Citation:

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S_2	$19° = 0.6$ hours,
M_2	$19° = 0.7$ »
N	$21° = 0.7$ »
K_2	$21° = 0,7$ »

so that the difference amounts to 42 minutes of time, whilst the amplitude may be assumed to be twice as large at *Sabang* — at least in the back parts of the bay — as on the road of *Oleh-leh.*

The mono- and semi-annual variations are for both places somewhat different; from the three series of observations at *Oleh-leh* it appears however that in these regions the monthly mean values of the sea-level widely differ for different years, so that a better agreement might be expected only if the observations extend over a long series or at least over simultaneous periods.

It is of some importance to remark that, whereas the semidiurnal tides at *Sabang* are nearly twice as strong as near *Oleh-leh,* the mono-diurnal tides seem to be amplified in a far less degree.

This point, concerning the way in which both tides propagate and are enlarged or diminished, is of great importance for the understanding of the mechanism of tides and requires a thorough investigation.

With a view of elucidating this point tide-gauges ought to be established at the entrance and in the back parts of bays and estuaries: for these experiments however stations should be chosen where the mono-diurnal tides are better marked than at *Sabang* so that an accurate determination of the characteristic constants is possible.

An analysis of tides at different parts of a river in which a tidal wave of mixed description propagates would also afford useful data for this purpose.

Hydrography. — "*On the relation between the mean sea-level and the height of half-tide.*" By H. E. DE BRUYN.

The mean sea-level is the mean of the height of the water observed at short intervals i. e. every hour.

Observations have proved, that the mean of 3-hourly observations does not practically deviate from this; in this way the mean sea-level in the years 1884—1888 has been determined by the Royal Geodetical Commission (Annual report of the Commission 1889).

Generally it is admitted that there is a constant difference, between the mean sea-level and half-tide (the mean of high and low water), during several years or months.

This has been done by the above-mentioned Commission, in their calculation of the mean sea-level for several years for Den Helder. Dr. H. G. VAN DE SANDE BAKHUYZEN also in his communication "On the variation of latitude," to the meeting of the Royal Academy (24th of Febr. 1896), assumed that the mean value of that difference during a month was a constant quantity at Den Helder. In both cases this was perfectly justified, as this value for the annual means is very nearly constant at Den Helder, and in the last case differences that may exist, are eliminated by the method of determination. However, the supposition that the difference is constant is not true for the annual means at all stations, and is certainly not so, for the monthly means at some stations.

I intend to trace those causes, which produce a difference in this value, and to find its range for one tide-gauge. I took Delfzyl for the observation-station, as at Delfzyl the difference between half-tide and the mean of the sea-level, is greater and more variable than at any other station in our country. From another point of view, Delfzyl would not be so advantageous, as there a comparison with tide-gauges in the neighbourhood is not possible.

Before proceeding further, a few words, to point out the importance of the law of the variation of that difference, are necessary. The knowledge of the mean sea-level is not only important for the annual means, but also for the monthly means, as we can deduce from them the annual variation, and also because an exact knowledge of the monthly means, assists in the detection of the unavoidable changes of the zero's in the automatic tide-gauges, and the determination of their values. As the high and low water marks are always determined in the first place, their mean is naturally known; therefore it saves much trouble, if it is possible to deduce from that mean value, the true mean sea-level, as the hourly observations can be then neglected. Besides, in the event of interruptions, which happen frequently in using the automatic tide-gauges, it is much easier to guess, the positions of high and low water, than the hourly heights, as high and low water are independent of the exact time. Moreover meteorological circumstances have, by the retardation or the acceleration of the tide, a greater influence on the hourly heights than on high and low water.

The mean sea-level can therefore be deduced more exactly from the height of half-tide, the difference of both being known, than from the hourly observations when some of these must be guessed.

Further let the difference between the mean sea-level and the height of half-tide be A, the mean sea-level Z, high water V, low water E. Half-tide is $\frac{1}{2}(V+E)$ and the range of the tide $(V-E)$.

The causes, which have an influence on the value of A, are four in number:

1st. the range of the tide $(V-E)$;

2nd. the mean sea-level (Z);

3rd. the time of the year;

4th. the presence of ice.

In the last mentioned case, I am not alluding to the fact of the ice preventing the working of the tide-gauge, for I consider this to be an interruption, but to the fact, that the presence of ice at a certain distance from the tide-gauges, deforms the tide-curve. This deformation is, in my opinion, one of the most interesting researches on tides.

I propose to solve the following question. What corrections are wanted for Delfzyl in the value A, deduced from a certain number of years, in order to find that quantity for separate months?

The data, which I had at my disposal were the values of V and E for 18 years (July 1881—July 1899) the values of Z for 7 years (1884—1890) viz. the height at 2, 5, 8 and 11 o'clock, and in addition the height at 2 and 8 o'clock for 8 years (1891—1898).

The mean range of the tide at Delfzyl is, according to these data 2750 m.m., the mean sea-level Z is according to the calculations of the above-mentioned commission 128 m.m., reckoned from the zero of the tide-gauge during the years 1884—1890. The mean value A during these 7 years is 193 m.m., so we find that the mean of half-tide is 128—193 m.m. = — 65 m m. Tide-curves of spring and neap-tides accompany this paper.

It is difficult to determine, how much each of the four causes, influences A at Delfzyl, as they often modify A in the same direction. So, during the year, the correction for each of the three first-named causes, is generally a sinusoide of about the same amplitude and the same phase.

It is therefore necessary, to adopt a certain definite value for one of these causes. I assume that the correction, due to the first cause, is proportional to the difference of the mean range of the tide and the observed value of that range $V-E$ and that their proportion is equal to the ratio of the mean values A and $V-E$, or ceteris paribus, A is always proportional to $V-E$. In substance this will be he case. I adopted as this proportion $^1/_{15}$, the correction is therefore $^1/_{15}$ $(V-E-2750)$ mm.

As concerns the second correction we see that when the sea-level is higher, A is smaller than the mean value, and vice-versa.

We do not know exactly the law governing these small changes, because various unforeseen circumstances e.g. storms influence them. The only thing that can be done, in my opinion, is to take for this correction a quantity proportional to the deviation of the mean sea-level. As it was my intention to deduce the value of Z from that of half-tide, which is known, I adopted for the value of that correction a quantity proportional to the deviation from the mean height at half-tide (— 65 mm.).

After comparing the same months of different years, I found that this correction amounts to about $^2/_{20}$ or $^2/_{30}$ of the value of that deviation i. e. $^1/_{20}$ or $^1/_{30}$ of $(V+E+130)$. I have adopted $^1/_{25}$ $(V+E+130)$.

That the height of Z has an influence on the form of the tide-curve, is probably due to the mud-banks in the Dollard. The surface that must be covered, constantly changes with the level of the sea, and so for equal tide-ranges, the quantity of water flowing in and out of the Eems at Delfzyl, is much greater for high sea-levels than for low.

Both corrections being applied another annual correction is still wanted. For this correction I adopted an annual sinusoide, the amplitude and the phase of which can be easily determined. The amplitude is in round numbers 10 mm. and the greatest positive value occurs about the 1st of July. From the observations in the seven-yearly period, there is no evidence of the existence of a half-yearly sinusoide. Considering also the heights at 2 and 8 o'clock for the period 1891—1898, there appears to be a semi-annual sinusoide, but the amplitude is very small, and it is questionable whether the sinusoide derived from those observations is not different from the mean sinusoide. It is better to entirely neglect this correction.

After applying these corrections, the values of the sea-level for some months, still show great negative divergencies. It is obvious that these are exactly the months in which we have a very low temperature, and in which there must have been ice. But as the mean temperature of a month is not an exact proof of the presence of ice, I adopted as a datum the thickness of the ice according to the observations at Den Helder (see the Proc. Kon. Inst. van Ingenieurs) as quoted in the following table.

Thickness of the ice in mm.

	1884	1885	1886	1887	1888	1889	1890
January	22	387	201	298	310	204	43
February	53	34	323	266	360	156	171
March	6	24	211	110	223	124	93
November	46	74	6	43	145	27	267
December	95	58	96	121	51	199	811

I took for the months in question the four ones in which the ice has the greatest thickness, and two other months in which the thickness too was great, following immediately on two of the former, as we may suppose the ice still existed during that time. The selected months are underlined in the table.

I found that in these months A is too small. It is difficult to find a cause for this, as, excepted at Delfzyl and Statenzyl, there are no tide-gauges in the Eems and the Dollard, and the gauge at Statenzyl does not work when the tide is low. Probably it is due to the ice on the mud-banks of the Dollard. Generally the effect of the ice is to raise high-watermark at the mouth of the river, but this is not the case at Delfzyl. On the contrary, the range of the tide is less in the months with ice. Probably both V and E are increased, but E more than V, and therefore the range is smaller and half-tide considerably higher, the mean sea-level is less increased than half-tide and hence the difference A is smaller.

The heights at 2 and 8 o'clock in the months January 1891 and February 1895, when there was much ice, give also corresponding results.

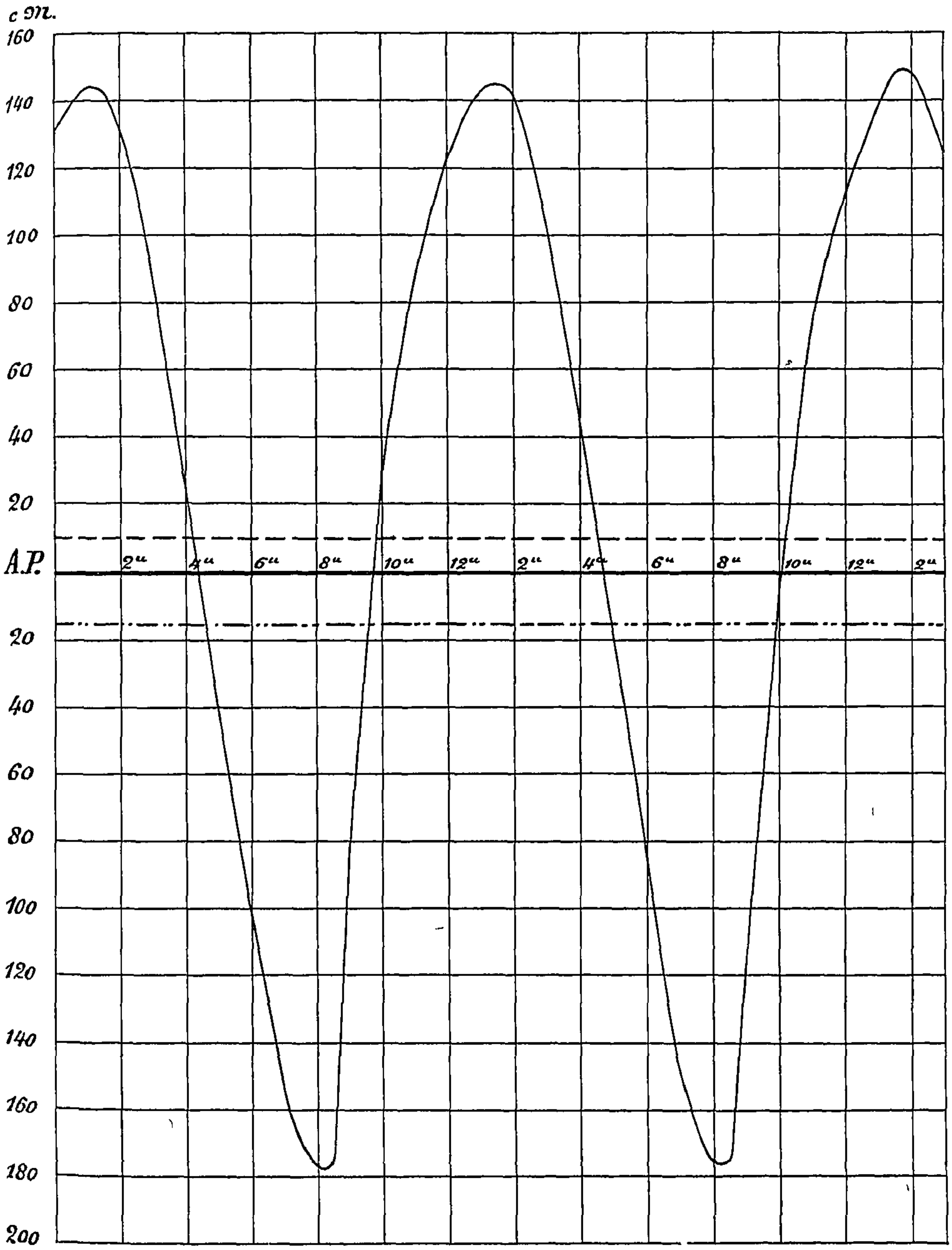
In the following table are given the values A, the corrections and the remaining differences, for the 5 months in which A is a maximum, the 5 months in which A is a minimum, the months with ice and two other months in which the error or the remaining difference is greater than 15 mm.

14*

Month	Value *A*.	Correction for			Remaining Difference.
		Tide-range.	*Z*.	Time of the year.	
		Maximum values of A.			
April 1884.	223^5	9	14^5	2^5	4^5
May 1886.	220^5	7^5	9^5	7	3^5
April 1888.	220	12^5	10	2^5	2
April 1889.	229	9	10	2^5	14^5
May 1889	225^5	10	10^5	7	5
		Minimum value of A.			
October 1884.	148	—5	—23	-2^5	-14^5
December 1884.	158	-4^5	—15	-9^5	—6
February 1889.	158	-3^5	-20^5	—7	—4
January 1890.	160	—5	-15^5	-9^5	—3
October 1890	149	-1^5	—25	-2^5	—15
		Months with ice			
January 1885.	178	—1	12	-9^5	-16^5
February 1886.	188^5	4	17^5	—7	—19
March 1886.	186	0^5	14	-2^5	—19
February 1888.	164	-1^5	15^5	—7	—36
March 1888.	183	—1	10	-2^5	-16^5
December 1890.	167	—7	21	-9^5	-30^5
		Months with differences greater than 15 mm.			
February 1890.	189	0^5	23^5	—7	—21
December 1888.	199^5	-2^5	-5^5	-9^5	24

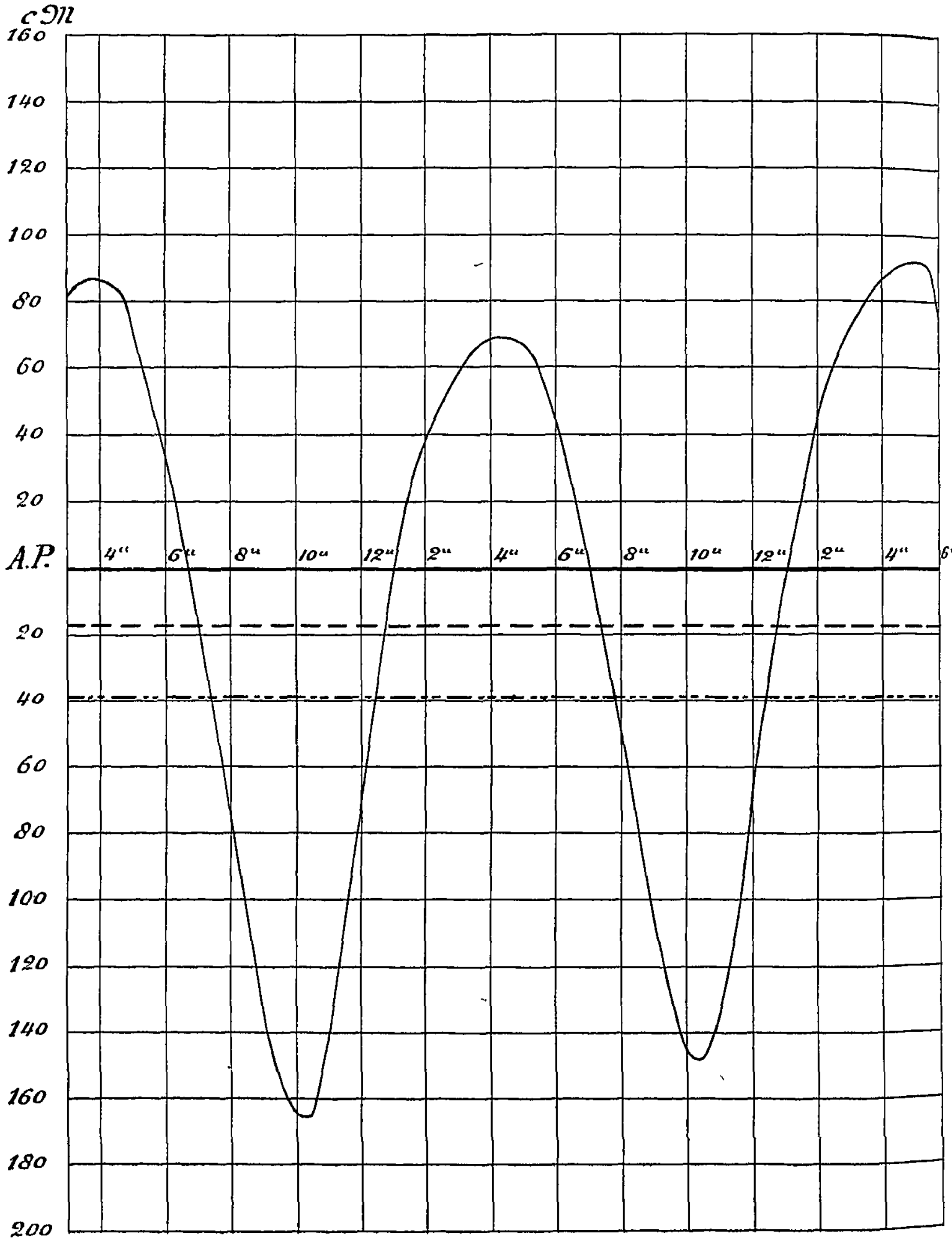
From this it appears that the corrections and the errors are positive in the months with maximum values of *A*, negative in those with minimum values. We find concerning February 1890 that there also has been ice during a portion of this month, that the mean height was lowest of all months and that as low water occurred on the 28th a few minutes before midnight, it had to be considered as occurring in March. The great negative difference can probably be explained by these circumstances.

Tidecurve at Delfzijl (Spring-tide). I.



– – – – Mean Sea-level. —..—..— Half-tide.

Tidecurve at Delfzijl (Neap-tide). II.



– – – – Mean Sea-level. –..–..– Half-tide.

The remaining differences for all months are given in the following table, in which half mm. have been neglected; the months with ice are underlined.

Month	1884	1885	1886	1887	1888	1889	1890
January	11	<u>—17</u>	11	5	—2	—13	—3
February	9	12	<u>—19</u>	—7	<u>—36</u>	—4	—21
March	6	—13	<u>—19</u>	2	<u>—16</u>	—3	—1
April	4	—0	0	—8	2	14	—7
May	—3	1	3	—3	2	5	0
June	—1	—11	—1	—5	—1	—8	—3
July	4	—4	—3	1	7	3	—6
August	5	3	—5	—1	4	11	6
September	0	3	—4	12	—2	—12	—1
October	—14	5	0	—2	2	6	—15
November	—6	—2	7	14	—9	3	3
December	—6	3	12	14	24	0	<u>—30</u>

The mean error of the sea-level for all months, those with ice excepted, is 6,0 mm. I computed this mean error also on the supposition that the second correction is $^1/_{20}$ or $^1/_{30}$ of $V + E + 130$. The mean error was found to be respectively 6,3 and 6,1 mm. When the value of Z is deduced from the heights at 2 and 8 o'clock and a mean correction $^1/_4$ (height at 5 + height at 11 — height at 8 — height at 2) is applied, the mean error of Z has been found to be 8,8 mm. It seems therefore that the deduction of Z from the height at half-tide gives more exact values, than the deduction from the heights at 2 and 8 o'clock.

Hence the value A for Delfzyl, after applying the above-mentioned corrections, the sun's longitude being φ and the correction for the presence of ice Y, is:

$$A = Z - {}^1/_2 (V + E) = 193 + {}^1/_{15} (V - E - 2750) - {}^1/_{25} (V + E + 130) + a_1 \cos(\varphi - \gamma) + Y,$$

γ being a constant angle.

Or: $$A = 5 + {}^2/_{75} V - {}^8/_{75} E + a_1 \cos(\varphi - \gamma) + Y.$$

Partly neglecting the variations of the monthly tide-ranges in different years, which are at a maximum 120 mm. and give only a maximum error of 3 mm. in the value of A, we can put $^2/_{75} \times 2750 + a_2 \cos(\varphi - \gamma_2)$ instead of $^2/_{75}\ V - {^2/_{75}}\ E$ mm.

The formula then becomes:

$$A = 78 - 0{,}08\, E + a_3 \cos(\varphi - \gamma_3) + Y.$$

The errors calculated from this formula do not differ much from the above-mentioned, for now the mean error is 6,4 mm. and the formula is therefore as exact, while the computation is much more easily carried out.

In conclusion I will add the following remarks.

First I should mention that I found some errors in the tables containing the observed height of the sea-level at Delfzyl during the months in which the greatest differences occurred and in those of two other tide-gauges during five months. For instance at Delfzyl I found a month in which one height had been read from the half hour-mark instead of the hour-mark, and also one reading with a wrong sign. After making the correction the great divergence was very much reduced. Although this is no proof, we may suppose that the greatest differences very nearly give the limit of precision.

Further I notice that the second correction mentioned above does not agree with the principle on which the method of harmonic analysis is founded, so that this method cannot give exact results in the reduction of the observations at Delfzyl. Still, I do not affirm that any other is better.

This want of agreement is demonstrated by the term 0,08 E in the preceding formula. For the same month in two different years (February 1889 and 1890) the difference of the two values of E is 583 mm., so that 0,08 E = 47 mm., and although this difference would not be of much importance for a single observation, it is far too great for an error of the monthly mean.

Mathematics. — Prof. JAN DE VRIES reads for Prof. L. GEGENBAUER at Vienna a paper entitled: "*New theorems on the roots of the functions* $C_n^{\nu}(x)$".

Up to this moment we know of the roots of the coefficients $C_n^{\nu}(x)$ of the development of $(1 - 2\,\alpha + \alpha^2)^{-\nu}$ according to ascending powers of α only this, that they are all real and unequal, are situated