Citation:

Zwiers, H.J., The system of Sirius according to the latest observations, in:
KNAW, Proceedings, 2, 1899-1900, Amsterdam, 1900, pp. 6-19

Astronomy. — *„The system of Sirius according to the latest observations"*. By Mr. H. J. ZWIERS. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

In N⁰. 3336 of the *„Astr. Nachr."* I have deduced the system of elements of the companion of Sirius so as to have an example for the application of my new method of computing the orbits of double stars. I have found:

Elements I.

$$T = 1893.759$$

$$\mu = -7^\circ.04486 \text{ (Period} = 51.101 \text{ year)}$$

$$e = 0.6131$$

$$i = 44^\circ\ 36'.0$$

$$\Omega = 37\quad 3.6\ (1900.0)$$

$$\lambda = \pi - \Omega = 223\quad 36.6$$

$$a = 7''.77$$

The observations which served as a basis for this orbit, extend from 1862 till the spring of 1890, when the companion was seen for the last time at Lick-Observatory by BURNHAM. For about six years it then disappeared in the rays of the principal star, till, towards the end of 1896, new measurements could be obtained again at Mount Hamilton. The absolute positions of Sirius, as observed in the meridian of Leyden, were reduced to the centre of gravity of the system by aid of the elements just given, combined preliminarily with the distance of the principal star to this centre, as found by AUWERS. In pursuance of the same object I immediately after the reappearance of the component took the computation once more in hand. For the computation of the final values I thought it advisable however to await a few further oppositions. As soon as Messrs. KEELER and AITKEN of Lick-Observatory had kindly communicated to me by letter, in February and March of this year, the results of their measurements in the recent winter, I have derived the final equations [1]). The error of my ephemeris amounting

[1]) An observation received in the beginning of May from Prof. HUSSEY arrived too late to be included in the computation.

in the winter of 1896—97 to over 4° and decreasing the following winter to somewhat over 2°, now proved to be reduced to 0° in the last opposition [1]). A total of 16 serviceable measurements after the periastronpassage seemed sufficient to venture on a correction of the elements of this interesting system.

The space not allowing me to enumerate here all the separate measurements, I must refer to the *Astr. Nachr.* 3084—85, for the observations up to 1890, where Prof. AUWERS communicates them in extenso. Here and there only do the positions used by me differ a little from his on account of assigning slightly different weights to the results of the separate nights. Just as Prof. AUWERS I had formerly been obliged to derive a measurement of HALL in 1888 from the compilation given by Prof. BURNHAM in *Monthly Notices* lviii 6 without knowing its source. In the 2nd part issued since then of the *Observations of double Stars, made at the U. S. Naval Observatory by* ASAPH HALL, I find: 1888.248 $p = 23°27$; $s = 5''777$, with remarks as *faint*, *very faint*, *extremely faint* for the separate nights. Not being able however to make the angle of position agree in any way with the surrounding measurements I have now also excluded this measurement. Farthermore Mr. HALL gives a few yearly means differing slightly from his previous statements in *M. N.*, *A. N.*, and *A. J.* I considered the last values the best and have modified the previous data accordingly.

The communication of the separate measurements after the periastron-passage would demand too much space; I therefore restrict myself to the following table of the mean numbers for each observer [2]).

The observed angles of position have already all been reduced to the meridian of 1900.0 by applying the correction for precession.

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[1]) It may be mentioned here that the orbit of Prof. AUWERS leaves the following deviations (*Obs. — Comp.*): $+15°.24$; $+13°.87$; $+11°53$. These are indeed greater, but they also indicate that the assumed time of revolution is nearer the truth than mine.

[2]) As a rule all the observations of one and the same observer during one opposition are contracted into a single mean. With the relatively great changes in r however, the motion of the angle in this part of the orbit is far from regular and the 2nd differences (with an ephemeris from year to year) amount to several degrees. I have therefore not dared to join into means the observations 5 and 11, 7 and 12, 13 and 15, 14 and 16; in every case the difference in time amounts to half a year nearly.

(*In passing the proofs through the press*). In *M. N.* lviii 6 is still communicated the following measurement of LEWIS at Greenwich with the 28-inch: 1898.214, $\theta = 179°2$, $r = 1''68$ (1 night). This had been overlooked but would have received at all events the weight 0, the deviation in the angle of position amounting almost to 10°, *i. e.* to more than 0''5 in arc of the great circle (according to elements II: $\Delta\theta = +9°93$; $\Delta r = +0''63$).

N°.	Date.	Observer.	Observation.		Number of nights.	n	Δ_1		Δ_2	
			θ	r			θ	r	θ	r
1	1896.920	Schaeberle [1]	189°20	3″73	4	3	+4°60	−0″35	−0°27	−0″06
2	97.017	Aitken [2]	187.03	3.84	8 : 5	4	+3.81	−0 27	−0.83	+0.03
3	.206	Hussey [3]	186.62	3.78	1	1	+6.04	−0.39	+1 84	−0.06
4	.216	Brenner [4]	189.07	3.68	2	0	+8.63	−0.49	+4.46	−0.16
5	1897.802	See [5]	173.89	4.63	4	3	+1.19	+0.29	−1.52	+0.67
6	.818	Aitken [6]	174.78	4.03	4 : 3	3	+2.28	−0.32	−0.38	+0.07
7	.828	Boothroyd [7]	173.66	4.95	2	2	+1.29	+0.60	−1.35	+0.99
8	.839	Schaeberle [8]	175.18	3.95	3	2	+2.94	−0.40	+0.34	−0.02
9	.940	Hussey [9]	175.04	4.01	3 : 2	2	+4.07	−0.37	+1.72	+0.02
10	1898.151	Aitken [10]	170.82	4.22	2	2	+2.44	−0.22	+0.63	+0.18
11	.273	See [11]	168.93	4.79	3	2	+2.02	+0.31	+0.52	+0.73
12	.276	Boothroyd [11]	170.74	4.86	3	2	+3.87	+0.38	+2.37	+0.79
13	1898.737	Aitken [10]	161.68	4.22	3	2	+0.14	−0.40	−0.21	+0.04
14	.785	Hussey [10]	162.10	4.18	2	2	+1.10	−0.45	+0.86	−0.02
15	1899.177	Aitken [10]	154.30	4.55	1	1	−2.44	−0.20	−1.76	+0.24
16	.286	Hussey [10]	154.63	4.40	3	2	−0.96	−0.38	−0.04	+0.06

[1]) A. J. 394; mean having regard to the weights. — [2]) A. N. 3465; 1st, 7th and 8th nights weight 2. — [3]) A. J. 427. — [4]) A. N. 3421; excluded for unreliability of the method — [5]) A. N. 3469; every night weight 1. — [6]) A. J. 424 and 429; mean having regard to the weights. — [7]) A. N. 3469; the two nights equal weight. — [8]) A. J. 420. — [9]) A. J. 427; every night weight 1. — [10]) Received in MS.; mean having regard to the weights. — [11]) M. N., lviii 7; all the nights weight 1.

The manner in which the weights n have been deduced shall be stated farther on; in both columns Δ_1 are contained the differences from my elements of A. N. 3336 in the sense *Obs. — Comp.*

My first work was to investigate anew the personal errors of the observers. These attaining considerable amounts especially in the distances I resolved to found the correction of the orbit exclusively on the angles of position. With the exclusion of the evidently unsuccessful measurements the means were taken of the differences *Obs. — Comp.* for every opposition, a diagram of these was made, the points being connected by a curve in the best way possible. According to the method of Prof. Auwers I also assigned weights of the form $q = mn$, where m depends on the telescope and n on the number of nights. I assumed preliminarily:

$m = 2$ for: Dearborn Obs. (β, HOUGH), Mt. Hamilton (both refractors), Princeton (23-inch), Virginia Univ., and the 26-inch of Washington.

$m = 1.5$ for: Cambridge (Mass.), Cincinnati, Glasgow (Mo.), Malta, RUTHERFORD & WAKELY, PERROTIN, BIGOURDAN, $O\Sigma$, RUSSELL and the small refractor at Washington.

$m = 1$ for all the other observers at refractors of at least 9-inch or reflectors of at least 20-inch aperture.

Farthermore:

$n = 4$ for more than 6 nights.

$n = 3$ for 4, 5 or 6 nights.

$n = 2$ for 2 or 3 nights.

$n = 1$ for 1 night.

Σq was multiplied for every yearly mean by the (computed) distance r in order to reduce to arcs of the great circle and so to obtain comparable weights. Finally to every yearly mean *Obs.—Comp.* a weight $p = \frac{r \Sigma q}{100}$, rounded off to tenths, was assigned. Observations deviating more than 0″5 (in arc of the great circle) were always excluded.

By comparing *Obs.—Comp.* for every observer with the corresponding ordinate of the curve, corrections were deduced whose mean furnished the following personal corrections (the weights, according to the number of nights, being taken into consideration).

Observer.	$\Delta\theta$	Weight.	Observer.	$\Delta\theta$	Weight.	Observer.	$\Delta\theta$	Weight.
Bigourdan	+0°77	3	Hall	−0°33	2 and 4	Stone	+1°79	3
Bond	−0.09	3	Holden	+1.12	4	Struve	−0.53	3
Burnham	−0.28	4	Hough	+0.24	4	Wilson	+1.10	3
Dunér	+0.16	2	Howe	+0.07	3	Winlock	+0.56	3
Engelmann	−0.45	2	Newcomb	+0.09	2 and 4	Young	−0.15	2 and 4
Foerster	+0.05	2	Peirce	−0.94	3			
Frisby	−0.96	4	Pritchett (C.W.)	−0.71	3			

The measurement of STRUVE at Rome gets the weight 2. BURNHAM's measurement in 1881.85 at the 12-inch at Mt. Hamilton is united to his measurements at Dearborn Obs.; likewise the measurements of ENGELMANN at the 7½-inch and the 8-inch at Leipzig and those of NEWCOMB at the small and the great refractor at Washington. For YOUNG and HALL the corrections obtained for the small refractor were united with half weight to those for the great one. Wherever in the last column two weights are given, the former refers to the smaller instrument.

Observers whose personal corrections could not be deduced received as a rule a weight that was 1 smaller than otherwise would have been their due with a view to the nature of the instrument. A weight 3 was assigned to LEAVENWORTH, PERROTIN, PETERS and WATSON; 2 to CHACORNAC, FUSS, LASSELL, MARTH, H. S. PRITCHETT, RUTHERFORD & WAKELY, SEARLE, UPTON and WINNECKE; the others received a weight 1.

After applying these corrections we could pass to the formation of the definitive yearly means *Obs.—Comp.* The assigned weights were again of the form *gn*, where *n* was assumed as before.

The following table contains in the first column the mean date, in the second the preliminary means *Obs.—Comp.* which have served for the construction of the curve of the errors referred to before, in the third the definitive means corrected for personal error. The last column furnishes in the same way as before the value of

$$\frac{1}{100} \, r \, \Sigma \, (g \, n) \, ,$$

Date.	$\Delta_1 \theta$	$\Delta_2 \theta$	p	Date.	$\Delta_1 \theta$	$\Delta_2 \theta$	p
1862.21	+0°57	+0°49	2.1	1878.12	—0°02	+0°12	6.8
1863.22	+0.13	+0.33	1.7	1879.13	—0.16	0.00	6.8
1864.20	—0.74	—0.93	2 8	1880.16	+0.28	+0.38	8.8
1865.21	—0.21	—0.14	2.3	1881.17	+0.18	+0.04	9.0
1866.21	—0.02	+0.10	2.3	1882.21	—0.27	—0.07	10.1
1867.20	+0.23	+0.21	2.8	1883.15	—0.14	—0.32	7.5
1868.19	—0.42	—0.54	3.6	1884.18	—0.46	—0.43	7.2
1869.19	0 00	—0.26	2.2	1885.19	—0.21	—0 15	4.4
1870.17	—0.65	—0.73	3.7	1886.14	—0.32	—0.23	4.0
1871.22	—0.70	—1.12	1.3	1887.19	—1.12	—1.07	2.8
1872.18	—0.03	—0.55	3.0	1888.970	......	—0.16	0.6
1873.22	—0.74	—0.92	1.4	1890.275	......	—1.44	0.3
1874.18	—0.49	—0.47	4.2	1897.004	......	+4.38	1.3
1875.22	—1.14	—0.89	4.7	1897.971	......	+2.43	3.2
1876.14	—0.48	—0.19	4.0	1898.844	......	+0 01	0.9
1877.19	—0.44	—0.22	4.4				

For the last position the measurement of HUSSEY in April 1899 could not be taken into account. Of all the measurements after 1888.0 the means are taken without regard to personal correction, this not being independently deducible and the use of the value deduced above for BURNHAM being prohibited on account of the entirely different appearance of the system.

That the number of the normal positions might not be unnecessarily great I formed normal places by uniting the yearly means two by two according to their weights for the whole of the period 1862—1880 when the changes in distance were still very slight and the motion of the angle therefore pretty regular and moreover very small. An exception was only made for the first three, of which only one position was formed. In order to simplify still further the following computations, the value of $log \sqrt{p}$ was rounded off to tenths; these modified values are indicated by $log \sqrt{p'}$ to distinguish them from the preceding. In this manner the following 21 normal deviations were obtained:

N⁰.	Date	$\Delta\theta$	$l.\sqrt{p'}$	N⁰.	Date	$\Delta\theta$	$l.\sqrt{p'}$	N⁰.	Date	$\Delta\theta$	$l.\sqrt{p'}$
1	1863.31	—0°154	0.4	8	1877.75	—0°014	0.5	15	1886.14	—0°23	0.3
2	1865.71	—0.020	0.3	9	1879 84	+0.217	0.6	16	1887.19	—1.07	0.2
3	1867.76	—0.212	0.4	10	1881.17	+0.04	0.5	17	1888.970	—0.16	9.9
4	1869.80	—0.555	0.4	11	1882.11	—0.07	0.5	18	1890.275	—1.44	9.7
5	1871.89	—0.722	0.3	12	1883 15	—0.32	0.4	19	1897.004	+4.38	0.1
6	1873.94	—0.582	0.4	13	1884.18	—0.43	0.4	20	1897.971	+2 43	0.3
7	1875.64	—0.568	0.5	14	1885.19	—0.15	0.3	21	1898.844	+0.01	0 0

As has already been stated the observations after the periastron-passage could not be treated in the same way as the previous ones, because for that part of the orbit the data are far from sufficient for a satisfactory deduction of the personal corrections. This statement however does not imply that the corrections found *before* 1888 are not at all subject to doubt. Whoever's task it was to investigate the critical problem of these corrections will immediately admit, that in a part of the orbit where e. g. two of the observers have a predominating influence, there can be no question about a complete elimination of the personal errors, even apart from the fact that the accidental errors are often many times greater than the constant ones. Hence the determination of the latter may be very uncertain.

Moreover it is a fact that the personal error often varies greatly with the angle of position itself, especially when the latter, as is the case with Sirius, gradually falls from 90° to 0°, so that the connecting line passes from the horizontal to the vertical position. However I did not feel at liberty to pass over the entire question; the indications of systematic differences were often too clear for doing so.

With regard to the last three normal positions I have still to remark that to the 24-inch refractor of Lowell Observatory the same weight 4 is assigned as to the 36-inch of Mt. Hamilton. The differences $\Delta\,\theta$ have been laid down in the following diagram and have been joined by right lines.

That the remaining errors might vanish as nearly as possible the differential relations were derived between the differences in the angle of position θ and the several elements. Without difficulty we find:

$$\Delta\,\theta = \Delta\,\Omega - \frac{\sin i}{\cot w + tg\, w \cos^2 i}\,\Delta\, i + \left(\frac{R}{r}\right)^2 \cos i\,\Delta\,\lambda +$$

$$+ \left(\frac{a}{r}\right)^2 \sin E \cos i\,(2 - e^2 - e\cos E)\,\Delta\,\varphi + \left(\frac{a}{r}\right)^2 \cos i \cos \varphi\,\Delta\, M_0 +$$

$$+ \left(\frac{a}{r}\right)^2 \cos i \cos \varphi\,(t - T_0)\,\Delta\,\mu\ .$$

In this expression

w indicates the distance from the node, measured in the plane of the orbit,

E the excentric anomaly,

r the apparent, and R the true distance of the companion,

φ the angle of excentricity.

The epoch T_0, for which M_0 stands, may be chosen arbitrarily; I have placed it somewhere in the middle of the period of observation namely at 1880.0.

The equations of errors obtained were treated in the well known manner according to the rules of the method of the least squares; to make the coefficients less unequal the following substitutions were made (logarithmically):

$$x = 0.6\,\Delta\,\Omega\,; \qquad y = 0.0\,\Delta\, i\,; \qquad z = 0.7\,\Delta\,\varphi\,; \qquad u = 1.8\,\Delta\,\mu\,;$$

$$v = 0.4\,\Delta\,\lambda \qquad w = 0.5\,\Delta\, M_0\,; \qquad n = 0.7 \text{ degrees.}$$

For the sake of brevity I state only the normal equations found (numerical coefficients)

$$+7.54570\,x - 5.39749\,y + 2.20722\,z + 0.63518\,u + 9.67691\,v + 4.10538\,w = -0.39532$$

$$-5.39749\,x + 10.82040\,y - 2.44634\,z + 2.60262\,u - 7.12051\,v - 0.31135\,w = +2.69237$$

$$+2.20722\,x - 2.44634\,y + 3.76221\,z - 1.35865\,u + 2.10846\,v - 0.23716\,w = -2.15710$$

$$+0.63518\,x + 2.60262\,y - 1.35865\,z + 1.89029\,u + 1.00294\,v + 1.96012\,w = +1.73341$$

$$+9.67691\,x - 7.12051\,y + 2.10846\,z + 1.00294\,u + 12.66065\,v + 5.51165\,w = -0.26101$$

$$+4.10538\,x - 0.31135\,y - 0.23716\,z + 1.96012\,u + 5.51165\,v + 3.73403\,w = +1.34712$$

These equations furnished the following values (logarithmically):

$$x = 0.820019 \qquad z = 0.330168_n \qquad v = 0.790540_n$$
$$y = 9.055875 \qquad u = 0.615761_n \qquad w = 0.628304$$

from which were deduced:

System Ia

$$☊ = 45^\circ\ 22'.7 \qquad \mu = -7^\circ.37278$$
$$i = 45^\circ\ 10'.2 \qquad M_0 = 103^\circ.6656 \qquad (T = 1894.0696)$$
$$e = 0.5832 \qquad \lambda = 211^\circ\ 17'.5$$

I thought it more advisable however to deduce the two elements μ and T directly from the observations, rather than from the above values. With the corrected elements ☊, i, e and λ the mean anomalies were deduced from the first and the last three angles of position; these were then united with suitable weights into 2 mean numbers, from which was easily deduced:

Ib. $\mu = -7^\circ.314775 \qquad T = 1894.0367$

With these elements the following errors were left in the normal positions:

1 : −0°.131	7 : −0°.422	13 : +0°.003	19 : −0°.207
2 : +0 .169	8 : +0 .114	14 : +0 .451	20 : +0 .151
3 : +0 025	9 : +0 .362	15 : +0 .597	21 : −0 .220
4 : −0 .319	10 : +0 .224	16 : +0 .119	
5 : −0 .513	11 : +0 .163	17 : +2 .103	
6 : −0 .409	12 : −0 .006	18 : +2 .550	

These errors are also represented in the lower diagram and connected by interrupted lines. Especially the last two positions before the periastron are now badly represented, a fact not to be wondered at, considering the large amount of the corrections of the elements.

Although these positions have but the weights 0.6 and 0.3 I have yet proceeded to a second approximation. For the new 2nd members of the normal equations I found :

+0.32052 +0.66856 +0.51950 +0.20554 +0.30590 +0.37168.

After a new solution of the normal equations μ and T were again determined as above; the system of elements obtained is:

System II.

$$T = 1894.0900 \qquad i = 46^\circ\ 1'9$$
$$\mu = -7^\circ.37069 \qquad \Omega = 44\ 30.2 \quad (1900.0)$$
$$e = 0.5875 \qquad \lambda = 212\ \ 6.4$$

The deviations left by this system in the normal positions are as follows. They have been connected by dotted lines in the diagram.

1 : —0°.203	7 : —0°.433	13 : —0°.521	19 : —0°.300
2 : +0 .209	8 : +0 .034	14 : —0 .205	20 : +0 .158
3 : +0 .082	9 : +0 .182	15 : —0 .218	21 : —0 .087
4 : —0 .250	10 : —0 .032	16 : —0 .925	
5 : —0 .455	11 : —0 .161	17 : +0 .773	
6 : —0 .380	12 : —0 .420	18 : +0 .098	

The outstanding errors are unimportant, but a certain regularity is unmistakable. The characteristic curvature in the original curve of errors before the periastron, is found back all but unchanged in the diagram of systems I^b and II. The cause may be sought in a perturbation by a third (invisible) member of the system ; the supposition however that not entirely eliminated personal errors have been at work seems to me more plausible. A third possibility remains : the not perfect accuracy of the coefficients of the equations of errors in the 2nd approximation might be the cause. Strictly speaking these ought to have been recalculated with the elements of system I^b.

But this supposition is already very improbable *a priori*. Tc at certainty on this subject without an entirely new and prol putation, I made use of the method of KLINKERFUES based angles of position. The ratio of the planes of triangles in th rent orbit to those in the true orbit being always as $\cos i : 1$

$$\frac{\sin (v_2-v_1) \sin (v_3-v_6)}{\sin (v_3-v_1) \sin (v_2-v_6)} = \frac{\sin (\theta_2-\theta_1) \sin (\theta_3-\theta_6)}{\sin (\theta_3-\theta_1) \sin (\theta_2-\theta_6)}$$

and two other similar equations in which the indices 4 an successively to be substituted for the index 3. For the epoch normal positions 2, 6, 10, 14, 17 and 20 the deviations normal positions were united with those of the two neighbouri according to the weights. We thus obtained :

$$\theta_1 = 76^\circ.219 \qquad \theta_2 = 59^\circ.650 \qquad \theta_3 = 45^\circ.476$$

$$\theta_4 = 33^\circ.573 \qquad \theta_5 = 13^\circ.213 \qquad \theta_6 = 173^\circ.079.$$

The second members of the equations may be denoted by α, β

$$\alpha = + 0.481680 \qquad \beta = + 0.297904 \qquad \gamma = + 0.1200$$

I started successively from 4 hypotheses : 1° system II ; 2° ΔM_0 = 3° $\Delta \mu = + 0^\circ.03$; 4° $\Delta e = + 0.01$.

From the three anomalies deduced from these I computed

	1st hypothesis.	2nd hypothesis.	3rd hypothesis.	4th hypc
α	+ 0.468089	+ 0.465082	+ 0.464792	+ 0.47
β	+ 0.294009	+ 0.290553	+ 0.290125	+ 0.30
γ	+ 0.119778	+ 0.117508	+ 0.116272	+ 0.13

from which the following equations ensued :

$$- 0.003007 \Delta M_0 \quad - 0.003297 \Delta \mu \quad + 0.006753 \Delta e \quad = + 0$$

$$- 0.003456 \Delta M_0 \quad - 0.003884 \Delta \mu \quad + 0.010131 \Delta e \quad = + 0$$

$$- 0.002270 \Delta M_0 \quad - 0.003506 \Delta \mu \quad + 0.015346 \Delta e \quad = + 0$$

The solution of these equations furnished the following entirely improbable values:

$$\Delta M_0 = + 51^\circ.8590 \qquad \Delta \mu = - 2^\circ.011252 \qquad \Delta e = - 0.07627$$

The question of course remained in how far these values might be brought within admissible limits by small allowable modifications in the assumed angles of position. Moreover, on account of their being arithmetical means, the corrections assumed for the six epochs were not exactly situated on the curve which connects the deviations in the best way possible. I have constructed therefore the curve of errors for the Elements II on a relatively large scale and I have deduced by its aid, for the same epochs as above, the following angles of position:

$$\theta_1 = 76^\circ.281 \qquad \theta_2 = 59^\circ.581 \qquad \theta_3 = 45^\circ.446$$

$$\theta_4 = 33^\circ.503 \qquad \theta_5 = 13^\circ.029 \qquad \theta_6 = 172^\circ.924$$

From these I computed:

$$\alpha = + 0.484570 \qquad \beta = + 0.299769 \qquad \gamma = + 0.120475.$$

The solution of the equations now led to:

$$\Delta M_0 = + 57^\circ.0261 \qquad \Delta \mu = - 2^\circ.23501 \qquad \Delta e = - 0.0854$$

It seemed to me that this proved sufficiently how impossible it is to cause the disappearance of the observed systematic course by a purely elliptic motion and I therefore stopped at System II, taking this to be the best which can be deduced for the present from the observations.

Finally I have determined the semi-axis of the orbit for each observer who had given more than three measurements of distance. As a rule measurements leaving a greater error than 0''.5 were excluded. This fate befell, besides one unsuccessful observation of SECCHI in 1863, only 5 other measurements of $O\Sigma$. This is not to be wondered at, if we consider the low position of Sirius at Pulkowa. The results obtained are compiled alphabetically in the following table where the column n gives the number of measurements from which a is deduced.

Observer.	a	n	Observer.	a	n
Aitken	7″.805	5	Hussey	7″.594	4
Bigourdan	7 .507	5	Newcomb	7 .747	4
Burnham	7 .404	10	Peirce	7 .576	4
Dunér	7 .417	5	Pritchett (C.W.)	7 .668	5
Frisby	7 .776	4	Stone	7 .423	5
Hall	7 .533	18	Struve	7 .812	14
Holden	7 .911	7	Wilson	7 .314	4
Hough	7 .358	8	Young	7 .579	7

From all the measurements of the above observers I find in the mean 7″594 for the semi-major axis. The complete system of elements by the side of which I introduce for the sake of comparison the one found by Prof. AUWERS in 1892, runs as follows:

	System II. ZWIERS	*System V*.* AUWERS
T	1894.0900	1893.615
μ	−7°37069	−7°2877
P	48.8421 year	49.399 year
e	0.5875	0.6292
i	46° 1′9	42° 25′6
☊	44 30.2 (1900.0)	37 30.7 [1]) (1850.0)
π—☊	212 6.4	219 56.5
a	7″594	7″568

I have also investigated for systematic deviations the *distances* found in the various years. To each observer of the above table the weight 1 was given (with the exception of the 6 measurements, mentionned above), the remaining ones were given the weight ½, evident failures being excluded. After the periastron-passage the observations of SEE and BOOTHROYD were omitted. As appears from the table

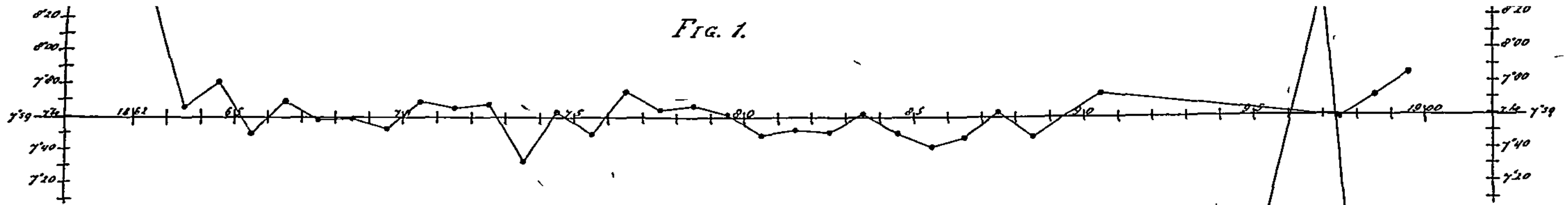
[1]) Reduction to 1900.0 + 16′.9.

on page 8 where in the columns Δ_2 the various values of *Obs.—Comp.*, as resulting from a comparison of the observations with System II, have been given already, these observations deviate in distance from 0″67 to 0″99 (in the same direction) from the computed ones, whereas the other distances, measured at Mt. Hamilton, fairly oscillate round them. The following consideration proves *a priori* that the latter must come nearer to the truth. The area of the sector traversed yearly is already known with great approximation from the first part of the orbit. So in each new orbit $r_1 r_2\, arc\,(\theta_1-\theta_2)$ must have about the same value as in the old one. Now $\theta_1-\theta_2$ is equal to 27°424 as appears from the normal positions 19 and 21, and equal to 23°052 according to the old orbit. Half the difference of the logarithms is 9.96229 = *log.* 0.9168, so that the old distances must be diminished on an average by 8.32%. This gives for 1897.0, 1898.0 and 1899.0 respectively 3″8, 4″0 and 4″3 (compare the ephemeris below), whilst the observations at Lowell Observatory gave much greater values.

The following table gives the yearly means obtained for *a* with their weights. It is easy to understand that from 1887 an error in *r* must appear magnified in *a*.

1862	8″33	(1½)	1873	7″33	(4)	1884	7″50	(7½)
1863	7.65	(2)	1874	7.63	(3½)	1885	7.42	(4)
1864	7.81	(2)	1875	7.49	(5)	1886	7.47	(5)
1865	7.49	(2½)	1876	7.75	(4½)	1887	7.62	(3)
1866	7.69	(6½)	1877	7.64	(4)	1888	7.47	(2)
1867	7.57	(3)	1878	7.66	(5)	1890	7.74	(1)
1868	7.58	(4½)	1879	7.61	(7½)	1897	7.58	(5)
1869	7.53	(4½)	1880	7.49	(8)	1898	7.72	(3)
1870	7.69	(3)	1881	7.53	(10½)	1899	7.85	(2)
1871	7.65	(4)	1882	7.51	(8)			
1872	7.67	(5½)	1883	7.62	(6)			

In the upper figure of the diagram accompanying this paper these values are laid down for the middle of the year and have been connected by right lines. One can see that the deviations are but



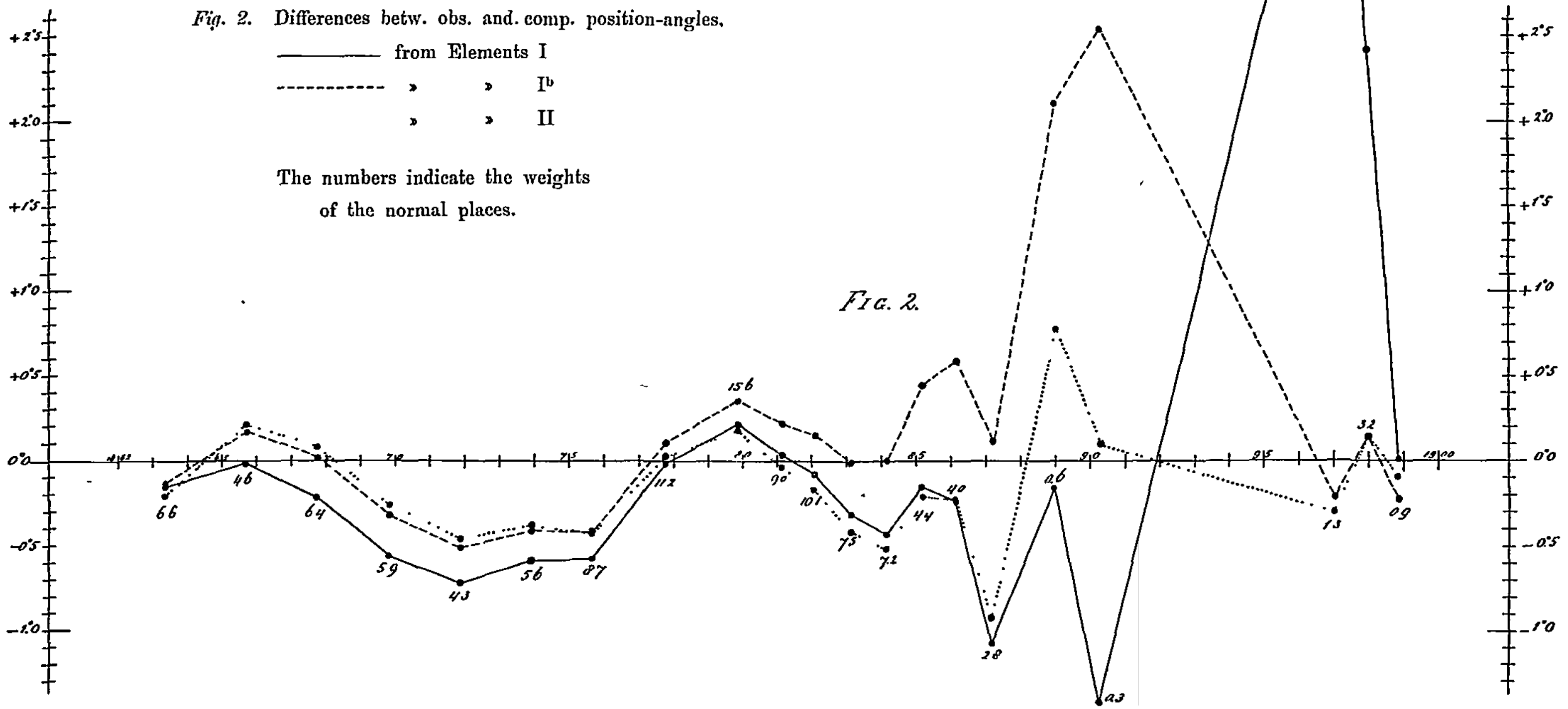


Fig. 1. Values of *a*.

Fig. 2. Differences betw. obs. and. comp. position-angles,

———— from Elements I

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» » II

The numbers indicate the weights of the normal places.

relatively very small (the weight of the value found for 1862 is in fact about zero) and that the values continually oscillate round the horizontal line of 7″59. Sixteen times the latter is intersected by the connecting lines, fourteen times this is not the case. There is no indication of systematical errors of any importance and I believe I am justified in declaring that system II satisfies all just claims.

For a comparison with future observations I have deduced an ephemeris, an extract of which follows in the subjoined table:

Date	θ	r	Date	θ	r	Date	θ	r
1896.0	205°53	3″60	1900.5	140°64	4″77	1905.0	107°17	6″80
.5	196.61	3.71	1901.0	135.60	4.97	.5	104 67	7.03
1897.0	188.14	3.80	.5	130.97	5.18	1906.0	102.34	7.26
.5	180.08	3.90	1902.0	126.71	5.40	.5	100.14	7.49
1898.0	172.42	4.00	.5	122.78	5.63	1907.0	98.07	7.71
.5	165.17	4.12	1903.0	119.16	5.86	.5	96.12	7.93
1899.0	158.36	4.26	.5	115.82	6.09	1908.0	94.27	8.14
.5	152.00	4.41	1904.0	112.72	6.33	.5	92.52	8.35
1900.0	146.10	4.58	.5	109.85	6.56	1909.0	90.85	8.56

The parallax of Sirius has been determined very accurately by the heliometer measurements of GILL and ELKIN at the Cape in the years 1881—83 and 1888—89. If we take with GILL 0″374 ± 0″006 for the mean according to the weights (M. N., Jan. 1898, p. 81), we shall find for the sum of the masses of the two stars 3.51 times that of the sun, of which, according to AUWERS (l. c. page 231) somewhat over $^2/_3$ is due to Sirius itself.

Physics. — "*Measurements on the magnetic rotation of the plane of polarisation in oxygen at different pressures.*" By Dr. L. H. SIERTSEMA. (Communication N°. 49 from the Physical Laboratory at Leiden, by Prof. KAMERLINGH ONNES).

The results of my measurements on the magnetic rotation of the plane of polarisation in some gases, made at a pressure of about 100 atm., agreed fairly well with those made by KUNDT and RÖNTGEN [1]).

[1]) Arch. Néerl. (2) 2, p. 378 (1899); Comm. Phys. Lab. Suppl. 1.