The percentage of water contained in crystallized indican is, therefore:

I 14.3 pCt.

II 15.4 pCt.

III 17.2 pCt.

the formula $C_{14}H_{17}NO_6 + 3H_2O$ requires 15.5 pCt.

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Mathematics. — "*A special case of the differential equation of* MONGE." by Prof. W. KAPTEYN.

To the communications inserted in the Proceedings of Nov. 25th and Dec. 30th 1899 we here add the results of our investigation of a case where the equation of MONGE consists of three terms.

If the equation of MONGE has the form

$$s + \lambda t + \mu = 0$$

this equation will admit of two intermediate integrals, only when

$$\lambda = \frac{1}{\varrho}\frac{\partial}{\partial q}(p\varrho + v) \quad , \quad \mu = \frac{1}{\varrho}H(p\varrho + v),$$

where ϱ represents any function of x, y, z, q, and the function v satisfies the differential equation

$$\frac{1}{\varrho}\frac{\partial\varrho}{\partial q}\frac{\partial v}{\partial y} + \left(1 + \frac{q}{\varrho}\frac{\partial\varrho}{\partial q}\right)\frac{\partial v}{\partial z} - \frac{1}{\varrho}H(\varrho)\frac{\partial v}{\partial q} = \frac{\partial\varrho}{\partial x}$$

and H denotes the operation $\dfrac{\partial}{\partial y} + q\dfrac{\partial}{\partial z}$.

Then one of the intermediate integrals is

$$p\varrho + v = f(x),$$

where f denotes an arbitrary function, the second being found by connecting the two integrals of the complete system

$$H(\varrho)\frac{\partial V}{\partial q}-\frac{\partial \varrho}{\partial q}\frac{\partial V}{\partial y}-\left(\varrho+q\frac{\partial \varrho}{\partial q}\right)\frac{\partial V}{\partial z}=0,$$

$$H(v)\frac{\partial V}{\partial q}-\frac{\partial v}{\partial q}\frac{\partial V}{\partial y}-\varrho\frac{\partial V}{\partial x}-q\frac{\partial v}{\partial q}\frac{\partial V}{\partial z}=0.$$

Special cases:

I. $s+\lambda t=0$

Here $\mu=\frac{1}{\varrho}H(p\,\varrho+v)=0$; therefore

$$H(\varrho)=0 \text{ and } H(v)=0,$$

whilst the differential equation for v reduces to

$$\frac{\partial v}{\partial z}=\frac{\partial \varrho}{\partial x}.$$

Let σ be any function of x, q, $u=z-qy$. We then find here

$$\lambda=\frac{1}{\frac{\partial \sigma}{\partial u}}\left[p\left(\frac{\partial^2\sigma}{\partial u\,\partial q}-y\frac{\partial^2\sigma}{\partial u^2}\right)+\frac{\partial^2\sigma}{\partial x\,\partial q}-y\frac{\partial^2\sigma}{\partial x\,\partial u}\right].$$

The differential equation $s+\lambda t=0$ now possesses the two intermediate integrals

$$p\frac{\partial \sigma}{\partial u}+\frac{\partial \sigma}{\partial x}=f(x) \text{ and } y\frac{\partial \sigma}{\partial u}-\frac{\partial \sigma}{\partial q}=f(q),$$

where f represents an arbitrary function.

These results are more general than those formerly communicated sub IV; we find these back by putting

$$\varrho=e^{-\int\phi(q)\,dq}\,,\qquad v=-\int e^{-\int\phi(q)\,dq}\,\psi(x,q)\,dq.$$

II. $s+\mu=0$.

Here $\lambda=\frac{1}{\varrho}\frac{\partial}{\partial q}(p\,\varrho+v)=0$; therefore

$$\frac{\partial \varrho}{\partial q}=0 \text{ and } \frac{\partial v}{\partial q}=0,$$

whilst the differential equation for v reduces to

$$\frac{\partial v}{\partial z}=\frac{\partial \varrho}{\partial x}.$$

Let σ be an arbitrary function of x, y, z. We shall now find

$$\mu=\frac{1}{\frac{\partial \sigma}{\partial z}}\left[p\left(\frac{\partial^2 \sigma}{\partial y\, \partial z}+q\frac{\partial^2 \varrho}{\partial z^2}\right)+\frac{\partial^2 \sigma}{\partial x\, \partial y}+q\frac{\partial^2 \sigma}{\partial x\, \partial z}\right].$$

The differential equation $s+\mu=0$, for which may be written

$$\frac{d^2 \sigma}{dx\, dy}=0,$$

possesses as intermediate integrals

$$\frac{d\sigma}{dx}=\frac{\partial \sigma}{\partial x}+p\frac{\partial \sigma}{\partial z}=f(x),$$

$$\frac{d\sigma}{dy}=\frac{\partial \sigma}{\partial y}+q\frac{\partial \sigma}{\partial z}=f(y),$$

where f denotes an arbitrary function.

These results differ in form only from those formerly communicated sub V.

Mathematics. — "*On the locus of the centre of hyperspherical curvature for the normal curve of n-dimensional space*". By Prof. P. H. SCHOUTE.

At the close of the preceding paper we have pointed out that the characteristic numbers of the locus of the centre of hyperspherical curvature are lowered if some of the points of the given rational curve lying at infinity coincide. At present we wish to trace for a special case the amount of those lower numbers, viz. for the case where the given curve is the "normal curve" N_n^n of the n-dimensional space S_n, in which it is situated. It is known that this curve is represented on rectangular coordinates by the equations

$$x_i=t^i,\quad (i=1,\ 2,\ \ldots\ n)\ ,\quad \ldots\ldots \quad (1)$$