

Citation:

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The frequency of the Pulsations was tested in vain like that of the Spasms for a concordance with the tropical or synodical revolution of the Moon.

In the yearly values of the frequency of these Pulsations no parallelism with the numbers for the sun-spots can be found, and in the monthly values a not very distinct yearly undulation appears, which however is quite different from that of the Spasms. But, curiously enough, the daily variations in the frequency agree, without being however quite equal, as appears from the following table of the epochs of the maxima etc.

	Spasms	Pulsations
Principal maximum	11—12 P.M.	0—1 A.M.
» minimum	7— 8 A.M.	7—8 A.M.
Secondary maximum	± 2 P.M.	± 1 P.M.
» minimum	6— 7 P.M.	5—6 P.M.

The Electrograms at Batavia show nothing remarkable during the occurrence of Pulsations, which means that no simultaneous changes in the Potential can be observed. As regards the slope of Electric Potential in the lower strata of the atmosphere, I think this will not have any influence on the magnet.

In concluding this preliminary communication I will point out, that a magnetic calm favours the development of the Pulsations, which is connected directly with the quiet of night, as shown by the magnetograms at Batavia. This nightly calm is clearly indicated by the diurnal variation of the above mentioned „deviations”, and the epoch of the minimum (1 A.M.) practically coincides with the maximum epoch of the Pulsations. But also the minimum epoch of the deviations (3 P.M.) coincides with the epoch of the secondary maximum, and this makes the connection less clear.

Physics. — Dr. FRITZ HASENOEHRL. “*The dielectric-coefficients of liquid nitrous oxide and oxygen.*” (Communication N^o. 52, from the Physical Laboratory at Leyden by Prof. H. KAMERLINGH ONNES).

(Read September 30th 1899).

Measurements of the dielectric-coefficients of liquid gases have been made up to the present only by LINDE¹⁾ and by DEWAR and

¹⁾ LINDE, Wied. Ann. 56 p. 546.

FLEMING¹⁾. The measurements of LINDE are concerned with those gases which become liquid under high pressure at relatively high temperatures and are not in direct relationship with the following work. On the contrary DEWAR and FLEMING have sought the dielectric-coefficient of liquid oxygen under the same condition as I, namely at the temperature of the normal boiling point under atmospheric pressure.

The gases were liquefied in the cryogenic laboratory of the University of Leyden, the arrangement of which is described in another place²⁾. I shall hence confine myself to mentioning here the special arrangements used in the determination of the dielectric-coefficients. During the experiments the cryogenic apparatus was under the personal care of Prof. KAMERLINGH ONNES, through whom alone my research was brought to a satisfactory conclusion. I wish to express here, for this and much other valuable assistance, my warmest and most sincere thanks.

1. The Method.

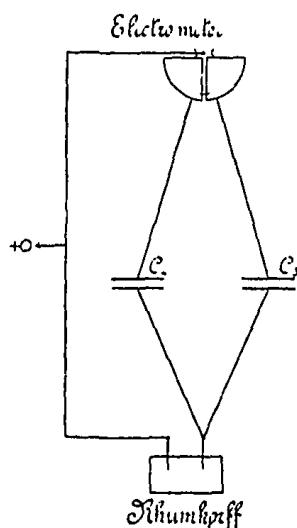


Fig 1

The method I used was a modification of GORDON's, the principle of which is clearly and diagrammatically shown in Fig. I. The inner surfaces of two condensers C_1 and C_2 are connected to the quadrant pairs of a THOMSON electrometer, and the outer surfaces to one pole of an induction coil the other pole which is earthed together with the needle of the electrometer. Then, if the capacities C_1 and C_2 are equal, the needle will not be deflected on starting the coil. If C_1 is an adjustable condenser, then the capacity of C_2 with different media can be obtained and hence immediately the dielectric coefficients of these media. But this assumes

that the electrometer is constructed symmetrically and that the capacity of the leads and also of the non-inductive parts of the condenser (i. e. other than the plates) are the same on the two sides. The simultaneous elimination of these two sources of error offers considerable difficulties.

If we call γ_1 and γ_2 the capacities of the quadrant pairs together

¹⁾ DEWAR and FLEMING, Proc. R. S. Lond.

²⁾ KAMERLINGH ONNES, Comm. Phys. Lab. Leiden N^o. 14. MATHIAS, Le Laboratoire cryogène de Leyde, Rev. Gén. d. Sciences, 1896 p. 381. And especially KAMERLINGH ONNES Methods and apparatus used in the cryogenic laboratory I. loc. cit. No. 51.

with their leads; p_1 and p_2 the characteristic constants of their action on the needle (the differential quotients of the mutual induction coefficients by the rotations); C_1 and C_2 the capacities of the two condensers to be compared; c_1 and c_2 the capacities of the non-inductive parts of the condensers connected to the electrometer; then the equation for the equilibrium of the needle is¹⁾

$$\left(\frac{C_1}{c_1 + \gamma_1}\right)^2 p_1 = \left(\frac{C_2}{c_2 + \gamma_2}\right)^2 p_2 \dots \dots \dots (1)$$

If however this equation is fulfilled the equality of C_1 and C_2 does not immediately follow. We can first make $p_1 = p_2$ by some known method. In this case equation (1) takes the form

$$\left(\frac{C_1}{c_1 + \gamma_1}\right)^2 = \left(\frac{C_2}{c_2 + \gamma_2}\right)^2 \dots \dots \dots (1')$$

We can now reverse C_1 and C_2 thus changing their influence and, keeping p constant, alter γ through a capacity such that the equilibrium of the needle is not further altered by this reversal. Hence besides (1') we have

$$\left(\frac{C_2}{c_2 + \gamma_1}\right)^2 = \left(\frac{C_1}{c_1 + \gamma_2}\right)^2 \dots \dots \dots (2)$$

And from (1') and (2) it follows that we must have $\gamma_1 = \gamma_2$ and $c_1 = c_2$ if we wish to arrive at

$$C_1 = C_2 .$$

It would however be difficult to make these quantities p and γ equal with the necessary accuracy. Further they alter with every change in the nullpoint of the electrometer needle, and hence these equalizations would have to be often repeated and would be certainly very tedious and lengthy. On account of these difficulties I have modified the method as follows.

C_2 remains permanently unaltered, C_1 is an adjustable condenser, and is so arranged that the needle will not move when the coil is started. The condenser, of which the capacity is required, is now put in parallel with C_1 . To bring the needle back to zero, the capacity C_1 must be decreased by a measurable amount which is equal to the required capacity. In this way the symmetry of the electrometer etc. requires no attention. The sole condition is that the wires, which put the unknown capacity in parallel with C_1 , have not themselves

¹⁾ See MAXWELL Electricity and Magnetism, Vol. I. p. 219.

a measurable capacity. Naturally this is not easily attained, but the respective corrections may be determined fairly simply as explained below.

2. Description of the separate apparatus.

The electrometer was a THOMSON'S in its original form. When this is set up in the neighbourhood of the working pumps of the cryogenic laboratory it must be little sensitive to vibration. For this purpose I obtained the damping by air (after TÖPLER), instead of by sulphuric acid, and replaced the bifilar suspension of two cocoon fibres by a platinum wire about 70 cm. long and 30μ in diameter which can carry a much greater weight.

After these changes had been made the vibrations were less than 0.1 mm. on a scale at 3.5 m. distance, even when the pumps of the cryogenic laboratory were working at a distance of 10 m., while with the original arrangement it was quite impossible to make observations under the same conditions.

The induction coil was worked by an alternating current making 200 vibrations per second, the spark distance at the ends of the secondary being about 0.05 mm.¹⁾

For the condenser an apparatus was used which was constructed according to NERNST²⁾ viz. two metal plates between which a glass plate can be displaced. Theoretically the alteration in the capacity is proportional to the position of the glass plate. In practice however the condenser must first be calibrated. This can be done with the help of a specially constructed condenser such as NERNST'S Trough condenser³⁾. The one used and shown in Fig. 2 only differs from this

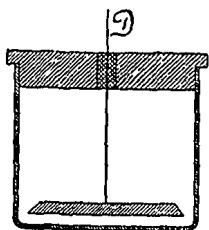


Fig. 2

in that the ebonite cover has been replaced by a metal one; and the metal tube, to which the other plate is fastened, by a stiff wire *D* of 2 mm. diam. The latter is insulated from the metal cover by a small thickness of ebonite. In a condenser constructed thus the non-inductive capacity is exceedingly small.

In the construction of the experimental condenser there are two points to be primarily considered; as large a capacity as possible must be introduced into a somewhat limited space, and the non-inductive capacity which is connected to the electrometer must be as small as possible. These

¹⁾ Preliminary experiments showed that the passage of sparks of several millimeters long did not explode liquid nitrous oxide though this is endothermic.

²⁾ NERNST, Zeitschrift für Physik. Chemie XIV. 4.

³⁾ NERNST, loc. cit.

conditions were fulfilled in the manner clearly shown in Plate I, which represents three different sections of the condenser together with the beaker of the cryostat, made to receive the liquefied gas, in which it is placed.

The two outer plates p_1 and p_2 are connected by the nuts to one another and by the pin t with the earth. The three screws s are placed in suitable glass tubes on which the five plate condenser is itself mounted. The plates themselves have a radius of 3 cm. and are separated from one another by small glass rods 1 mm. long. To reduce the errors so introduced to a minimum these glass rods must be made as small as possible, and it appeared to be best to cut them from a 1 mm. glass tube with a wall of $\frac{1}{5}$ mm. and then to grind them exactly equal. The above mentioned error cannot finally be more than 0.1 % since it enters equally into the numerator and denominator of the expression for the dielectric coefficient. The 1st, 3rd and 5th plates are connected with a pole of the induction coil, the 2nd and 4th with the electrometer. The necessary wires for this, d_1 and d_2 , are fastened to the 1st and 2nd plates respectively; they are drawn through small openings in the superimposed plates, and continue above through the glass tube g .

By means of a pin t the whole condenser is fastened to the cover of a hollow cylinder of brass m in which the liquid to be investigated is placed. The hollow cylinder is earthed together with the two outer plates ²⁾.

The method of filling the condenser with liquid gases must now be discussed, but some points should be first considered. The condenser and hollow cylinder must be protected as much as possible from external heat to prevent the formation of bubbles of vapour, whence they are immersed in the beaker B_1 under the liquid gas. The liquid gas must be employed in sufficient quantity to cool the condenser and to keep it cold, it must be kept from the deteriorating action of atmospheric air, and finally care must be taken that the vapours drawn off are not lost. All these conditions can be best obtained by the aid of a cryostat i. e. a boiling glass with its cases, such as is used in the cryogenic laboratory for measurements with liquid gases. The description of the latter can be found in another place ³⁾.

The following must also be considered. The principle of the method

¹⁾ The section is taken partly through the tube g and partly through the tube N .

²⁾ By the wire ϕ . See Plate I, Comm. N^o. 51.

³⁾ KAMERLINGH ONNES, Comm. N^o. 51 § 3.

is to compare the capacity of the same condenser in air and in a given medium when all dimensions are kept strictly constant. When the capacity of the air condenser is obtained at the room temperature, and that of the liquid condenser at the temperature of boiling gases the geometrical proportions are probably altered by thermic expansion and deformation, and in consideration of the large difference of temperature it is possible that an appreciable error may thus enter.

For these reasons the condenser was arranged in the hollow cylinder so that this enclosed space could be evacuated and cooled in liquid gas. Then, in the manner described below, this evacuated space containing the condenser could be filled with liquid gas from the beaker.

It must be noticed that, however thin the leads may be they always will represent a measurable capacity and hence that the smallest displacement of the apparatus will produce considerable errors. Especially to avoid this it is desirable to mount the apparatus as in ONNES' method for the use of liquid gases in measurements. According to this method, the apparatus to be dipped in and filled with liquid gas is mounted in the closed boiling case, in which the liquid can be immediately poured out, so that the operations of exhausting cooling and filling with the liquid gas allow the position of the condenser and the leads to remain unaltered.

The manner in which the above mentioned hollow brass cylinder is mounted with the experimental condenser in the boiling glass is given in Comm. N^o. 51 § 2 ¹⁾. The further arrangement of the covering of the condenser is shown in Plate I. The inside of the cylinder communicates with the exterior in two ways. One is the fine copper tube *r* which communicates with the part of the beaker unoccupied by the cylinder. This can be opened or closed at pleasure from without by means of a cock in which the pin *h* is moved by the rod *h*₂ and handle *h*₃. The other outlet is the glass tube *g*, through which the expanding vapours can be drawn towards *u* to be collected in the caoutchouc bags ²⁾.

¹⁾ Plate I is a detailed drawing of the cryostat containing the condenser. Plate IV is a diagramatic representation of the nitrous oxide circulation and the cryostat, for the oxygen circulation see MATHIAS l. c.

²⁾ Further explanations: *a*₂ are small screws to fasten packing *a*₁, *b* wooden block to support glass tubes *z*₁ which the wires *d*₁ and *d*₂ pass through, *c* wooden block in two parts to support outflow tube without conducting of heat, *f* soldered in screw used in boring the canal for the liquid gas, *k*₁ packing under the level, *k*₂ brass mount for screwing on the same, *k*₃ leather cushion, *e* caoutchouc tube to connect the glass tube *g* with the brass covering of the condenser and protected by fishglue against the liquid gas. *e*₂ brass bands to make all tight with the help of the screws

Measurements are obtained with the above as follows. First the cock h is shut and the liquid gas allowed to stream into the boiling glass. When a sufficient quantity has collected there, and we may consider that the whole of the condenser has assumed the temperature of the gas, the inside of the hollow cylinder is evacuated through the glass tube g and the capacity of the condenser determined. The cock h is then opened so that the liquid gas streams into the inside of the cylinder in consequence of the pressure. When this operation is finished the cock h is again shut and the capacity of the condenser redetermined, this time with the liquefied gas as a medium¹⁾.

In order to be certain that the cylinder was full of liquid gas the gauge N was found to be necessary, in which the level of the meniscus indicates outside the level of the liquid in the cylinder or the glass tube g adjacent to it. The glass tube l which connects g to N allows an equalization of pressure²⁾.

It was unfortunately impossible for the electrometer and above mentioned auxiliary apparatus to stand in the room where the refrigerating machinery is installed, partly from want of space and partly because of the inevitable vibration caused by the working of and attendance on the pumps. Hence I had simply the choice between, placing the boiling flask containing the condenser in the neighbouring room where the electrometer already stood; or keeping the boiling flask in the cryogenic laboratory and connecting the condenser to the electrometer by adequately long leads. In the former case the liquefied gas would have to be conveyed to the boiling flask through about 5 m. of tubing into which it would be very difficult to prevent the entrance of heat, and a successful termination of the research would be doubtful. This consideration caused me to primarily favour the latter arrangement, although the sensibility of the method is undoubtedly diminished owing to the non-neglectable capacity represented by the long leads. Finally I considered that the advantages of the latter arrangement outweighed its disadvantages.

It appeared to be necessary to eliminate the influence of the long

indicated; $j, p, q, u_1, u_2, w_1, w_2$ see Comm. N^o. 51 § 2; t_2, t_1 copper mountings to fasten steel pins v_1, v_2 . The lateral opening of the glass tube g can be clearly seen in front of the side tube u_2 of the brass T tube.

¹⁾ In the first of the three sections given the beaker and condenser are empty, in the second the beaker is full and in the third both are full of liquid gas.

²⁾ The gauge has been turned in the second section in order to show l . As remarked above the gauge has not been drawn in Plate I. Comm. N^o. 51 § 2, it is observed through K_2 (Pl. I, loc. cit.).

leads, for which auxiliary capacities were required. These were made on the same principle as the experimental condenser; metal plates which are connected by glass tubes and separated by small glass rings. The so formed condenser is then insulated by a layer of paraffin and placed in a card-board box coated with tin foil. The foil is put to earth so that the capacity is quite invariable and independent of the presence of neighbouring bodies.

Such an auxiliary condenser is also useful when experimenting on a substance with a large dielectric-coefficient, for then the glass plate of NERNST'S adjustable condenser may not be sufficiently long to give the required change of capacity. In such a case the auxiliary capacity should be put in parallel with the adjustable condenser.

3. Arrangement of the apparatus

In order to avoid the errors arising from the change of capacity of the leads, the whole apparatus must be immovably and permanently fixed. For the same reason the condensers should be very carefully switched on and off. This requires the use of a switch board to which the taunt wires are fixed and the capacity of which is as small as possible. Indeed for this purpose I employed small ebonite plates provided with mercury cups which could be connected by small metal bars. A diagrammatic representation of the arrangement is shown in Fig. 3.

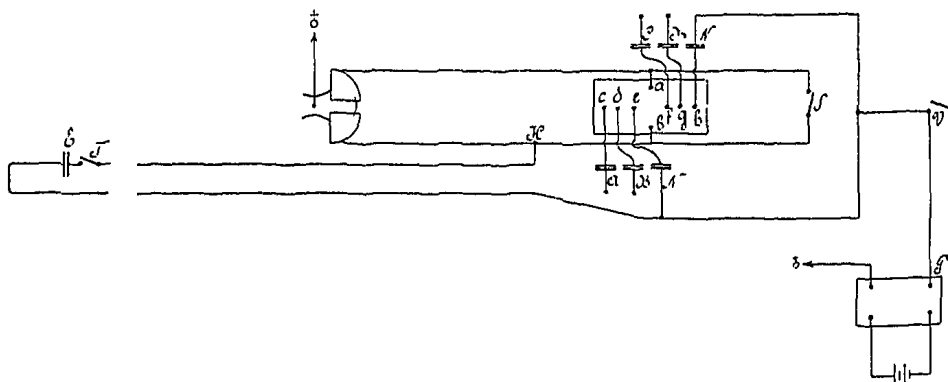


Fig. 3.

From the two quadrant pairs of the electrometer two wires proceed to the mercury cups *a* and *b*, and then to the small key *S*, by which a metallic connection between the quadrants can be made and the zero reproduced. From the six other mercury cups *c*, *d*, *e*, *f*, *g*, *h*, leads go to the inner plates of six condensers N_1 , N_2 , *A*, *B*, *C*, *D*, of which N_1 , N_2 are adjustable NERNST condensers and *A*, *B*, *C*, *D* are auxiliary condensers as described above. The outer plates of these

last four condensers can be connected as required to the earth or to the pole P of the Rumkorff coil to which the outer plates of N_1 and N_2 are always connected. When the key V is shut the condensers are earthed together. From H a wire proceeds into the neighbouring room, and can be connected by the key T to the experimental condenser E ; the outer plate of which is always connected to the coil.

Although only one adjustable condenser is required by the theory of the method, the second N_2 is used partly for convenience, partly for the calibration of the former ¹⁾ as described below.

4. Calibration of the adjustable condensers.

The adjustable condenser was, as mentioned above, an instrument constructed after NERNST's design. The alteration in the capacity would be proportional to the movement of the glass plate if the apparatus were theoretically exact. This is naturally not the case, so the condenser must first be calibrated. The method employed is also due to NERNST. The calibration condenser is put in parallel with the adjustable condenser as described above, and the displacement determined which is required to re-establish equilibrium.

When this is done the calibration condenser is again cut off, and equilibrium re-obtained with the help of the other adjustable condenser. The former condenser is again put in parallel with the calibration instrument, the necessary displacement measured and the process repeated until the glass plate is exhausted. In this way one finds the different positions of the glass plate which correspond to equal differences of capacity. The application of this method of calibration depends upon the supposition that the capacity of the non-inductive parts of the calibration condenser is negligible, as well as that of the wire which puts it in parallel with the adjustable condenser. The first condition is almost absolutely fulfilled by the condenser described above (see fig. 2). The use of the wire can moreover be avoided as shown by NERNST ²⁾. The condenser is insulated by an ebonite plate and is so arranged that the 2 mm. wire D , which projects above the cover of the condenser, stands at the same height as the wires of the adjustable condenser (Fig. 3). The calibration condenser can now be switched on and off by shifting it for about 2 mm. in a horizontal direction, so that the wire D is brought into contact with the lead. The respective positions of the

¹⁾ See NERNST l. c.

²⁾ l. c.

leads, condenser plates etc. will be so little altered by this, that an error from this change is scarcely to be feared.

The whole of the connecting wire is hence reduced to the above mentioned wire *D* which has already a small capacity. The error is still further reduced, as *D* is inductively affected by the metal cover of the calibrating condenser and hence may be considered as part of the inner plate of this.

The results of a calibration performed in the manner described above are given in the second column of Table I.

TABLE I.

Capa- city	Position of the glass plate	Diff.
0	8.40	6.65
1	15 05	6.40
2	21.45	6.20
3	27.65	5 95
4	33.60	6.10
5	39.70	6.20
6	45.90	6.35
7	52 25	6.58
8	58.83	6 70
9	65.53	6.78
10	72 31	6 41
11	78 72	6.16
12	84.88	5.59
13	90.47	5.25
14	95 72	4.96
15	100.68	4.54
16	105.22	4.17
17	109 39	4.13
18	113 52	3.62
19	117.14	

These numbers are the means of four series of observations in which the maximum difference was 0.2 mm. They form the basis of

the following measurements. As only the differences of capacity are required we put the capacity in the position 8.40 as 0, in the position 15.05 as 1 etc., i. e. we take that of the calibrating condenser as unity. The corresponding numbers are given in the first column of the above table, from which intermediate values can be obtained by graphic interpolation.

As a control of the accuracy of the calibration values given in Table I the following work was undertaken. Measurements were made with another condenser in the same manner as before with the calibration condenser, but the connecting wire was about 12 cm. long and had hence a relatively large capacity. Let c be the capacity of this condenser, d that of the wire and γ that of the electrometer together with its leads, which were unaltered throughout the experiment. Further let $x_0, x_1, x_2 \dots$ be the capacities of the adjustable condenser which correspond to the different positions of the glass plate obtained in the calibration. Hence, when the condition that the needle is in equilibrium is fulfilled, we have

$$\frac{x_0}{\gamma} = \frac{x_1 + c}{\gamma + d}; \quad \frac{x_1}{\gamma} = \frac{x_2 + c}{\gamma + d}; \quad \dots \quad \text{etc.} \quad (3)$$

from which we have immediately

$$\frac{\gamma + d}{\gamma} = \frac{x_2 - x_1}{x_1 - x_0} = \frac{x_3 - x_2}{x_2 - x_1} = \dots \quad \text{etc.}$$

hence the consecutive displacements of the glass plate are proportional to one another. The numbers given in Table II were obtained in this manner and each is the mean of three values as satisfactory

T A B L E II.

Position of the glass plate.	Capacity observed	Diff	Capacity calculated.	Δ
52 25	7.000		—	—
61 80	8.439	1.439	8.440	0 001
71 80	9 914	1 475	9 933	0 019
81 77	11.487	1.573	11.485	0.002
91.02	13 099	1.612	13.099	0.000
99.61	14 783	1 684	14.775	0 007
107 46	16.526	1 743	16 516	0.010
114.81	18.330	1.809	18 325	0.005

as in the former observations. The second column contains the capacities corresponding to the respective positions and obtained by the above graphic interpolation.

Column (3) contains the differences of the capacities in column (2) which should be proportional to one another. From the various consecutive quotients the geometric mean is obtained and the numbers calculated which are actually proportional to one another and which also agree as much as possible with the numbers in column (3). From these "calculated differences" column (4) is obtained by addition. The differences again between columns (4) and (2) are found in column (5) which we may safely call errors of observation. The greatest is 0.019 of the capacity of the calibrating condenser and is equivalent to a displacement of 0.13 mm. of the glass plate in the adjustable condenser. We can take this as the highest limit of attainable accuracy, being that with which a single capacity is itself determined.

Tables III and IV were obtained in the same manner. The former relates to the part of the glass plate which was not used in Table II, while the latter represents the results of an experiment in which the former method was carried out with a considerably greater capacity.

It will be seen that these tables show "observation errors" of the same magnitude as Table II, and the remarks made above concerning the accuracy of the determinations hold also with these values. On the magnitude of the errors we may notice the following. The accuracy of a single adjustment was at a maximum 0.1 mm. as shown by a number of observations, in which the readings

T A B L E III.

Position of the glass plate.	Capacity observed.	Diff	Capacity calculated.	Δ
52.25	7.000	—	—	—
43.15	5.555	1.445	5.545	0.010
34.55	4.160	1.395	4.164	0.004
26.70	2.850	1.310	2.854	0.004
18.90	1.595	1.255	1.611	0.016
11.25	0.425	1.170	0.431	0.006

T A B L E IV.

Position of the glass plate.	Capacity observed.	Diff.	Capacity calculated.	Δ
27.15	2.920		—	—
52.25	7.000	4.080	6.988	0.012
80.10	11.220	4.220	11.233	0.013
103.70	15.661	4.445	15.663	0.002

with one adjustable condenser were repeated *ceteris paribus*. Each of the numbers given in the above four tables is deduced from three readings, two with the condenser N_1 and one with N_2 . The values which are given in the second column of Tables II, III and IV have a larger inaccuracy, since the errors of direct reading are added to the eventually similar errors of the calibration curve. But all the values are the mean of four series of three observations, so the required error is reduced to 0.12 mm. This is in accordance with the maximum error seen in the Tables II to IV which is 0.13 mm., while most are much smaller.

These considerations are not however very precise, for the calculation of a capacity from the calibration curve implies the use of more than one observation, and the values „Capacity calculated” are derived from all the observations. Hence the calculated error must be greater than 0.12 mm., and therefore the maximum error of 0.019 in the capacity — or of 0.13 mm. in the position of the glass plate — is entirely explicable from the reading errors. This result leaves no doubt as to the accuracy of the principle of the experiments.

5. *Measurements.*

The final determinations were made in the following manner. The adjustable condenser N_1 was set at 21.45 and the key T (Fig. 3) opened, equilibrium was then attained by connecting up the auxiliary capacity and adjusting N_2 . The key T was then closed and N_1 readjusted until equilibrium was again established. If now we call x_0 and x_1 the capacities of N_1 in the first and second positions respectively, γ the capacity of the electrometer and its leads including the wire from H to the key T , d that of the wire from T to the

experimental condenser, c that of the experimental condenser with vacuum as a medium. Then we have as above in equation (3)

$$\frac{x_0}{\gamma} = \frac{x_1 + c}{\gamma + d}$$

or

$$x_0 - x_1 = c = x_0 \frac{d}{\gamma} \dots \dots \dots (4)$$

When the experimental condenser is now filled with a dielectric fluid its capacity will be altered to c' and a considerable movement of the glass plate will certainly be required to compensate it.

If we call the capacity of the condenser in this third position x_2 , then we have in analogy with the foregoing

$$x_0 - x_2 = c' - c = x_0 \frac{d}{\gamma} \dots \dots \dots (5)$$

The differences $x_0 - x_1$; $x_0 - x_2$ can be immediately read off. In the same way we may express the difference $c' - c$ as a measurable quantity from (4) and (5). From this we can determine the value of c'/c if we know the value of c , which can be obtained by measurements in which the value of γ is purposely varied. But we find that the differences of the readings occurring in the observations are so small that the value of c cannot be determined to a greater accuracy than 10 %, and is hence useless. A determination of the value of $x_0 d/\gamma$ is required, and this can be made once for all by cutting off the lead close to the condenser and fixing it in almost exactly the same position as before by means of wax, but insulated from the condenser. One can now convince oneself that the error produced in fixing the wire does not reach 0.2 mm., by loosening and refixing it, moreover the error occurs equally in the numerator and denominator of the expression for the dielectric-coefficient. In the same way the capacity of the wire can be determined, by proceeding with the lead in exactly the same way as with the whole condenser, and thus arriving at the equation

$$x'_0 - x_0 = x_0 \frac{d}{\gamma} \dots \dots \dots (6)$$

and hence

$$\frac{x_0}{\gamma} = \frac{x'_0}{\gamma + d}$$

in which x'_0 is the capacity of the adjustable condenser after the last alteration.

From (4), (5) and (6) one obtains immediately for the dielectric coefficient

$$\frac{c'}{c} = \frac{(x_0 - x_2) + (x'_0 - x_0)}{(x_0 - x_1) + (x'_0 - x_0)} = K \dots \dots \dots (7)$$

In the following account of the results of my measurements let y_1 be the position of the glass plate of the adjustable condenser after switching on the experimental condenser with a vacuum as medium and at the temperature of the liquid gas; y_2 be the position when the medium is the liquid gas. The capacities corresponding to these positions are, as before, indicated by x_1 and x_2 . Before the determinations the glass plate always stood at 21.45 (so $x_0 = 2.000$).

1. *Nitrous oxide.*

19 June:

$y_1 = \dots \dots \dots$ (This reading was omitted)
 $y_2 = 106.2; 106.5; 106.2.$

20 June:

$y_1 = 58.85; 58.85; 58.90.$
 $y_2 = 107.90; 108.10; 107.95.$

These determinations are not however really trustworthy, as it appeared that the nitrous oxide became quite impure.

The following results are final.

9 July:

$y_1 = 56.30, 56.40; 56.35$ Means 56.32
 $y_2 = 106.00; 105.90; 106.00; 106.20$ 106.03 (± 0.17).

The corresponding capacities are:

$(x_0 = 2.000).$
 $x_1 = 7.640.$
 $x_2 = 16.191.$

2. *Oxygen.*

10 July:

$$\begin{array}{ll}
 y_1 = 57.70; 57.70; 57.75 & \text{Means } 57.72 \\
 y_2 = 86.15; 86.10; 86.05; 85.85; 85.95 & 86.02 (\pm 0.17)
 \end{array}$$

The corresponding capacities are.

$$\begin{array}{l}
 (x_0 = 2.000) \\
 x_1 = 7.843 \\
 x_2 = 12.200
 \end{array}$$

This gives the above defined difference.

$$x_0' - x_0 = 3.731.$$

As the glass plate can only be moved from 21.45 to 8.40, an auxiliary condenser must be employed; it is however scarcely worth while to communicate the resulting data in extenso.

From the above results and formula (7) we obtain the following values for the dielectric-coefficients:

$$\left. \begin{array}{l}
 K_{N_2O} = \frac{14.191 + 3.731}{5.640 + 3.731} = 1.912 \\
 K_{O_2} = \frac{10.200 + 3.731}{5.843 + 3.731} = 1.455
 \end{array} \right\} \dots \dots \dots (8)$$

To these values we can make a correction; we must consider that not only the experimental condenser but also the leads were immersed to a definite height in the liquid gas, while the entire length of the leads was 88 cm. The value

$$x_0' - x_0 = x_0 \frac{d}{\gamma} = 3.731$$

only applies rigidly when the whole lead is in air. When the capacity of the condensers filled with liquid gas is required the quantities d and $(x_0' - x_0)$ must be multiplied by

$$\left(\frac{83}{88} + K \cdot \frac{5}{88} \right)$$

where K is the dielectric-coefficient of the medium. For this correction the approximate values given by (8) are quite sufficient, and

the influence of the surrounding tubes is negligible in view of the accuracy obtained.

We arrive finally at the following values for nitrous oxide:

$$K_{N_2O} = \frac{14.197 + 3.731 \left(\frac{83}{88} + 1.912 \frac{5}{88} \right)}{5.640 + 3.731} = 1.933$$

for oxygen:

$$K_{O_2} = \frac{10.200 + 3.731 \left(\frac{83}{88} + 1.455 \frac{5}{88} \right)}{5.843 + 3.731} = 1.465 .$$

As one sees the values of y_2 are a little more variable than those which refer to our condensers when filled with air (see § 4). This may be due to variations of temperature or small impurities. When we take this into account and assume for the other numbers the accuracy arrived at above, we find that the maximum error (if we assume that the errors are additive) in the dielectric-coefficient of nitrous oxide is 0.5 % and in that of oxygen 0.7 % while the error probably can be smaller.

The value 1.491 given by DEWAR and FLEMING for the D.-C. of oxygen at the normal boiling point, differs from my value by 1.8 %, an agreement which may be considered as satisfactory, if we take into account the deviations in the various values arrived at by different workers, even where the experimental substance can be more easily produced than liquid gases.

6. *Application of the CLAUSIUS-MOSOTTI formula to the results.*

An obvious application of the above results is employing them to test the CLAUSIUS-MOSOTTI formula.

This is usually expressed

$$\frac{K + 2}{K - 1} \cdot d = \text{Const.} = D$$

where K is the dielectric-coefficient, d the density.

This equation enables us to calculate the D.-C. of a substance in the liquid state when we know the D.-C. in the gaseous state and the densities of both aggregates.

This is unfortunately not possible for nitrous oxide as the density at the normal boiling point is not accurately known. However as it is very interesting to see how my value agrees with the observations

of LINDE¹⁾, we will take 1.15 for the density of nitrous oxide after NATTERER²⁾. Hence we obtain 4.85 as the value of D for liquid nitrous oxide at its boiling point, while LINDE found 5.42 for the same at 0° C. For gaseous nitrous oxide we obtain $D = 5.103$ assuming $d = 1.969 \times 10^3$, $K = 1.001158$ after KLEMENCIC.

With oxygen also only an approximate test of the formula is possible as the required data are inaccurate. More especially the value of the D.-C. of gaseous oxygen is unknown, and there exists only a well grounded supposition by DEWAR and FLEMING³⁾ that it will not differ sensibly from that of air, which was found by both BOLTZMANN and KLEMENCIC to be 1.00059 at 0° C. and 760 mm.

The density of gaseous oxygen is 1.4292×10^{-3} at 0° C. and 760 mm.⁴⁾, that of liquid oxygen is 1.124 after OLSZEWSKI⁵⁾, 1.1375 after DEWAR⁶⁾ and 1.134 after LADENBURG and KRÜGEL⁷⁾.

If we then assume 1.00059 for the D. C. of gaseous oxygen, and 1.1375 for the density of the liquid we arrive at 1.556 as the D.-C. for the latter, which value agrees as far as the order of magnitude with the values found by DEWAR and FLEMING. Conversely assuming the D.-C. of the liquid oxygen we obtain the value of 1.00051 for the gas instead of 1.00059.

From the uncertainty of the data employed a better agreement cannot be expected. The experiments are at least not contrary to the CLAUSIUS-MOSOTTI formula, while the further consideration of its application to oxygen must be deferred for the present.

Physics. — *“The HALL-effect and the increase of Magnetic Resistance in Bismuth at very low Temperatures”*. — By Dr. E. VAN EVERDINGEN Jr. (Communicated by Prof. H. KAMERLINGH ONNES).

(Will be published in the Proceedings of the next meeting.)

¹⁾ l. c.

²⁾ Pogg. Ann. 62 p. 134.

³⁾ l. c.

⁴⁾ LANDOLT and BÖRNSTEIN, p. 116.

⁵⁾ Ztschr. f. phys. Chem. XVI, 383.

⁶⁾ Proc. Royal Instit. 96.

⁷⁾ Ztschr. f. Compr. Gase 99 p. 77.

(November 22, 1899.)

