution of ponderable bodies the values of λ_m and the energy μ per unit space may be taken to depend. We have spoken of the dimensions, the masses and the electric charges of the electrons, or of a particular kind of electrons. These might be the same through all nature, and besides these there is the mean kinetic energy ω of a molecule at the temperature T. Now we may conceive different ways, in which λ_m and μ could be derived from these quantities. For instance, a given electric charge e, taken together with a given amount of energy ω, may determine a definite *length*. This follows at once from the „dimensions" of e and ω, but we may explain it as well by remarking that, if a charge e is uniformly distributed over a sphere of radius R, there will be an electrostatic energy

$$\frac{1}{2}\frac{e^2 V^2}{R}$$

(e being expressed in electromagnetic units). Hence, if we desire this energy to have the value ω, the radius must be

$$R = \frac{1}{2}\frac{e^2 V^2}{\omega} \quad . \; . \; . \; . \; . \; . \; . \; . \; (17)$$

This is a length, entirely determined by e and ω, and it may be that λ_m bears always a fixed ratio to R. As to the energy per unit volume, it will probably be determined by some such condition as this, that the energy, contained in a cube whose side is λ_m, is in all cases the same multiple of ω [1]).

We may add that ω varies as T, and that therefore the line R, calculated by (17), will vary as $\frac{1}{T}$. Hence, the length of λ_m, if determined in the way we have indicated, will be found inversely proportional to the temperature, as we know it to be. Moreover, in accordance with BOLTZMANN's law, the energy in unity of volume would become proportional to T^4, if a cube, whose side varies as $\frac{1}{T}$, contained an amount of energy, which is itself proportional to the temperature.

I shall conclude by mentioning that Prof. PLANCK, after having found for the function $f(T, \lambda)$ the form

$$\frac{2\, V^2 b}{\lambda^5}\, e^{-\frac{aV}{\lambda T}},$$

has calculated from experimental data the coefficients a and b contained in it, and has used these coefficients, together with the velocity of light and the constant of gravitation, for the purpose of establishing units of length, mass, time and temperature that are given by nature, without it being necessary to choose some standard body.

If the above considerations are to be trusted, this universal system of units would be based on the velocity of light, the constant of gravitation, the mean kinetic energy of a molecule and the properties of the electrons, present in all ponderable matter.

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[1]) What multiple this is, may be deduced from the observations on radiation, combined with what we know about the mass and the kinetic energy of a molecule. It is also implicitly contained in the considerations by which PLANCK terminates his last paper. By his formula, which, as he shows, agrees with the results of the kinetic theory of matter, I find that the energy of radiation in a cube whose side is $\bar{\lambda}$ (§ 4) amounts to a little more than 5,5 ω.