the addition need be but very small and the degree of hardness not very great: the question here is only to impede cleavage or gliding.

But then similar phenomena we know excellently well in alloys, in which very small additions may considerably increase the hardness of the main substance, and these additions in themselves need not be very hard. This is a known fact of iron, but the quotient of copper (its theoretical hardness) is equally great as that of iron; so copper by the addition of small quantities of other elements must be able to acquire not only the hardness of iron, but also that of steel. So by these additions we must impede cleavage or gliding in copper. For this purpose it is preferable to choose elements, which are not too near akin to copper, because they may possess the same cleavage or form an isomorphous mixture.

What has been said of copper as a matter of course, holds for all metals, not one, as we may say, entirely lacking cleavage or translation. So not a single metal will reach its theoretical hardness. In the first place however this may be said of the metal beryllium, which yields the very high quotient 0,233. According to its quotient (theoretical hardness) it should be able to attain an experimental hardness, which greatly exceeds that of steel.

Astronomy. — *On the luminosity of the fixed stars.* By Prof. J. C. KAPTEYN.

1. *Mean parallax of stars of determined magnitude and proper motion.*

In a paper published elsewhere [1]) I found for the mean parallax $\bar{\pi}_{\mu\, m}$ of stars of a determined proper motion μ and a determined magnitude m (Potsdam system) the formula

$$\bar{\pi}_{\mu\, m} = \varepsilon^{m-55} \sqrt[p]{A\,\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The values of the constants were derived as well for the whole of the stars as for the stars of the first and second spectral type (Secchi's notation) separately.

[1]) Publ. of the Astr. Labor. at Groningen No. 8, On the mean parallax of stars of determined proper motion and magnitude.

I found

	Type I	Type II.	All the stars.	
A	0.116	0.0262	0.0387	
p	1.11	1.54	1.405	. . (2)
ε	0.905	0.905	0.905	

The spectra were there, as they are in this paper, taken from Pickering's „Draper Catalogue". Exceptions to this rule will be expressly stated. This catalogue will be denoted by the letters D.C. The relation (1) was derived:

1st From directly measured parallaxes, almost exclusively using the longest and most reliable series of such measures.

2nd From the mean parallax of stars of different magnitudes, according to the determination which was communicated to the Academy in the meeting of October 1897 [1]).

A further confirmation of the values of the parallax given by formula (1) for the stars with extremely small proper motions was found in the strong condensation towards the milky way of the bright stars with very small proper motions (see Proceedings Jan. 1893), as compared with the condensation for the whole of the fainter stars.

The values for type I are, comparatively speaking, very uncertain. This is explained by the fact that for this type large proper motions are exceedingly scarce, in consequence of which the parallaxes of very few stars of this type, and these exclusively very bright ones, have been directly determined.

For type II the circumstances are much more favourable. Still the values given for this type and for the whole of the stars must only be considered as preliminary results, which may be altered somewhat by the here following considerations.

2. *Probability that a star's parallax exceeds its mean value in a given proportion.*

In the paper quoted I also tried to derive the probability that the parallax of any arbitrarily chosen star shall exceed its mean

---

[1]) The only alteration made in the figures there given is a small correction, which has been applied to the mean magnitude of the stars 0—3.5, in order to bring them in better accordance with the best photometric determinations.

value, computed by formula (1), in a given proportion. This determination, which necessarily must be very rough, was based on the hypothesis that the quantities

$$z = log \frac{\pi}{\pi_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where π is the true, and π_0 the *probable* parallax, are distributed according to the law of errors.

By this hypothesis the determination of the required probability was reduced to the derivation of a single quantity, for which the probable amount (ϱ) of z was chosen.

The relations between the probable parallax π_0 and the mean parallax $\bar{\pi}$ is given by the formula

$$\pi_0 = \bar{\pi} . e^{-\frac{\rho^2}{4\, mod^2\, (0\ 47694....)^2}} = \bar{\pi} . e^{-5.827\rho^2} \quad . \quad . \quad . \quad . \quad (4)$$

The value of ϱ was derived from the observed parallaxes in different ways. The value which was finally adopted is

$$\varrho = 0.19 \pm 0.02 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Introducing this value, (4) becomes

$$\pi_0 = 0.810\, \bar{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The true uncertainty of this value of ϱ is somewhat larger probably than is indicated by the p. e.; it can not be doubted however that the true value of ϱ must be very small. It thus appears that the proper motion, combined with the magnitude of a star, affords a very good criterion of its distance. It is not difficult by means of the value (5) of ϱ to compute a table giving the probability that the parallax of an arbitrarily chosen star exceeds a times its mean value $\bar{\pi}$. Such a table is given in the paper quoted above. It appears that the probability is 0.5 that the parallax of a star taken at random shall be included between

$$0.523\ \bar{\pi} \text{ and } 1.255\ \bar{\pi} \text{ or between } \frac{\pi_0}{1.55} \text{ and } 1.55\ \pi_0,$$

where π_0 is the probable parallax computed by the formulae (1) and (6).

The accuracy of all these determinations (with the exception perhaps of those for type I) is already so considerable as to justify an attempt to determine from these data, combined with the known number of stars of determined magnitude and proper motion, the number of stars of determined apparent magnitude within a given distance from the solar system, and from these numbers again to conclude the relative frequency of stars of determined absolute luminosity.

3. *Data for proper motion, magnitude, and number of stars of a given magnitude.*

For the northern hemisphere the necessary data about the proper motions of stars brighter than 6.5 can be derived from AUWERS BRADLEY.

To the proper motions derived from this source I have applied the following corrections:

a. A correction originating in a correction to the constant of precession of — 0.000446 of its amount.

b. A correction to the motions in declination of — 0″.008 for declinations south of + 51° 30′ and of — 0″.001 for more northern declinations.

These values of the corrections are not yet the best which can be derived, but they differ very little from them. For the fainter stars the data about proper motions are much more uncertain.

Still I think I have succeeded in collecting even for these stars such data as will suffice to furnish a good check on the results derived from the brighter ones.

To derive data for the number of stars of a given magnitude the following sources were used:

a. *Gore*. The hundred brightest stars. Knowledge Sept. 1900 p. 202.

b. *Kobold*. (Vierteljahrsschr. der A. G. Vol. 34 p. 213).

From these two sources I could derive directly the numbers of stars of different magnitudes up to 5.5, according to photometric determinations. A correction of + 0 ^{m} 17 (see Potsdam Obs. Vol. 13, p. 459) has been applied to reduce the Harvard results to the Potsdam scale.

c. For fainter stars the data of the B. D. were used. The corrections which are necessary to reduce the magnitudes of this work to the Potsdam scale are now known with tolerable accuracy. For the magnitudes 3.0—7.0 these corrections are given in the Potsdam D. M. (Potsd. Obs. Vol. 13, p. 454); for the magnitudes 6.5—9.0 by the investigations of SEELIGER (Betracht. üb. die raümliche Vertheilung der Fixsterne. Abh. der K. bayer. Ak. der Wiss. 2e Cl. 19er Bd. 3e Abth. S. 21). The mean was taken of SEELIGER's values for the declinations 0°—49°. SEELIGER's data, when reduced to the

Potsdam scale, agree very well with the values which have been found in Potsdam for the magnitudes 6.5 and 7.0.

From all these data I find the following comparison of the magnitudes of the B. D. with the Potsdam photometric magnitudes. For the latter we have as is well known

$$\log \frac{\text{intensity of star of mag } m}{\text{intensity of star of mag } m+1} = 0.4. \quad . \quad . \quad (7)$$

BD.	Potsdam.
3.0	3.38
4.0	4.25
5.0	5.08
6.0	6.01
6.5	6.59
7.0	7.13
7.5	7.68
8.0	8.18
8.5	8.77
9.0	9.37

. . . . (8).

It was assumed that the magnitudes of AUWERS BRADLEY are homogeneous with those of the B.D.

For the numbers of stars I find from the just mentioned sources, after a careful reduction to rounded off values of the magnitude according to the Potsdam scale:

Potsd. mag.	total	Typ. $I + II$ only.
brighter than 1.50	18	17
1.50—2.50	51	47
2.50—3.50	145	133
3.50—4.50	466	456
4.50—5.50	1 508	1 476
5.50—6.50	4 944	4 839
6.50—7.50	15 370	15 043
7.50 - 8.50	45 530	44 561
8.50—9.50	153 830	150 557

. (9)

The numbers of stars belonging exclusively to the first and second spectral types, which are given in the last column were derived: for the magnitudes 0—3.5 from Mc. CLEAN's determinations; for the other magnitudes by multiplying the numbers of the foregoing column by $\frac{46}{47}$.

This ratio was found by actual countings in the D.C.

4. *Numbers of stars whose proper motion is included between given limits.*

In the following table are given the numbers of stars which I found between different limits of proper motion and magnitude. The reason why not the total numbers, but only those for types I and II are given, is simply that the latter could be more easily derived from other countings which had previously been made. The difference is practically of no importance for the present investigation.

The stars brighter than 1.5 are entirely omitted; of all these stars the parallaxes have been measured. For the formation of the following tables they can be taken directly from the observations.

Magnitudes 1.5—3.5. The proper motions were taken from NEWCOMB's Fundamental Catalogue. The spectra of those stars, which are too far south for the DC. were taken from Mc. CLEAN's *Spectra of Southern Stars* (London 1898).

Magnitudes 3.5—6.5. The corrected proper motions of AUWERS BRADLEY [1]) were counted, the magnitudes having been previously corrected by (8). In all there appeared to be of the magnitudes 3.6—4.5, 4.6—5.5 and 5,6—6.5 respectively 297, 652 and 1017 stars. Thus, in order to get the numbers (9) for the whole of the sky, the numbers of stars in BRADLEY had to be multiplied by the the respective factors 1.535, 2.264 and 4.756. (Consequently to get the numbers of stars which are actually in BRADLEY, the numbers of Table 1 must be divided by these same factors).

Magnitudes 6.5—9.5. Different sources (AUWERS BRADLEY, AUWERS *A G C*, BOSS *A G C*, PORTER's catalogue of proper motions, combined with countings in the catalogues of LALANDE and BESSEL) were consulted to determine what fraction the numbers of the stars with proper motions 0″00—0″10, 0″10—0″15, 0″15—0″20 . . . . .

1) Rejected were all the stars which have been incompletely observed by BRADLEY, and a few others. There remained 2640 stars in all.

TABLE 1. Number of stars Type I + Type II in the whole sky.

μ	Mag. / Mean μ	1.5–2.5	2.5–3.5	3.5–4.5	4.5–5.5	5.5–6.5	6.5–7.5	7.5–8.5	8.5–9.5
		2.1	3.1	4.1	5.1	6.1	7.1	8.1	9.1
0".000—0".009	0".005	6	5	22	90	343	1504	9010	38257
.010— .019	.015	4	15	52	194	638	1896	7313	28184
.020— .029	.025	1	10	41	177	595	1910	5882	21225
.030— .039	.035	3	16	27	188	542	1910	4768	15809
.040— .049	.045	3	12	27	93	461	1499	3877	12045
.050— .059	.055	5	13	22	86	357	1249	3342	9184
.060— .069	.065	1	4	25	77	252	963	2540	6775
.070— .079	.075	2	2	18	71	247	752	1827	4968
.080— .089	.085		5	25	45	209	692	1337	3614
.090— .099	.095	1	9	17	54	200	646	802	2409
.100— .149	.125	5	10	57	152	424	963	1649	4373
.150— .199	.175	5	9	34	79	181	420	1070	2033
.200— .299	.25	5	11	32	73	200	315	670	919
.300— .399	.35	1	3	17	43	105	75	178	422
.400— .499	.45		5	12	18	34	121	133	196
.500— .599	.55	1	3	8	7	24	38	53	60
.600— .699	.65	2		2	7	5	24	32	36
.700— .799	.75			9	5	10	17	27	26
.800— .899	.85	1		3	2		12	13	16
0.900— 0.999	0.95				2		6	9	10
1.000— 1.199	1.1		1	3	5		7.5	9	10
1.200— 1.399	1.3			3	2	5	12.0	9	7.6
1.400— 1.599	1.5				2		4.5	4.4	6.0
1.600— 1.799	1.7						1.5	4.4	1.5
1.800— 1.999	1.9			2	2		1.5	0.0	1.5
2.000— 2.999	2.5		1				6.0	8.9	6.0
3.000— 3.999	3.5					5	3.0	4.5	6.0
4.000— 4.999	4.5				2		1.5		3.0
5.000— 5.999	5.5					5		4.5	
6.000— 6.999	6.5						1.5		4.5
7.000— 7.999	7.5								
Total		46	134	458	1476	4842	15 042	44 576	150 607

are of the whole number of all the stars. The subdivision of the proper motions 0″000—0″100 was then made by the aid of certain plausible conditions, which are certainly or probably fulfilled by the numbers of small proper motions. Further explanation about this point will be given in a subsequent more detailed publication. The numbers given in the table were derived by multiplying the total numbers (9) by the fractions which have been found in this way.

In order to simplify the computations without sensibly impairing the accuracy, all the stars of which the magnitude is between 3.5 and 4.5 between 4.5 and 5.5 etc., were reckoned to be of magnitude 4.1 [2]) respectively 5.1 etc. Similarly for all the proper motions between 0.″000 and 0.″009; 0.″010 and 0.″019 etc. the mean proper motions 0.″005, 0.″015 etc. were substituted.

For each magnitude and each proper motion occurring amongst the arguments of table 1, the mean parallax was now computed by the formula (1). I thus found *e. g.* for

$$\text{p. m. } 0''045, \quad \text{mag } 6.1, \quad \bar{\pi} = 0''0102,$$

which value thus represents the mean parallax of the 461 stars of which according to table 1 the p. m. and magnitudes are included between the limits 0″040 and 0″050, respectively $5^m,5$ and $5^m,6$. By the aid of the table which was quoted above it is now easy to compute the number of stars amongst these 461, of which the true parallaxes are included between given limits. We thus find:

---

[2]) If the number of stars of magnitude m or brighter is $A_m = K. a^m$, then the mean magnitude $\bar{m}$ of the stars, whose apparent magnitudes are included between the limits m and m + 1, will be $\bar{m} = m - \frac{1}{l\,a} + \frac{a}{a-1}$. For photometric magnitudes I find in the mean $a = 3.266$. This gives $\bar{m} = m + 0^m 596$, for which I have taken $m + 0.6$.

Limits of π			fraction of the whole	Number.
0″00000	and	0″00100	0.001	0
00100	»	00158	.004	2
00158	»	00251	.028	13
00251	»	00398	.097	45
00398	»	00631	.209	96
00631	»	0100	.275	127
0100	»	0158	.226	104
0158	»	0251	.116	54
0251	»	0398	.0358	16.5
0398	»	0631	.0068	3.1
> 0″0631			.0009	0.4
			1.000	461

Repeating the same computations for all the numbers of table 1, the following summary is obtained [1]).

In the second column are given the mean parallaxes $\bar{\pi}$, which were computed by the formula

$$\frac{5}{3}\,\frac{1}{\bar{\pi}}=\frac{\dfrac{1}{\pi_2^5}-\dfrac{1}{\pi_1^5}}{\dfrac{1}{\pi_2^3}-\dfrac{1}{\pi_1^3}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

The mean parallax $\bar{\pi}$ given by this formula satisfies the condition that the absolute magnitude (see next §) computed from it corresponds to the mean of the absolute magnitudes of all the stars whose parallaxes have values between π_1 and π_2.

In the last column are given the volumes of the spherical layers

[1]) A similar table was published by me last year. (Publ. of the Astr. Lab. at Groningen, No. 1, p, 93). Since that time I found occasion to repeat the whole investigation with greater care, so that the present results must be considered as more trustworthy.

TABLE 2. Number of parallaxes in the whole sky (Type I + Type II).

π	Mag / $\bar{\pi}$	—0.9	0.1	1.1	2.1	3 1	4.1	5.1	6.1	7 1	8.1	9 1	Vol.
0″.00000—0″.00100					0.4	0.5	3.0	18 0	84 0	440.0	3016 0	15363 8	
.00100— .00158	0″.00118				1.0	2.0	8.0	34.0	152 0	667.0	3658 0	16197.0	3 140 000
.00158— .00251	.00187			1	1.9	2.7	15.2	76.0	306.0	1216 0	5419.0	21953.0	788 000
.00251— .00398	.00296				2.9	7.9	29.1	141.0	531.0	1942 0	6912 0	25578.0	198 000
.00398— .00631	.00469				3.8	14 9	48.7	211.0	770.8	2642 0	7880.0	26044 0	49 700
.00631— 0100	.00743	1	I	1	5.1	20.2	64 7	255 5	901 0	2899 7	7352 0	20977 0	12 500
.0100 — .0158	.0118				6.1	23.7	74.6	248.8	833.5	2403.2	5278.5	14067 5	3 140
.0158 — .0251	.0187				7.3	21.8	74 5	209.4	614.3	1556.9	5003 9	6789.5	788
.0251 — .0398	.0296			2	6.5	17.6	59.2	142.4	367.5	780.5	1338 0	2568.4	198
.0398 — .0631	.0469		1	2	5 2	11.5	40 1	78 8	177.2	322.5	496.4	792.1	49 7
.0631 — .0100	.0743		1	1	3.3	6.4	23.3	37.7	69.1	114.6	157.3	207 9	12 5
.0100 — .158	.118		1	1	1 6	3.0	11 4	15.3	22 9	39.2	45.8	50.0	3.14
>.158	.204	1	1	2	0 9	1.8	6.2	8.1	12.7	18 4	19 6	18.8	1.05
		2	5	10	46.0	134.0	458.0	1476.0	4842.0	15042 0	44576.5	150607.0	

included between two spheres of which the radii correspond to the parallaxes of the first column. As unit of volume I took the volume of a cube, of which the side is the distance corresponding to a parallax of 0″1. The numbers entered in the columns of the apparent magnitudes —0.9, 0.1 and 1.1 show simply the number of stars whose *measured* parallax lies between the corresponding limits in the first column.

6. *Absolute luminosity and absolute magnitude.*

As unit of luminosity I will adopt the total luminosity of the sun. It is true that our knowledge of the relation between the quantities of light which we receive from the sun and from certain fixed stars is still very imperfect. This is however of little importance, because, when this relation will be better known, it will only be necessary to multiply all our results by a certain constant, in order to bring them into accordance with the new determination.

I will here adopt: light of the sun $=$ 40.000.000.000 $\times$ light of Vega [1]. According to the Potsdam measures the apparent magnitude of Vega is 0.41. From these data it can be easily derived that the sun, when transferred to a distance corresponding to the parallax $\pi = 0''10$, would have the apparent magnitude 5.48. I will adopt $5^m,5$, which accidentally agrees exactly with the mean magnitude of the Bradley stars. If we put further:

L = luminosity, or total illuminating power of a star of apparent magnitude m and parallax $= \pi$,

we find easily by the relation (7):

$$\log L = 0.2000 - 0.4\,m - 2\log \pi \qquad (11)$$

We further define the *absolute magnitude* (M) of a star, of which the parallax is π and the distance r, as the apparent magnitude which that star would have if it was transferred to a distance from the sun corresponding to a parallax of 0″1. It is easily seen that

$$M = m - 5\log r = m + 5 + 5\log \pi = 5.5 - 2.5\log L \quad . \qquad (12)$$

For the sun $L = 1$; the formula thus gives for the absolute magnitude of the sun $M = 5.5$, in accordance with what has been said above.

[1]) Young, General Astronomy p. 213.

TABLE 3. Log. number of stars per unit of volume.

Mean π	r \ $M.$	−7.55	−6.55	−5.55	−4.55	−3.55	−2.55	−1.55	−0.55	0.45	1.45	2.45	3.45	4.45	5.45	6.45	7.45	8.45	9.45	10.45	Density.
	log. L	5.22	4.82	4 42	4.02	3.62	3.22	2.82	2.42	2.02	1.62	1.22	0.82	0.42	0.02	9.62	9.22	8.82	8 42	8.02	
0″.00118	84.7	3.51	3.81	4.40	5.033	5 685	6.326	7.068	7.713												0.122
.00187	53.5	4.11	4.38	4.53	5.286	5.984	6.589	7.188	7.838	8.446											0.234
.00296	33.8			5.18	5.60	6.167	6.852	7.428	7.992	8.543	9.111										0.418
.00469	21.3				5.88	6.477	6.991	7.628	8.190	8.726	9.201	9.719									0.656
.00743	13.5		5.90	5.90	5.90	6.613	7.209	7.714	8.310	8.858	9.366	9.769	0.225								0.869
.0118	8.47						7.28	7.878	8.377	8.899	9.423	9.884	0.225	0.651							0.985
.0187	5.35							7.97	8.441	8.975	9.423	9.892	0.295	0 580	0.935						1.031
.0296	3.38							8.00	8.52	8.949	9.476	9.857	0.269	0.596	0.830	1.113					1.000
.0469	2.13							8.32	8.60	9.021	9.364	9.907	0.200	0.552	0.812	0.999	1.202				0.917
.0743	1.35								8.90	8.90	9.422	9.709	0.270	0.479	0.742	0.962	1.100	1.220			0.829
.118	0.85									9.51	9.51	9.71	9.980	0.560	0.688	0.863	1.097	1.164	1.201		0.742
.204	0.49									9 98	9.98	0.219	0.004	0.173	0.663	0.864	1.041	1.210	1.266	1.257	0.648

7. *Derivation of the star-density and the luminosity-curve.*

By the aid of (11) and (12) table 2 can be so altered that the argument: apparent magnitude is replaced by the argument $\log L$ or M.

If this is done, and if further the numbers of the table are divided by the volumes given in the last column and logarithms are taken, we get the following table: (p. 669)

The numbers of the last row of this table require some explanation. If this row had been derived in the same way as the others the resulting numbers would have been those corresponding to the values 1.94, 1.54, 1.14 ... etc. of $\log L$. For the sake of uniformity I derived from these, by interpolation between the logarithms, the values corresponding to the values 2.02, 1.62, 1.22 ... etc. of $\log L$.

It is still possible to enter this table with the argument: apparent magnitude; for the logarithms belonging to the same apparent magnitude are now placed in an oblique line descending towards the right. In order to facilitate such an entering of the table, the logarithms belonging to the apparent magnitudes 3.1—7.1 have been included between heavy lines. This enables us to judge more readily of the weight of the several numbers tabulated. Thus it is seen at once that the numbers which are in the table to the left of the heavy lines must have a very small weight because they are relative to the stars of the magnitudes 2.1, 1.1 ... which are exceedingly few in number. Similarly, though for a different reason, the numbers which are outside the heavy lines on the right hand side, and which belong to the magnitudes 8.1 and 9.1, have a small weight, at least for the smaller values of the parallax.

The table virtually is nothing else than a table for the logarithms of the relative densities of stars of different absolute magnitudes (or absolute luminosity). The absolute density, *i. e.* the *total* number of stars per unit of volume, can not be determined of course, because we know nothing about the very faintest stars. We can however determine that density expressed in its value at a certain distance from the sun as unit.

For this distance I will provisionally adopt the distance corresponding to a parallax of 0″ 0296. I will adopt the hypothesis that the luminosity-curve is the same for different distances from the sun. Luminosity-curve I call the curve which for every absolute magnitude gives the number of stars per unit of volume, or in other words, which gives the proportion in which the stars of different apparent magnitudes would be distributed over the sky, if they were all placed on the surface of the sphere whose radius corresponds to the parallax 0″1. In the following tables I will give not the numbers of stars of each absolute magnitude, but the logarithms of these numbers. As a consequence of the hypothesis which has been mentioned the ratio of the *absolute* densities is necessarily the same as that of the densities for the separate absolute magnitudes.

If the density was constant, the numbers in each vertical column of table 3 should be identical. For the middle of the table this condition is roughly satisfied; for the large and for the small distances however it is not.

The manner in which the densities given in the last column are determined, is perhaps best explained by an example.

The number of stars (not the logarithm) per unit of volume for the stars of the four absolute magnitudes —6.55, —5.55, —4.55, —3.55 together is:

for π = 0″00118, (app. mag. 2.5—6.5) 0.000 0623
„ „ = 0″00187, (app. mag. 1.5—5.5) .000 1215

We thus get for the ratio of the densities Δ_1 and Δ_2

$$\frac{\Delta_1}{\Delta_2} = 0.513$$

As a second determination we have for the stars of the absolute magnitudes —6.55 to —2.55:

for π = 0″00118 (app. mag. 2.5—7.5) 0.000 2743
„ „ = 0″00187 (app. mag. 1.5—6.5) 0.000 5095

from which we get

$$\frac{\Delta_1}{\Delta_2} = 0.538.$$

The mean was taken of the two values 0.518 and 0.538, giving to the first value (which depends chiefly on the stars of the apparent magnitudes 6.1 and 5.1) twice the weight of the second value (which depends chiefly on the stars of the apparent magnitudes 7.1 and 6.1).

In the same way the ratio was found of the densities at consecutive distances from the sun (π = 0″00118, 0″00187, 0″00296 etc.) These ratios, together with the adopted density 1.0 for the distance corresponding to a parallax of 0″0296, gave the values of the last column of table 3.

If now from the logarithms of each row of the table we subtract the corresponding logarithm of the density, or in other words, if the whole is reduced to the density for π = 0″0296, the following table is derived:

45

TABLE 4. Log. number of stars per unit of volume, reduced to $\pi = 0''0296$.

Mean π \ M. / log L	—7.55	—6.55	—5 55	—4.55	—3.55	—2.55	—1.55	—0.55	0.45	1.45	2.45	3 45	4 45	5.45	6.45	7.45	8.45	9.45	10.45
	5.22	4 82	4.42	4.02	3.62	3.22	2 82	2.42	2 02	1.62	1.22	0.82	0.42	0.02	9.62	9.22	8 82	8.42	8.02
0″.00118	4 42	4 72	5.31	5.947	6.599	7.240	7.982	8.627											
.00187	4.74	5.01	5.16	5.917	6.615	7.220	7 819	8.469	9.077										
.00296			5.56	5 98	6.546	7.231	7.807	8 371	8 922	9.490									
.00469				6.06	6.660	7 174	7 811	8.373	8.909	9.384	9 902								
.00743		5.96	5.96	5.96	6.674	7.270	7.775	8 371	8.919	9.427	9 830	0.286							
.0118						7.29	7 885	8 384	8.906	9.430	9.891	0.232	0.658						
0187							7.96	8.428	8.962	9.410	9.879	0.282	0.567	0.922					
.0296							8.00	8 52	8.949	9.476	9.857	0.269	0.596	0.830	1.113				
.0469							8.36	8.64	9 059	9.402	9.945	0.238	0.590	0.850	1.037	1.240			
.0743								8 98	8.98	9.503	9.790	0.351	0 560	0.823	1.043	1.181	1.301		
.118									9.64	9.64	9 84	0.110	0.690	0.818	0.993	1.227	1.294	1.331	
.204									0 17	0.17	0.41	0.19	0.361	0.851	1.052	1.229	1.398	1.454	1.445
Mean.	(4.42)	4.72	5.27	5.943	6.601	7.222	7.809	8.376	8.920	9.431	9.879	0.264	0.583	0.830	1.024	1.229	1.398	(1.45)	(1.44)
	(4.38)	4.68	5.27	5.931	6.589	7.215	7.806	8.372	8.909	9.410	9.863	0.261	0.601	0.857	1.048	1.239	1.408	(1.46)	(1.45^5)

The numbers given in this table evidently define what we have called the luminosity-curve.

In taking means the following weights were given:

A. In the columns of abs. mag. —6.55, —5.55. . . . . to + 0.45

B. „ „ „ „ „ „ 1.45, 2.45 . . . . 8.45

apparent magnitude	*A.*	*B.*
brighter than 2.5	0	0
3.1	2	2
4.1	5	5
5.1	14	7
6.1	21	7
7.1	3	1
fainter than 7.5	0	0

These weights are roughly proportional to the numbers of stars in BRADLEY which have contributed to the formation of the numbers of table 4.

It is evident that the final means depend only in a very small measure on the values which were found for the densities. They can be derived quite independently of these densities. So we find *e. g.* directly from table 3 for

$$\log \frac{\text{number of stars of absol. mag. } -1.55}{\text{„ „ „ „ „ „ } -2.55}$$

the following values (the assigned weights are added in brackets)

0.599 (3)
0.576 (5)
0.637 (3)
0.505 (1)
Mean. 0.591.

If in the same way the ratios are derived of the numbers of stars belonging to any two consecutive magnitudes, it is only necessary, for a complete knowledge of the luminosity-curve, to derive the number of stars per unit of volume for *one* absolute magnitude. This number was obtained by effecting the best possible agreement with

45*

the curve which has already been found. The resulting values are given in the last row of table 4. It will be seen that the discrepancies of the values derived by the two methods are very small.

8. *Influence of the uncertainty of the constants of formula* (1).

In order to investigate in how far the values here derived are affected by uncertainties in the fundamental quantities ϱ, ε, π_{55}, I have altered these quantities by amounts which are almost certainly outside (in some cases *far* outside) the limits of the true uncertainties.

As far as the values of π_{55} (i. e. the mean parallax of stars of mag 5.5 and p. m. μ) are concerned, the principal causes of uncertainty are: 1st. the remaining uncertainties in the measured parallaxes and 2nd. the remaining uncertainty in the linear velocity of the sun, by the aid of which the mean secular parallaxes derived from the parallactic motion were reduced to parallaxes in the usual acceptation of the word.

Now it is evident that, if every parallax is multiplied by the same factor not much different from unity, then also all densities will be multiplied by a certain constant factor not much different from unity. Thus, if provisionally we do not aim at the most refined precision in the absolute values of the densities, it is clear that we can make these two uncertainties to bear either wholly on the large and directly determined parallaxes, or wholly on the small parallaxes derived from the parallactic motion.

I chose the latter course, and consequently I took care that the directly measured parallaxes were as well represented by the new formula as by the old one.

In deriving the formula (1) the value [1]) $h = 16.7 \pm 1.15$ kilometer per second was used for the velocity of the solar system. A few months [2]) ago CAMPBELL, derived from the material given by his own observations, which is much more extensive than that from which the above value was derived, the value $h = 19.9 \pm 1.52$ kilometer. For the mean linear velocity of the stars he finds 34.1 kilometer. From this latter value we get, by the method explained in Proceedings October 1897, another value, which cannot differ much from $h = 18.3$. As the final value from CAMPBELL's observations we must thus adopt about $h = 19.0$. Everything considered the value:

[1]) Proceedings October 1897.
[2]) Astrophys. Journ. Jan. 1901, p. 81 599.

$$h = 18.45 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

appears to me to be the most probable value which can at present be adopted.

I now derived anew the values of $\pi_{\mu m}$ in formula (1) in the following suppositions, to which I add the constants which were formerly found (sol. I):

Sol.	h	A	p	$\bar{\varepsilon}$	ρ
I	16.7	0.0387	1.405	0.905	0.19
II	16.7	0.0387	1 405	0.905	0.00
III	16 7	0.0387	1.405	1.000	0.00
IV	20 2	0.0454	1 30	0.905	0 19
V	18.45	0.0419	1.355	0.87	0 19

. (14)

In these solutions the stars fainter than 2.5 and in some of them also those brighter than 7.5, which have no influence on the result, were, for brevity's sake, omitted.

These different solutions give for the densities (Δ):

TABLE 5. Density Δ.

π Limits.	π Mean.	I	II	III	IV	V
0".00100—0".00158	0" 00118	0.122			0.187	0.162
.00158— .00251	.00187	0.234			0.345	0.292
.00251— .00398	.00296	0 418	0.223	0.184	0 568	0.465
.00398— 00631	.00469	0.656	0.592	0.571	0.789	0.684
.00631— .0100	.00743	0.869	1.072	1.294	0.968	0.852
.0100 — .0158	.0118	0.985	1.191	1.507	1.040	0.945
.0158 — .0251	.0187	1 031	1 122	1.403	1.050	0.984
.0251 — .0398	.0296	1.000	1.000	1.000	1.000	1.000
.0398 — .0631	.0469	0.917	1.045	0.889	1.007	0.980
.0631 — .100	.0743	0 829	0.771	0.497	0.875	0 957
.100 — .158	.118	0.742	0.728	0.406	0.813	0.933
> 0.158	.204	0.648	0.627	0.220	0.780	0.929

For the luminosity-curve we find (after adding to the values I, II and III the constants -0.081, $+0.056$ and -0.104)

TABLE 6. Luminosity-curve.
(Log. number of stars per unit of volume for $\pi = 0''0296$).

Log. *L*.	*M*.	I — 0 081	II + 0 056	III — 0 104	IV	V
5.22	— 7.55	(4.34)				
4.82	— 6.55	4.64			4.65	4.60
4.42	— 5.55	5.19	(5.71)	(5.82)	5.29	5.17
4.02	— 4.55	5.862	5.71	5.94	5.960	5.928
3.62	— 3.55	6.520	6.46	6.58	6.601	6.586
3.22	— 2.55	7.111	7.165	7.142	7.189	7.210
2.82	— 1.55	7.728	7.838	7.682	7.764	7.815
2.42	— 0.55	8.295	8.431	8.227	8.323	8.380
2.02	0.45	8.839	8.993	8.735	8.852	8.922
1.62	1.45	9.350	9.564	9.311	9.340	9.413
1.22	2.45	9.798	9.995	9.763	9.769	9.839
0.82	3.45	0.183	0.285	0.174	0.117	0.190
0.42	4.45	0.502	0.480	0.521	0.452	0.478
0.02	5.45	0.719	0.533	0.622	0.660	0.680
9.62	6.45	0.943	0.601	0.687	0.850	0.836
9.22	7.45	1.148	1.132	1.389	1.084	1.026
8.82	8.45	1.317	1.20	1.607	1.248	1.102
8.42	9.45	(1.37)	(1.3)	(1.7)		(1.10)
8.02	10.45	(1.36)	(1.4)	(1.8)		(1.11)

The discussion of the densities must be deferred to a subsequent communication, because it will be necessary in that discussion to keep the stars of different galactic latitudes separated *ab initio*. There seems to be reason to believe that this discussion, if therein we include some additional data furnished by observation, will lead to a better understanding of the real structure of the galactic system.

The table 5 might therefore have been omitted here but for the fact that it brings out clearly a defect of our solution I and indicates at the same time the means to correct it. This defect lies in the rapid decrease of the density Δ for the larger parallaxes. A graphical representation in which the densities are taken for ordinates, while the abscissae are not the parallaxes, but the distances from the sun, shews the enormous rapidity of this decrease. Such a rapid decrease appears entirely incredible, as compared with the slow and gradual change for larger distances. If we could actually use stars which are evenly distributed over the whole sky, instead of almost exclusively over the northern hemisphere, and if the density varies continuously with the position in space, then the mean density in the immediate neighbourhood of the sun must even be found constant.

The values of Δ_2 show the same decrease. A variation of ϱ has thus no influence. The Δ_3's on the other hand show a still more rapid decrease. It follows immediately that by a *diminution* of ε the defect in question can be corrected. It appears from the Δ_4's that a change of the distances in the direction which is made necessary by CAMPBELL's results, has also an effect in the desired direction.

It is easily inferred from the table that the defect must nearly disappear by a new computation, if therein we determine the parallaxes in accordance with the value (13) of the sun's velocity, and adopt

$$\varepsilon = 0.87. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

This alteration of ε is just inside the estimated limits of uncertainty of this quantity [1]).

The last column of table 5, which was computed with these data, shows that the density becomes indeed tolerably constant for all parallaxes larges than $0''.01$.

For this reason the solution V is the solution which in my opinion is to be preferred, though it remains possible that the subsequent discussion of the densities will necessitate further small changes in the values of the constants.

9. *Reliability of the results derived for the luminosity-curve.*

The values which were given to ϱ in solution II, and to ϱ and ε in solution III are outer limits, which were taken for the sake of the simplicity of the computations. With regard to ϱ the alteration is 9 to 10 times the p. e., for ε it exceeds nearly three times the estimated limit of uncertainty. Also the solution IV was made with values of the constants, the deviations of which from those of solution I probably exceed the real uncertainties of the latter.

Nevertheless the discrepancies between the different curves in table 6 are inconsiderable. It thus appears that the form of the curve is very little affected by errors in the constants of formula (1).

1) See Public. of the Astr. Lab. at Groningen No. 8, p. 10.

Excepting the extreme ends of curve, which for evident reasons are rather uncertain, errors of 0.1 in the values resulting from solution V must already be considered as unprobable. In the middle of the curve this corresponds to only about 0.2 of a magnitude.

Besides on the uncertainties of the constants, the correctness of the curve also depends on the greater or smaller degree of completeness and certainty of the data about the magnitudes and the proper motions which form the basis of the whole investigation. We can however easily estimate the effect of these causes, as it is possible to derive the curve:

1st. exclusively from stars of app. mag. 3.1, 5.1, 7.1;
2nd. " " " " " " 4.1, 6.1, 8.1.

These two determinations are absolutely independent of each other. The computation was carried out with the data of solution I [1]), in precisely the same way as that for the last row of table 4, *i. e.* entirely independently of the densities. The results are given in the following table

M.	I Mag. 3 1, 5 1, 7 1	II Mag. 4.1, 6 1, 8.1	I—II
—6.55	4 730		
—5 55	5 183	5.272	—0 089
—4.55	5.957	5.930	+ 027
—3 55	6.638	6.561	+ .077
—2.55	7.229	7.233	— 004
—1.55	7.815	7.883	— .068
—0.55	8.349	8 454	— .105
0.45	8.875	9.013	— .138
1.45	9.368	9.482	— .114
2.45	9 821	9.927	— .106
3.45	0.222	0.280	— .058
4 45	0.576	0 593	— .017
5 45	0.882	0 797	+ .085
6.45	1.163	0 968	+ .195
7.45	1,291	1.165	+ 126
8.45	1.509	1.269	+ .240
9 45		1.390	

1) Afterwards this computation was also made for solution V. The results are all but identical to those of sol. I.

Keeping in mind that we may legitimately expect that the errors in the adopted curve, so far as they depend on the uncertainties here considered, will range between limits only about half as wide as those of the differences I—II, we come to the conclusion that these differences are already very satisfactory. All things considered I think we may safely expect that (excepting the extreme ends of the curve) the values resulting from solution V will never be in error much more than 0.2, which corresponds to about 0.4 of a magnitude in the middle of the curve [1]).

Moreover our knowledge of the proper motions is increasing rapidly, so that we may reasonably hope that within a comparatively short time, we may be able to reduce still more the uncertainties of the curve.

Especially for the fainter end of the curve, which depends exclusively on the large proper motions of faint stars we will certainly soon have better data by which it can be corrected and continued.

From the above numbers the curve appears to reach a maximum about the absolute magnitude 10.5. Whether for fainter stars it will descend as rapidly or more rapidly, and whether it will soon reach a limit, below which no luminous stars exist, are questions to answer which a knowledge of the number of large proper motions of stars fainter than the ninth magnitude is required. It seems not at all impossible by the aid of photography to derive, even within a few years, an approximate knowledge of these proper motions for stars down to the 13^{th} or even somewhat higher magnitudes.

At the brighter end the continuation will cause more difficulties, as it must depend on an accurate knowledge of the extremely *small* proper motions, which can only be slowly attained in the course of years.

A number of conclusions can at once be drawn from our results, which however I will defer till after the discussion of the densities. I will here only illustrate the meaning of the curve by a few numbers. According to the curve V, there will be in a space which contains

---

1) The uncertainty resulting from errors in the adopted position of the Apex and in the corrections to Bradley's declinations, was left out of consideration here. I hope shortly to be able to give the alterations which result from these causes.

2.000.000	stars				of the same luminosity as that of the sun						
1	star	with	100.000	times	greater	»	than	»	»	»	»
38	stars	»	10.000	»	»	»	»	»	»	»	»
1800	»	»	1.000	»	»	»	»	»	»	»	»
36000	»	»	100	»	»	»	»	»	»	»	»
440000	»	»	10	»	»	»	»	»	»	»	»
over 5000000	»	»	10	»	smaller	»	»	»	»	»	»
7500000	»	»	100	»	»	»	»	»	»	»	»

Below this degree of luminosity it seems that the number of stars ceases to increase. The first and last numbers are of course very uncertain.

It may also be remarked that we find a total density which is much larger than is commonly assumed.

The mean parallax of the stars of magnitude 5.3 becomes 0″.0158 by the solution V. Inside a sphere with a radius corresponding to this parallax I find (by sol. V) already 43000 to 44000 stars whose luminosity is not smaller than $\frac{1}{15}$ of that of the sun. The number of the still fainter stars can not be determined. If on the other hand we adopt the usual approximation which assumes the same luminosity for all the stars, the number of stars inside the same sphere will of course be the number of stars of the apparent magnitude 5.3 and brighter. This number is (Potsdam system) only about 1730, that is only $\frac{1}{25}$ part of the number which was found above.

10. *Stars of the first and the second spectral type.*

Although the data relating to the separate spectral types are by far less certain than for all stars together, I will nevertheless mention the results which I derived from them, as they bear on the conclusions arrived at in a former paper.

The uncertainties are of two kinds:

1st. For Type I the constants (2) are very uncertainly determined.

2nd. Our knowledge of the spectra is far from so complete and accurate as could be wished.

For these reasons the following results, at least those for type I, do not deserve the same confidence as the preceding ones for all the stars together. With regard to the first point, it has already been mentioned that the total weight of the direct determinations of parallax, which were available for type I, is very small. It is not one sixth part of that for type II.

Moreover these parallaxes belong exclusively to bright stars of comparatively small proper motion. It would be of the highest importance for an investigation like the present, if observers, who devote themselves to the determination of parallaxes, would pay especial attention to the comparatively few stars of the first type with large proper motions.

As to the second point:

Having regard to the fact that the D.C. is only complete down to the stars of about the 6^{th} magnitude, it is to be feared that of the fainter stars which it contains a larger number proportionally will belong to the photographically brighter stars of the 1^{st} type than to the stars of the same (visual) magnitude of the second type. If this is so, our results will be systematically affected.

In order to get more certainty about this point I derived the ratio

$$P = \frac{\text{number of stars of Type II}}{\text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{I}} \quad . \quad . \quad . \quad . \quad (16)$$

for different magnitudes, not only from PICKERING's data [1]) but also from the spectroscopic Dm (Decl. — 1^0 tot + 20^0) of Potsdam [2]), which is complete down to the visual magnitude 7.5 and is thus of special value for our purpose.

PICKERING's results are given with the argument: photographic magnitude, while we require here visual magnitudes. The necessary data to effect this reduction are given in PICKERING's work; nevertheless the accuracy of the results is considerably impaired by this circumstance.

The result of my computations was:

Vis. mag.	Number stars in DC. Typ I.	Typ II.	P.
3.50—4.00	74	52	0.70
4.00—4.50	157	109	0.70
4.50—5.00	301	234	0.78
5.00—5.50	633	494	0.78
5.50—6 00	1348	1045	0.78
(6.00—6.50	2717	2220	0.82)

. . (17)

[1]) Annals of the Astrophys Obs of Harvard Coll. vol. 26, Part 1, p. 147.
[2]) Publ. des Astrophys. Obs. zu Potsdam 8er Bd. 3es Stuck.

The data for stars fainter than the 6th magnitude are included in brackets, because we cannot be sure that the two types are observed equally completely; there is even a strong probability to the contrary.

The Potsdam Dm gives a classification which differs from that of the D. C.

According to Harvard. Obs. 26, part I, p. 177 we have approximately:

$$\left.\begin{array}{l} \text{Class I of Vogel} = A + F + G \text{ Draper Cat.} \\ \;\;\;\text{"}\;\; \text{II}\;\; \text{"}\;\;\;\;\; \text{"}\;\; = K \qquad\qquad\;\; \text{"}\;\;\;\;\; \text{"} \end{array}\right\} \quad \ldots . \quad (18)$$

Now I find by countings in the Potsdam Dm.:

mag. BD,	Cl. I.		Cl II.		$\frac{\text{Cl. II}}{\text{Cl. I}}$		
0—3.5	40		10		0.25		
3.6—4.0	19	86	16	40	0.84	0.47	
3.1—4.5	27		14		0.52		
4.6—5.0	44		39		0.89		
5.1—5.5	68	276	39	151	0.58	0.55	
5.6—6.0	164		73		0.44^{5}		
6.1—6.5	300		185		0.62		(19)
6.6—7.0	552	1708	308	953	0.56	0.56	
7.1—7.5	856		460		0.54		
7.6—8.1	66		35		0.53		
8.1—8.5	38	113	16	57	0.43	0.50	
8.6—9.0	9		6		0.67		

It follows from these numbers that, at least down to the visual magnitude 7.5 the quotient $\frac{\text{Cl II}}{\text{Cl I}}$ does not sensibly vary with the magnitude.

Accordingly we have by (17) in the notation of the D. C.

$$\frac{K}{A+F+G} = \text{Const.} = \beta$$

from which

$$\frac{K+F+G}{A} = \frac{\text{Type II}}{\text{Type I}} = P = \beta + (\beta+1)\frac{F+G}{A}. \quad . \quad (20)$$

From the data of the D. C. reduced to visual magnitudes I find

vis. mag.	Number $F+G$	Number A	$\frac{F+G}{A}$
3.5—4.0	21	63	0.34
4.0—4.5	44	134	0.33
4.5—5.0	94	255	0.37
5.0—5.5	200	560	0.36^5
5.5—6.0	425	1269	0.33^5
6.0—6 5	908	2629	0.34^5

Thus also the last term of (20) appears to be eminently constant.

From the Potsdam Dm we thus derive the conclusion, which is in good agreement with the directly derived table (17), that the value of P does not sensibly vary with the magnitude (at least not down to mag. 7.5).

The numbers of stars of the two types were now derived as follows, the very few stars brighter than 2.5 being omitted:

1st. *Magnitudes* 2.5—3.5. All stars of these magnitudes in the whole of the sky were brought together, as explained above.

2nd. *Magnitudes* 3.5—6.5. The spectra of the Bradley stars [1]) were taken from the D.C. and the whole of the stars which are in this catalogue were counted between the same limits of proper motion and photometric magnitude as in table 1. From these countings the total number of these stars for the whole of the sky was then derived in the manner which has been explained above. Finally the numbers of stars of the different magnitudes (2.5—6.5) were multiplied by such factors (differing little from unity) that the con-

[1]) The stars which have been excluded have already been mentioned above.

dition $P =$ const. is fulfilled, while the total numbers (Type I + Type II) are left unchanged.

3rd. *Magnitudes* 6.5—7.5. To begin with the same method was used for these stars as for those of the magnitudes 3.5—6.5. The number of stars in Bradley belonging to these magnitudes is so small however, that the numbers for the individual proper motions necessarily run somewhat irregularly. Therefore I first divided the whole of the stars in two parts, *viz.* those with proper motions $< 0''10$ and those with proper motions $> 0''10$.

The ratio $a = \dfrac{\text{number of stars of } 1^{\text{st}} \text{ Type}}{\text{id. } 1^{\text{st}} \text{ Type} + 2^{\text{nd}} \text{ Type}}$ was then determined

separately for each of these parts and compared with the analogous ratio for the magnitude 6.1. It appeared that the factors, by which these ratios for the magnitude 6.1 must be multiplied to give those for the magnitude 7.1, were very near unity. These factors were then used to derive the ratios a for the *separate* proper motions $0''.00$—$0''.01$,, $0''.01$—$0''.02$ . . . etc. for the magnitude 7.1. Once these ratios a found, the table 1 furnishes the necessary numerical values.

In this way I found finally the numbers which are given in the following table: (p. 685)

It appears from this table that the numbers for type I show still considerable irregularities, which are still more apparent, if the table is condensed by taking wider limits of proper motion, and if then all the numbers are expressed as fractions of the analogous numbers of table 1. It appears in this way that *e.g.* the number of stars of large proper motion of the magnitude 5.1 is considerably smaller than might be expected from the same numbers for the magnitudes 4.1 and 6.1. At first sight such an irregularity is rather surprising, as it is not at once apparent how the spectrographic observations can be subject to systematic errors depending on the proper motion.

A closer scrutiny shows however that such a thing is not at all impossible in the present case.

In Astronomy and Astrophysics Vol. XII, p. 811 are given a number of corrections to the data of the D. C., which corrections I have duly applied.

The corrections bear exclusively on stars of large proper motion, whose spectrum has been reinvestigated on the indication of Mr. W. H. S. Monck. In how far these corrections influence the number of stars of the first type with large proper motions is apparent from the preceding

TABLE 7. Number of stars in the whole sky.

	TYPE I.					TYPE II.				
μ	3 1	4 1	5.1	6.1	7 1	3.1	4.1	5.1	6.1	7 1
0".000— 0" 009	2	15	76	220	914	3	8	17	121	590
010— .019	8	37	128	435	1227	7	16	68	199	669
.020— .029	8	28	126	332	1007	1	14	54	262	903
.030— 039	9	20	117	345	1154	7	7	73	194	756
.040— 049	7	19	47	284	872	5	8	45	174	618
050— .059	8	18	53	201	665	5	5	34	155	584
.060— 069	2	18	50	130	470	2	7	28	121	493
.070— .079		13	24	112	320	2	6	46	136	432
.080— .089	2	12	21	112	349	3	13	24	97	343
.090— .099	6	10	19	66	199	2	7	34	136	447
.100— .149	4	24	62	173	391	6	32	88	253	572
.150— .199	6	8	21(2)	51	117	2	24	56	132	303
.200— .299	3	10(3)	5(12)	19(19)	29	9	22	65	185	286
300— .399	1	2	2(4)	(5)		2	14	39	107	75
.400— 499	2		2	5	17	3	11	15	30	104
.500— .599						3	8	7	24	38
.600— .699				(5)			2	7	5	24
.700 — 799							8	5	10	17
.800— .899							3	2		12
0.900— 0.999								2		6
1.000— 1.199						1	3	5		7.5
1 200— 1.399							3	2	5	12.0
1.400— 1.599								2		4.5
1.600— 1.799										1.5
1 800— 1.999							2	2		1.5
2.000— 2 999						1				6.0
3.000— 3 999									5	3.0
4.000— 4.999								2		1.5
5.000— 5 999									5	
6 000— 6.999										1.5
7.000— 7.999										
	68	234	753	2485	7731	64	223	722	2356	7311

table. If the corrections had been neglected, the numbers of stars of the magnitudes 4.1, 5.1, and 6.1 would have been increased by the quantities which are added in brackets.

For the magnitude 7.1 the analogous increase could not easily be derived, owing to the particular manner in which the numbers for that magnitude were obtained. It will be seen that by these corrections the proportions are entirely changed in the case of the very large proper motions of type I [1]).

If the corrections which are still necessary are so considerable, we cannot expect very reliable results.

The manner in which I tried as completely as possible to remove the irregularities, will best be shown by an example: The total numbers of stars of type I for the magnitudes 4.1, 5.1 and 6,1 are 234, 753 and 2485. If now every number of the third column is multiplied by $\frac{753}{234}$, and every number of the fifth column by $\frac{753}{2485}$, these three columns are reduced to the same total number.

After this was done the numbers of the 3rd, 4th and 5th columns were added; the sums were divided by 3 and the resulting values were adopted as corrected values for the magnitude 5.1. In the same way the corrected numbers for the magnitudes 4.1 and 6.1 were derived. In order to be able to do the same for mag. 3.1 the numbers for mag. 2.1 were also derived. The numbers for mag. 7.1 were not altered.

In the case of type II the number of large proper motions is so considerable, and the influence of the corrections which have just been discussed is so small, that the numbers of table 7 were adopted as they stand.

In order to derive from table 7 (altered for type I as just now explained) the densities and the luminosity-curve in the same way as explained above, a first computation was made with the values (2). Afterwards a second computation was carried through in which the corrected value (15) of ε was used and the parallaxes were made to agree with the corrected velocity (13) of the solar system.

In the case of type I the alteration of ε had a large influence on the value of $\pi_{5.5}$ derived from the directly measured parallaxes, owing to the fact that these direct determinations belong exclusively to very bright stars.

---

[1]) According to Mr Monck 60 percent of the stars of type I, to which he called attention on account of their large proper motions, were actually altered to type II.

The following are the values of the constants for the two solutions (A and B):

Type	Sol.	h	A	p	ε	ρ	
I	A	16.7	0.116	1.11	0 905	0.19	(21)
"	B	18.45	0.0753	1.20	0.87	0.19	
II	A	16.7	0.0262	1.54	0.905	0.19	
"	B	18.45	0.0316	1.47	0.87	0.19	

With these data the following densities were found:

TABLE 8. Densities Δ.

π Limits.	π Mean.	Type I. Sol. A		Type I. Sol. B		Type II. Sol. A		Type II. Sol. B	
0″00100 — 0″00158	0″00118	0.280		0.278		0.070		0 102	
.00158 — .00251	.00187	.470		0.478		0.156		0.190	
.00251 — .00398	.00296	.738		0.726		0.254		0.314	
.00398 — .00631	.00469	1.006		0.986		0 440		0.474	
.00631 — .0100	.00743	1.202		1.172		0.622		0.655	
.0100 — .0158	.0118	1.215		1.171		0.802		0.790	
.0158 — .0251	.0187	1.189		1.283		0.960		0.933	
.0251 — .0398	.0296	1.000		1.000		1.000		1.000	
.0398 — .0631	.0469	0.822		0.826		0.993		1.186	
.0631 — .100	.0743	0.669		0.669		0.940		1.083	
.100 — .158	.118	0.338	0.583	0.505	0.619	0.883	0.908	1.059	1.072
> 0.158	.204	0.290		0.368		0.598		0.981	

46

For the luminosity-curve we find:

TABLE 9. Luminosity-Curve.

(Log. number per unit of volume for $\pi = 0''.0296$)

Log. L.	M.	Type I		Type II	
		Sol. A —0.039	Sol. B	Sol. A —0.078	Sol. B
4.82	—6.55	4.433	4.382	4.182	4.192
4.42	—5.55	4 951	5 000	4.702	4.678
4.02	—4.55	5.640	5.636	5.375	5.371
3.62	—3.55	6.223	6.262	6.104	6.131
3.22	—2.55	6.804	6.854	6.767	6.849
2.82	—1.55	7.358	7.422	7.443	7.491
2.42	—0.55	7.902	7.972	8.046	8.116
2.02	0.45	8.413	8.477	8.634	8.703
1.62	1.45	8.902	8.907	9.170	9.232
1.22	2.45	9.308	9.362	9.635	9.691
0.82	3.45	9.644	9.632	0.062	0.054
0.42	4.45	9.927	9.843	0.401	0.422
0.02	5.45	0.093	0.002	0.670	0.640
9.62	6.45	0.297	0.139	0.937	0.818
9.22	7.45	(0.50)	(0.08)	1.080	1.004
8.82	8.45			1.306	1.115

In table 8, Sol. A. we again find, for both types, a strong decrease of the density with diminishing distance. By the alteration of ε to 0.87 and the slight alteration to the distances in Sol. B. this decrease disappears practically entirely for type II. For type I the decrease has become somewhat less rapid, but it has not disappeared. The weight of this result is but very small however. The number of stars of type I whose parallax is $> 0''.063$, is so small that any conclusion based thereon is of necessity little reliable, especially in a case like the present where, as has been shown above, the adopted number of stars with large proper motions may be very materially in error. For reasons which have already been mentioned, it must be considered as probable that, as soon as more reliable data will be available, we will, for this type also, find the density not far from constant for parallaxes larger than $0''.02$.

As a consequence of this result some of the conclusions, at which I had previously arrived (Proceedings Jan. 1893), must be withdrawn, or at least considerably altered.

These conclusions were based on the result, derived by STUMPE, RISTENPART, and others, *viz.* that, if the stars are arranged in groups according to their proper motions, the mean parallaxes of these groups are approximately proportional to the mean proper motions. It is only subsequently that I found that this result was arrived at by an illegitimate reasoning and is certainly not in accordance with the facts.

For the stars with *large proper motions* (say larger than 0".10) it follows from the above that the variation of the quantity Q in the paper quoted, is, either entirely or at least to a large extent, a consequence, not of a condensation of the stars of type II in the neighbourhood of the sun, but of the fact that the number of faint stars of the first spectral type, as compared to the number of bright stars of the same type, is not so large as in the case of the second type.

Physiology. — H. D. BEYERMAN: "*On the influence upon respiration of the faradic stimulation of nerve tracts passing through the internal capsula.*" (Communicated by Prof. C. WINKLER).

In a recent publication WINKLER and WIARDI BECKMAN [1]), in stimulating with the faradic current the lateral part of the praecrucial circumvolution in a dog's brain, have proved the influence of this field of the cortex upon the respiratory movements. Acceleration of rhythm and an inspiratory position of the thorax were the effects generally obtained during the faradisation of this spot (fig. 1, compare the fields 11, 12, 15 and 16).

Repeating their experiments I found, that faradisation of the most proximal parts of the above mentioned spot (the fields 15 and 16) causes only acceleration of rhythm (or if respiration is very frequent, increase of the amplitude of each respiration), whereas faradisation of its caudal part (the fields 11 and 12) is followed by a forced inspiratory position of the thorax.

Hence there are to be adopted, two cortical spots regulating the respiration, one, proximal, accelerating rhythm, the other caudal, forcing the inspiration. Both are situated on the lateral end of the praecrucial circumvolution.

[1] WINKLER and WIARDI BECKMANN. Proceedings Vol. I, 25 March 1899.