Citation:

Sanders, C., Determination of the latitude of Ambriz and of San Salvador (Portuguese West-Africa), in:
KNAW, Proceedings, 2, 1899-1900, Amsterdam, 1900, pp. 398-406

has different values in different directions, the condition, which is thereby brought about, may be compared with a condition where the pressure has a different value in different directions. If we call the quantity which is to be considered as the pressure in the direction $\perp$ to the capillary layer, p_1 and that in the direction of the capillary layer, the following formula would hold:

$$p_1 + a\varrho^2 + c_2\varrho\frac{d^2\varrho}{dh^2} - \frac{c_2}{2}\varrho\left(\frac{d\varrho}{dh}\right)^2 = p_2 + a\varrho^2 + \frac{c_2}{2}\varrho\frac{d^2\varrho}{dh^2}$$

or

$$p_1 - p_2 = -\frac{c_2}{2}\varrho\frac{d^2\varrho}{dh^2} + \frac{c_2}{2}\left(\frac{d\varrho}{dh}\right)^2.$$

This difference in pressure taken over a surface normal to the bordering layer, with a length of 1 cm. and a breadth equal to the thickness of the capillary layer, furnishes the value of the capillary tension:

$$\int (p_1 - p_2)\, dh\ .$$

The work which is to be performed for enlarging the surface with 1 cm.^{2}, the temperature remaining constant, so the capillary energy is:

$$\sigma = \int (p_1 - p_2)\, dh = \int \left\{ -\frac{c_2}{2}\varrho\frac{d^2\varrho}{dh^2} + \frac{c_2}{2}\left(\frac{d\varrho}{dh}\right)^2 \right\} dh\ .$$

With the aid of this latter consideration we can easily show that the capillary energy is equal to the amount with which the thermodynamic potential of the bordering layer, taken over a cylinder whose section is 1 cm.^{2} and whose height is equal to the thickness of the capillary layer, exceeds the thermodynamic potential of the same mass in the homogeneous vapour- or liquid-phasis.

Astronomy. — "*Determination of the latitude of Ambriz and of San Salvador (Portuguese West-Africa).*" By C. SANDERS (Communicated by Dr. E. F. VAN DE SANDE BAKHUYZEN).

During a several years' residence on the West coast of Africa I spent as much as I could of my leisure in making observations for determining geographical positions. Till now, besides a rather inaccurate theodolite, which did not allow to read beyond full minutes, I had at my disposal only a sextant. Recently however, after consulting with Dr. E. F. VAN DE SANDE BAKHUIJZEN who for a long time already has rendered me valuable assistance in my endeavours to obtain useful results, I have bought a portable universal instrument, by means of which I hope that my future observations will attain a higher degree of accuracy.

Yet, among the earlier observations there are already some, of which the publication may prove desirable, with a view to the great uncertainty which still exists about the exact position of several places on the South-West coast of Africa.

I will here communicate my observations for the determination of the latitude of Ambriz and of San Salvador, both in Portuguese West-Africa.

I. *Determination of the latitude of Ambriz.*

The observations were made with a sextant of WEGENER with vernier on which can be read 10″, and an artificial horizon; besides I used a mean time chronometer. Observations referred to the sea-horizon, together with some made by means of the small theodolite mentioned above, are not communicated, because they are far less accurate.

The errors of graduation of the sextant were determined by Dr. KAISER at Leiden as follows:

at 0°	0″0	at 70°	+ 22″5
10	+ 5.5	80	+ 23.5
20	+ 9.5	90	+ 24.0
30	+ 13.0	100	+ 24.5
40	+ 16.0	110	+ 24.5
50	+ 18.7	120	+ 24.5
60	+ 21.0		

Before each set of observations I tested the adjustment of the mirrors and the telescope. If small deviations were found, they were immediately corrected. The index error was always determined before the observations by 4 till 6 pointings on the direct images; in the case of solar observations they were equally distributed over both limbs. This determination was often repeated in the same manner after the observations.

I assumed for the eastern longitude 13° 8′ or expressed in time 52 m. 32 sec. This value was deduced from determinations of the time (measurements of 8 till 10 altitudes near the prime vertical) and a comparison of the so found local time with that of Greenwich, as given by the chronometers on board several ships that touched at this port. The English Admiralty-chart (corrected up to 1897) gives also 13° 8′ for the Eastern longitude of Ambriz.

As provisional value of the latitude I assumed 7° 50′ south.

The observations were made before the old factory of the „Nieuwe Afrikaansche Handelsvennootschap" and consist of the three following series:

1. *Circummeridian altitudes of the sun on May* 10, 1893.

For the reduction of the observations I used the following formula:

$$\varphi = \delta - z + \frac{\cos\varphi\cos\delta}{\sin(\delta-\varphi)} \cdot \frac{2\sin^2\frac{1}{2}t}{\sin 1''} - \frac{\cos^2\varphi\cos^2\delta}{\sin^2(\delta-\varphi)}\, cotg\,(\delta-\varphi)\frac{2\sin^4\frac{1}{2}t}{\sin 1''}$$

z represents the northern zenith distance, whereas southern latitude is regarded negative. The term depending on $2\sin^6\frac{1}{2}t$ could be neglected, as its influence, even in the case of the greatest hour angles, was too small.

The following corrections and daily rates of the chronometer were found:

March	18	1893	$+$ 46m 56s 50	
				$-$ 0s 24
April	12	"	$+$ 46 50.40	
				$-$ 0.23
May	3	"	$+$ 46 45.48	
				$-$ 0.39
June	3	"	$+$ 46 33.26	

That I might use a constant value for the declination of the sun, the hour-angles were reckoned from the instant of the maximum altitude, computed from the formula:

$$t_0 = 0.255 \frac{d\delta}{dt}(tang\varphi - tang\delta)$$

in which t_0, the hour-angle of the maximum altitude, is expressed in seconds of time and $\frac{d\delta}{dt}$ stands for the variation of the sun's declination in one hour, expressed in seconds of arc.

The observed altitudes were corrected for refraction, parallax and semidiameter. The places of the sun etc. were taken from the Connaissance des Temps.

For the time of the observations and the used part of the sextant we find:

Corr. chronometer to mean loc. time	+ 46m 43s4
Indexcorrection sextant	— 1′ 55″
Corr. for error of graduation	+ 25″
Temperature	27° C.
Barometer	760 m.M.
Mean loc. time of transit of sun, T	11h56m 13s
Hour-angle of maximum altitude, t_0	— 4s5
Declination of the sun for $T + t_0$	+ 17° 44′ 43″5

The separate observations and their results are given in the following table, which needs no further comment.

Limb	Chron. time.	Hour-angle.	Reading Sext.	Latitude.
l	10h53m 40s	— 15m 45s1	127° 45′ 40″	— 7° 50′ 3″6
u	54 55	14 30.1	128 54 50	49 53.2
l	56 14	13 11.1	127 56 20	49 59.1
u	57 1 5	12 23.6	129 2 30	50 4.4
l	57 37.5	11 47.6	128 1 10	50 2.3
u	58 20	11 5.1	129 7 30	49 45.4
l	59 2	10 23.1	128 5 30	50 5.5
u	59 40	9 45.1	129 11 20	49 48.8
l	11 0 17.5	9 7.6	128 8 50	50 10.3
u	1 6.5	8 18.6	129 15 5	49 47.7

Hence:

Latitude from lower limb	— 7° 50′ 4″2
" " upper limb	49 51.9
Mean value	— 7° 49′58″0
Difference upper limb—lower limb	+ 12″3

Examining these results and also those of the 2nd series given below, it appears clearly that there is a perceptible constant personal error in esteeming the contact of the two images of the sun, and its mean value as deduced from the two series is 5″.3 If we correct the separate results for this amount, we obtain as mean error of a single pointing

± 5″.8 and as mean error of the final result ± 1″.8. We must also take into account however the error made in determining the index correction, which influences all the observations equally. If we assume that for this determination 5 pointings were made (compare above) and accordingly take the mean value of its error to be $\pm 5''.8 \times \sqrt{1/5} = \pm 2''.6$, the total mean error of the result becomes ± 3″.2.

Moreover, as the observations are not arranged symmetrically with respect to the meridian, an error in the correction of the chronometer also influences the final result. Probably this error is not large, as the rate of the chronometer was pretty regular in the period considered. Also the results from the first and the last pair of observations, viz. 58″.3 and 58″.6, agree well inter se. An error in the chronometer correction of 2^s would result in a variation of 3″.5 in the latitude.

2. *Circummeridian altitudes of the sun on May* 14, 1894.

The observations were reduced in the same way as those of May 10, 1893. The sun's places etc. were now taken from the Nautical Almanac; they are based on the same elements and values as those of the Conn. d. T. The following corrections and daily rates of the chronometer were found:

Jan. 12	1894	+ 43m 44s76	
			— 0s80
April 26	"	+ 42 21 30	
			— 0.99
May 22	"	+ 41 55 60	
			— 2.01
July 13	"	+ 40 11 20	

For the time of the observations and the used part of the sextant we find:

Corr. of the chronometer	+ 42m 8s6
Index correction sextant	— 1′ 50″
Correction for error of graduation	+ 25″
Temperature	27°
Barometer	760 mM.
Mean loc. time of transit of sun, T	11h 56m 9s4
Hour angle of maximum altitude, t_0	— 4s4
Declination of the sun for $T + t_0$	+ 18° 41′ 10″1

The separate observations and their results follow here:

Limb.	Chron. time	Hour angle.	Reading Sext.	Latitude.
l	11h 2m 41s	— 11m 20s 4	126° 10′ 20″	— 7° 50′ 5″3
u	3 30	10 31.4	127 16 20	50 1 2
l	4 17	9 44 4	126 14 50	50 8 8
u	5 18	8 43.4	127 21 10	49 58.7
l	6 9	7 52.4	126 19 50	49 54 1
u	6 54	7 7 4	127 25 10	49 43.0
l	7 42	6 19 4	126 22 20	50 9 7
u	8 24	5 37 4	127 27 40	49 46.8
l	9 5	4 56.4	126 24 40	50 3.9
u	9 42	4 19 4	127 29 0	50 0.0
l	21 3	+ 7 1.6	126 21 10	50 6.0
u	21 42	7 40 6	127 23 10	50 9.3
l	22 16	8 14.6	126 18 5	50 22.1
u	22 50	8 48.6	127 20 30	50 12.4
l	23 29	9 27.6	126 15 30	50 11.0
u	24 45	10 43.6	127 15 40	50 3.5
l	25 33	11 31 6	126 9 20	50 17 8
u	26 9	12 7.6	127 11 25	49 59.7
l	26 46	12 44.6	126 5 50	50 1.8
u	27 26	13 24.6	127 7 0	49 57.9

Hence:

Latitude from lower limb	— 7° 50′ 8″05
upper 〃	49 59.25
Mean result	— 7° 50′ 3″6
Difference upper limb—lower limb	+ 8″8

The value now found for the personal contact error agrees fairly well with that found from the first series.

If again we correct the separate results for the mean value of this error, viz. 5″.3, we now get as mean error of one pointing ± 8″.3

and for the mean of all $\pm 1''.9$. If we add to this as mean error of the index correction $\pm 8''.3 \times \sqrt{1/5} = \pm 3''.7$, the total mean error becomes $\pm 4''.1$. The uncertainty in the correction of the chronometer may be perhaps a little larger than for May 10, 1893, although the acceleration found after May 22 has probably not begun before that date, about which time the colder season began. But at any rate, through the symmetrical arrangement of the observations, an error of this kind will influence only very slightly the final result.

In reality the observations for index error will as a rule be preciser than the altitude determinations with the mercurial horizon and, if on the other hand we take into consideration the possible uncertainty of the chronometer correction, then I don't think the precision of my observations is overrated, if we assume $\pm 5''$ as mean error both of the result of May 10 1893 and of that of May 14, 1894.

3. *Meridian altitudes of the sun and of α Crucis.*

In the third place I measured several times the greatest altitude of the sun and once that of α Crucis.

The observations on the sun were always made on the lower limb, as in this way the maximum altitude from the mercurial horizon is easiest and most accurately measured; the images of the lower limb namely are separating before the culmination.

Here follow the observations and their results. The column „Corr." gives the sum of the index correction and the correction for errors of graduation. The declinations and the semidiameter and parallax of the sun were taken for 1893 from the Connaissance des Temps, for 1894 from the Nautical Almanac.

Date.	Object.	Reading.	Corr.	Temp.	Barom.	Latitude.
May 7 1893	Sun L. L.	129° 56′ 50″	— 1′ 30″	29°	759.5	— 7° 50′ 6″7
„ 8 „	„ „	129 24 20	1 30	29	758	50 4.4
„ 9 „	„ „	128 52 20	1 30	28	759	50 4.2
„ 10 „	„ „	128 21 0	1 30	27	760	50 1.5
June 2 „	„ „	119 20 15	1 31	27	761	50 1 1
May 10 1894	α Crucis.	70 42 0	1 13	25.5	760.4	50 9.6
„ 14 „	Sun L. L.	126 28 20	1 30	27	760	49 57.0

The results from the altitudes of the sun still remain to be corrected for the personal error; applying for this the value found previously, we obtain as final results:

May 7	1893		— 7° 50′ 1″4
〃 8	〃		49 59.1
〃 9	〃		49 58.9
〃 10	〃		49 56.2
June 2	〃		49 55.8
May 10	1894		50 9.6
〃 14	〃		49 51.7
		Mean	— 7° 49′ 59″0

As it is difficult to form an opinion about the relative precision of the observation of α Crucis and those of the sun, the same weight is given to all of them. The mean error of an observation is then found to be ± 5″.6, that of the mean ± 2″.1.

The results from the 3 series now must be combined. Although for the last series a smaller mean error was found, it did not seem advisable to assign to it a greater weight than to the others. For it is possible that for this kind of solar observations the personal error differs perceptibly from that in the determination of circum-meridian altitudes.

So we have:

Series I		— 7° 49′ 58″0
〃 II		63.6
〃 III		59.0
	Mean	— 7° 50′ 0″2

The three series agree fairly well inter se, and as final result for the latitude of the place of observation we may take:

$$-\ 7^\circ\ 50'\ 0''$$

which value will probably be exact within a few seconds.

The reduction of the latitude to that of the harbour light amounts, according to the map of "Port Ambriz" on the English Admiralty chart: "Cape-Lopez-bay to St. Paul de Loanda", to + 12″ ± 2″ (the map is not graduated), and so the latitude of the harbour light is found to be:

$$-\ 7^\circ\ 49'\ 48''$$

The value given on the Admiralty chart is: — 7° 52′ 9″ and accordingly 2′ 21″ too much south.

II. *Determination of the latitude of S. Salvador do Congo.*

A few observations have been obtained about the latitude of San Salvador, the old capital of the former kingdom of Congo. They were made before the factory of the "N. Afrik. Handelsvenn.", situated about 1 K.M. north of the centre of the hill on which the town is built (562 m. above the sea-level).

Only the following meridian altitudes were observed.

Date.	Object.	Reading.	Corr.	Temp.	Bar.	Latitude.
May 8 1895	Sun L. L.	132° 49′ 10″	— 1′ 10″5	27°	714	— 6° 15′ 18″2
„ 10 „	„	131 45 20	1 10	27.5	714	15.3
„ 11 „	α Crucis.	67 31 50	1 13	22	713.5	26.1

If to the results from the observations of the sun we apply the correction + 5″.3, the mean result for the latitude becomes:

$$-6^\circ\, 15'\, 16''$$

with an uncertainty of perhaps 10″ or still more.

Dr. Chavanne (Map of Justus Perthes 1886) found for the latitude of San Salvador (the hill extends over a few kilometers only)

$$-6^\circ\, 20'\, 10''$$

and for the longitude 14° 47′ 18″ East of Greenwich. Very probably there is also an error in the longitude of more than 20′, the true eastern longitude being smaller.

Physics. — "*Methods and Apparatus used in the Cryogenic Laboratory. II. Mercury pump for compressing pure and costly gases under high pressure*". By Prof. H. Kamerlingh Onnes.

(Will be published in the Proceedings of the next meeting).

Physiology. — "*Lipolytic ferment in human ascitic fluid*". By Dr. H. J. Hamburger.

(Will be published in the Proceedings of the next meeting.)