us now imagine that at the moment t_1 the molecule I collides with the molecule III. Between t_1 and $t_1 + \theta$ the value $\frac{d^2 a_1}{dt^2}$ will then be very great. Afterwards that quantity will be reduced to an average value; but at the moment $t + \frac{x_0}{V}$ the wave emitted by molecule I will arrive at molecule II and will cause $\frac{d^2 a_2}{dt^2}$ to have an abnormally great value between $t_1 + \frac{x_0}{V}$ and $t_1 + \frac{x_0}{V} + \theta$. Between the moments $t_1 + 2\frac{x_0}{V}$ and $t_1 + 2\frac{x_0}{V} + \theta$ the value of $\frac{d^2 a_1}{dt^2}$ will be again abnormally great and so on. This solution is certainly not contained in the function-solution.

Therefore before we assume the theory of GALITZIN of the broadening of the lines of the spectrum as proved, we should have to make a separate examination, in order to investigate in how far we may assume that the motion of the molecules may be represented approximately by the function-solution, and in how far the roots which GALITZIN has not found, have influence on the phenomenon. In this investigation we should have to start from the hypothesis about the way in which the molecules are made to vibrate.

I shall however, not occupy myself further with this problem, but I shall try to solve the more intricate problem of vibrators which are in motion, in order to investigate, if a connection may be found between the ponderomotoric forces of radiation and the molecular forces. The solution of this problem I have had in view from the very beginning.

Astronomy. — "*The 14-monthly period of the motion of the pole of the earth from determinations of the azimuth of the meridian marks of the Leiden observatory from* 1882–1896". By J. WEEDER (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

1. From the motion of the poles of the earth over the earth's surface results not only a variation of the geographical latitude of each place, but also a variation of the direction of the meridian; hence the azimuth of each direction varies accordingly.

If in seconds of arc x and y express the deviations of the north pole from its mean place in the direction of the meridian of Greenwich (x) and in the direction of 90° West of Greenwich (y), then we can represent the azimuthal deviation Δ of a meridian with regard to its mean direction for a place at geographical longitude λ (West of Greenwich) and latitude β, in seconds of time by the formula:

$$+ (x \sin \lambda - y \cos \lambda) \frac{\sec \beta}{15} = \Delta$$

This formula represents also the variable part of each azimuth when these azimuths are taken so as to increase from North to West.

2. Prof. TH. ALBRECHT [1]) has deduced from the variations of latitude of several places a continuous series of values for the co-ordinates x and y, beginning with 1890.0; these show that the path of the pole of the earth is geometrically rather intricate. In 1891 Dr. S. C. CHANDLER found a 14-monthly as well as a yearly period in the motion of the pole, but thought that two periodical terms of the periods mentioned would be insufficient to express the co-ordinates of this motion. Dr. E. F. v. D. SANDE BAKHUYZEN on the contrary has contented himself with using these two periodical terms [2]). According to his computation the results derived with regard to the motion of the pole from observations after 1858 can be brought to agree fairly well with the supposition that each of the co-ordinates x and y consists of 2 singly-periodical terms one having a period of about 14 months, the other of exactly a year. It appears then that the terms of the 14-monthly period may also be the components of a circular motion of the pole.

The most probable elements of this circular motion are according to Dr. E. F. v. D. SANDE BAKHUYZEN:

Period	430.66 days
Amplitude	0.″159
Epoch of the greatest latitude for Greenwich	Julian date 2408568 or 1882 May 2

and the components for the Julian date t corresponding to this:

$$x = + 0.''159 \cos. 2\pi \frac{t - 2408568}{430.66}$$

[1]) TH. ALBRECHT, Berichte uber den Stand der Erforschung der Breitenvariation, in December 1897, 98, und 99.

[2]) E. F. VAN DE SANDE BAKHUYZEN, Sur le mouvement du pôle terrestre, d'après les observations des années 1890—97, et les résultats des observations antérieures. *Archiv. Néerl. Série 2. T. II.*

$$y = -0.''159 \sin. 2\pi \frac{t - 2408568}{430.66}$$

From the terms of the yearly period follows a motion of the pole in an ellipse whose axes are 0.″121 and 0.″057; on September 28 the pole is in the major axis, in the meridian 19° East of Greenwich.

3. The following investigation intends to deduce the 14 monthly part of the motion of the pole from the results for the azimuth of the meridian marks of the Leiden observatory. The pier of the north mark was renewed in 1880, the pier of the south mark in 1882; the determinations of azimuth used for this research, were begun for the north mark only not until Juli 1882, for the mean azimuth of both marks in January 1884, and so in each case more than a year after the construction of the pier, when the masonry was for the greater part solidified.

The period of the observations used here ends July 1896, and so includes 14 years or 12 14-monthly periods so that the variations of azimuth of the latter period could be deduced independently of the yearly period. The material consists of transits of α Ursae minoris in both culminations and without using the artificial horizon. The observers were E. F. v. d. Sande Bakhuyzen and J. H. Wilterdink.

The following tables I and II show, for both observers and culminations separately, the number of the observations made and their distribution over the years and the months.

TABLE I.

July 1882 July 1883	83 84	84 85	85 86	86 87	87 88	88 89	89 90	90 91	91 92	92 93	93 94	94 95	95 96
Observer: E. F. v. d. S. BAKHUYZEN.													
Upp. Culm.	30	28	26	23	22	13	18	13	16	25	21	12	6	3
Low. Culm.	31	32	39	27	21	9	24	10	12	25	22	14	2	5
Observer: J. H. WILTERDINK.													
Upp. Culm.	37	23	23	28	17	11	37	19	15	23	20	11	8	2
Low. Culm.	32	30	35	29	26	18	25	17	17	19	15	9	8	6

TABLE II.

Months.	J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.	Total.
Observer: E. F. v. D. S. BAKHUYZEN.													
Upp. Culm.	15	28	27	30	17	17	5	16	30	28	24	19	256
Low. Culm.	3	20	37	50	28	43	25	25	17	11	11	3	273
Observer: J. H. WILTERDINK.													
Upp. Culm.	16	36	32	42	33	10	9	13	22	27	20	14	274
Low. Culm.	9	9	20	28	29	40	34	31	41	14	25	6	286

4. The following remarks may help to form an opinion about the value of this material for the investigation of the motion of the pole:

1st. The mean value of the accidental error in the azimuth, deduced from one transit-observation, is two or three times greater than the amplitude of the 14-monthly motion.

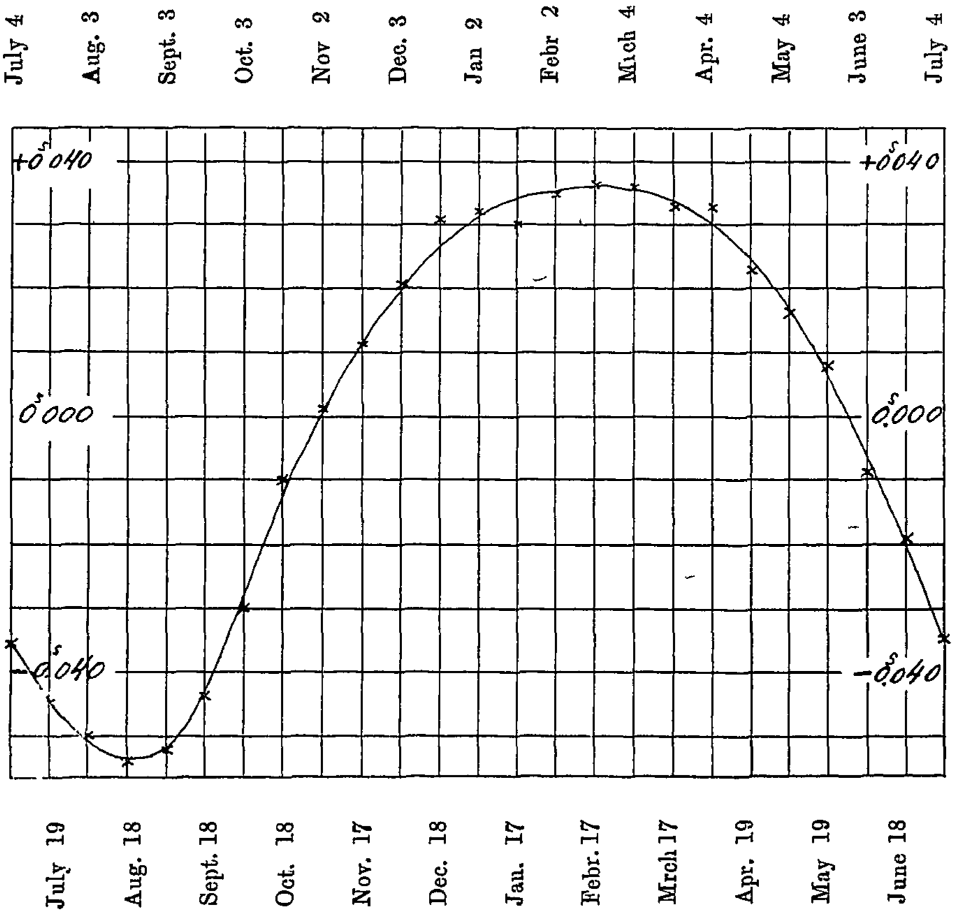
2nd. The directions of the marks in the horizontal plane are by no means absolutely stable, on the contrary their accidental variations throughout the whole period have been greater than the variation of the direction of the meridian resulting from the motion of the pole.

3rd. In spite of the remarks 1 and 2, the number of the observations and their distribution over a long period may lead us to expect a satisfactory determination of the periodical influences on the azimuth.

Yet for an independent determination of the length of the period, the time during which the observations were made was too short. Therefore I had to content myself with a determination of the amplitude and the epoch.

Besides the accidental variations the two directions of the marks are liable to systematic yearly-periodical motions in azimuth, as is clearly proved by the differences in azimuth of the two directions, which are of course independent of the motion of the pole. The graphical representation of this periodical variation (Fig. 1) has been deduced from the readings of the marks from 1883 Aug. 10—1898 Aug. 10.

Fig. 1.



One part of this periodical annual variation is caused by a motion of the direction of the north mark, another part by a similar motion in the direction of the south mark, and probably the mean azimuth of the two directions is also influenced by the same motion. As communicated in § 3 the determinations of azimuth from 1882 July to the end of 1883 concerned the north mark only; in order to be able to unite the azimuth of this period with those of the following years to a system as homogeneous as possible, the azimuths of the north mark before 1894 are diminished by half the difference in azimuth between the two directions from fig. 1; after having applied this correction, the remaining influence of the periodic motion on the azimuth of the north mark, is equal to that on the mean azimuth of both marks.

In the azimuths this influence is combined with that of the periodic motion of the meridian direction, resulting from the yearly-periodical part of the motion of the pole; as we cannot compute the two influences separately, we cannot compare these determinations of azimuth with the elements of the yearly motion of the pole.

As with regard to the 14-monthly motion of the pole there is no reason to expect changes of the same period in the directions of the marks, I have attributed the variation of azimuth to the meridian direction, and therefore to the motion of the pole only, and from the latter I have calculated, assuming a period of $430^2/_3$ days, the amplitude and epoch of the above-mentioned part of the motion of the pole.

5. About the reduction of the observations I will communicate as much as is required to understand the deduction of the numbers to be given below, and also their meaning with respect to the 14-monthly motion of the pole. The observed time of transit reduced to the middle thread, has been corrected for the inclination of the axis of rotation, the collimation constant, the clock-correction, and also partly for the azimuth of the telescope and the personal equation of the observer.

The azimuth constant of the transit instrument used for this reduction is deduced from approximate values of the azimuth of the north mark up to 1884, further for the mean azimuth of both marks, which approximate values are derived from the results of preliminary calculations and are taken so as to vary with the time as regularly as possible. A few corrections have been applied to these values:

1°. In agreement with the idea mentioned above, half the difference in azimuth of the north and south marks has been added to the azimuth of the north mark before 1884.

2°. periodical corrections have been applied to the azimuths over the whole period, which corrections result from a 14-monthly motion of the pole according to Dr. E. F. v. d. Sande Bakhuyzen, and are expressed by the formula:

$$\Delta = \frac{+\ 0.159\ sec\ \beta}{15}\ sin\left(2\pi\ \frac{t-2408568}{430.66} + \lambda\right)$$

β and λ being the latitude and longitude (West of Greenwich) of the Leiden observatory. (Proc. Vol. I, p. 202).

By applying the correction 2 for the periodic motion of the pole to the approximate values of the azimuth constant, the corrections of that constant deduced from the discussion of the whole series of observations are freed from the greater part of the influence of the 14-monthly periodic motion as proved by the results. By means of the azimuth-differences between the telescope and the north mark (before 1884) or the mean of the two marks (after 1884) as deduced from the micrometer readings on the marks, we have deduced the approximate values of the azimuth of the transit circle from those of the azimuths of the marks.

The personal equations of the observers which have been applied here are the differences of the Right Ascension of Polaris according to their observations and the Right Ascension of this star according to the "Fundamental Catalog der Astronomische Gesellschaft." These approximate values are deduced from former observations of Polaris, in the supposition that the personal equation for both culminations and under all circumstances is the same.

6. After the reduction mentioned in § 5 each time of transit o should represent, if the elements of reduction were exact, the apparent Right Ascension of Polaris c as deduced for the moment of observation from the mean place of the Fundamental Catalog. The apparent Right Ascensions are borrowed from the "Berliner Jahrbuch"; only in the years 1882—85 the mean Right Ascension of the "Jahrbuch" differs from that of the "Katolog" and then the apparent Right Ascension of the Jahrbuch have been reduced to the later system. The differences $o—c$ of the observed and computed Right Ascensions further have served to determine the corrections of the personal equation and the instrumental errors and principally those of the assumed azimuths of the marks.

7. For the discussion of the quantities $o—c$ the following course has been taken. First the mean of the values $o—c$ for three successive months have been formed for each observer and for each culmination separately in order to determine that part of the values $o—c$ which, independently of the influence of the accidental and systematical errors of the instrument, is the same for the two culminations. For the determination of this part the observations of the two observers at first have been treated separately. In so far as the two culminations were both observed during the same periods of three months half the sums of their mean results are taken; as it appeared that these values showed periodical annual variations, the periodic part has first been deduced and afterwards the annual means of the residuals are computed. It then appeared that the results for the two observers agreed fairly well, both with regard to the annual means and to the co-efficients of the periodic part. Therefore I have combined the corresponding results of the two observers in one system; and have represented graphically the annual means obtained in this way by a smooth curve. The sum of the ordinates of this curve and of the deviations resulting from the periodical annual variation form the part of the quantities $o—c$, common to both the culminations;

this part has been subtracted from them. Finally a constant correction for personal equation, different for each observer has been applied to the $o—c$, in order to reduce the mean value of the corrected $o—c$ for each observer to 0.

8. The quantities $o—c$ corrected as explained in 7 consist of the accidental errors of observation and the influences of systematical errors in the adopted values for the inclination of the axis, the constant of collimation and the azimuth. The azimuth-corrections might have been determined with greater precision if the observations had been or could be made free from the influence of the first mentioned systematical errors. The observations at my disposal do not contain any data from which to determine the systematical corrections of the inclinations adopted; some reflection-observations of POLARIS from 1882—84, made for this purpose, have not been considered as their number was too small for the determination of a satisfactory correction of the inclination. Somewhat different from this is the opportunity for correcting the constants of collimation from the observations themselves as they have been made in the two positions of the instrument. During the period of the observations the transit circle has been reversed 24 times which divide the whole interval into 12 periods in which the clamp was east and 11 during which it was west.

The longest of these periods lasted 28 months, the shortest nearly 2 months.

The influence of an error in the assumed amount of the constant of collimation on the azimuth-correction, calculated from the value of $o—c$, changes in sign by the reversal of the instrument; it can therefore be found when we compare the azimuth-corrections from observations of Polaris, immediately before and after the reversal, or, if that error proves to be constant during a longer time, when we compare the mean azimuth-corrections deduced from observations during a longer period before and after the reversal.

It now appeared that the observations of Polaris immediately before and after the reversals were not numerous and that moreover one single observation is not accurate enough to betray a small systematic error. The graphical representation of the mean azimuth-corrections during long periods, on the contrary showed clearly the influence of a small error in the assumed constant of collimation. These mean values during the same position of the instrument showed generally a regular variation; if however we would combine the means of all the periods in a regular chronological order, then the curve obtained disagreed for the greater part of the time with the idea of a regular variation of the azimuths of the marks.

9. That a small correction of the assumed constant of collimation for Polaris, which might give a regular variation of the azimuth during the whole period, is not impossible may be seen from what follows. This constant consists of two parts: the constant of collimation determined in the nadir by the reflection of the vertical thread on the horizontal mercury-surface, and the small correction for the flexure of the rotation axis expressed by the formula $b\,(1 + \cos z)$. The first part, the constant of collimation in the nadir, is probably very exact (only in the period 1884—1885 the degree of precision may be a little less, the level being less reliable), but the value of the flexure found by determining from time to time the constant of collimation in the horizontal position of the telescope by pointing at the marks before and after a reversal is less trustworthy, especially as it appeared that it varies distinctly, when the position of the cell of the object glass was altered.

Therefore I thought myself justified in deducing from the Polaris-observations and the readings of the meridian marks small corrections to the constant of collimation, which rendered the variation of the azimuth-correction more regular, and which, with a few exceptions were constant in the periods during which the cell of the object glass was in the same position.

In order to be able to judge in how far the motion of the pole in the 14-monthly period deduced from the observations is dependent on this correction, I have computed this motion supposing: 1st that the constant of collimation is left unchanged, 2nd that the small correction mentioned has been applied to it.

10. The values of $o-c$, during 14 years (1882 July to 1896 July) corrected according to 7, for each observer separately, are divided according to the time of the observations into 12 groups, each enclosing 430 days (the assumed value of the period of the pole-motion) and each of these periods is subdivided into 43 periods of 10 days [1]). The mean values of $o-c$, during these 10 days for each culmination separately, were now formed and to each mean was given the weight 1, independent of the number of values. So we obtained a series of numbers (at a maximum of 24), representing the values of $o-c$ in both culminations belonging to one and the same

[1]) In this computation the period is actually reduced to $430^2/_{30}$ days, by causing a period of 430 days to succeed two periods of 431 days. In the periods of 431 days one of the subdivisions consists of 11 days instead of 10 days.

phase of the pole-motion; after this the means of these values belonging to a same phase were taken, after changing the sign of $o-c$ at lower culmination, because the influence of a variation of azimuth on $o-c$ in both culminations has a different sign. Finally for each observer the deviations of these 43 numbers from their mean has been formed, which deviations after the reduction from differences in time of transit to differences in azimuth, represent the still remaining influence of a periodical change of the pole on the azimuth of the marks.

A great part of that influence has been removed by applying to the azimuth, assumed in the reduction, the periodic corrections mentioned in § 5 sub 2^0, computed after the formula deduced for the 14-monthly motion of the pole by Dr. E. F. VAN DE SANDE BAKHUYZEN. In order to obtain the whole influence of this periodic motion, this correction of the azimuth has again been added to those 43 values for each observer.

Of these values U_B and U_W for the two observers the mean has been taken independently of the number of the observations and these numbers are given in Table III.

TABLE III.

Ordinal number.	Before the correction of the constant of collimation.		After the correction of the constant of collimation.		
n	$\frac{1}{2}(U_B+U_W)$	$\frac{1}{2}(U_B-U_W)$	$\frac{1}{2}(U_B+U_W)$	$\frac{1}{2}(U_B-U_W)$	Obs.—Comp.
1	+ 0.017	+ 0.013	+ 0.010	+ 12	+ 0.003
2	+ 26	+ 01	+ 19	— 05	+ 10
3	+ 17	+ 07	+ 17	+ 06	+ 07
4	+ 12	+ 01	+ 14	— 08	+ 02
5	00	+ 04	+ 03	+ 04	— 10
6	— 12	+ 10	— 07	+ 07	— 21
7	— 02	— 02	+ 06	00	— 08
8	+ 18	— 03	+ 22	— 03	+ 07
9	+ 13	— 04	+ 16	00	+ 01
10	+ 25	+ 11	+ 29	+ 06	+ 15
11	+ 29	— 03	+ 32	+ 01	+ 18
12	— 05	+ 08	+ 07	+ 06	— 06
13	+ 16	+ 04	+ 11	+ 10	— 01

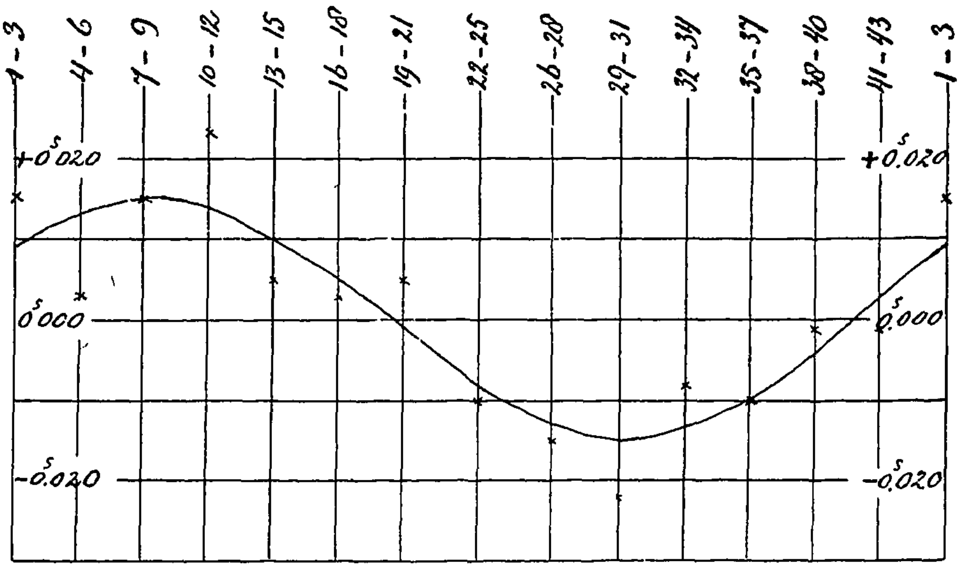
40*

Ordinal number.	Before the correction of the constant of collimation.		After the correction of the constant of collimation.		
n	$^1/_2(U_B+U_W)$	$^1/_2(U_B-U_W)$	$^1/_2(U_B+U_W)$	$^1/_2(U_B-U_W)$	Obs —Comp.
14	+ 02	— 03	— 06	— 01	— 16
15	+ 19	— 07	+ 11	— 07	+ 02
16	— 18	+ 16	— 23	+ 15	— 30
17	+ 21	+ 06	+ 20	+ 07	+ 15
18	+ 24	+ 05	+ 11	+ 06	+ 08
19	+ 20	— 14	+ 08	— 09	+ 07
20	+ 14	— 05	+ 09	— 07	+ 10
21	— 04	+ 01	— 02	+ 01	+ 02
22	+ 01	— 01	+ 03	00	+ 09
23	— 25	— 24	— 15	— 19	— 07
24	— 08	+ 08	— 07	+ 14	+ 02
25	— 24	+ 07	— 20	+ 05	— 09
26	— 20	+ 03	— 20	+ 05	— 08
27	— 11	— 01	— 07	— 02	+ 06
28	— 19	+ 02	— 17	+ 01	— 03
29	— 24	+ 21	— 30	+ 19	— 15
30	— 21	— 07	— 22	— 06	— 07
31	— 11	— 04	— 14	— 09	+ 01
32	— 20	— 12	— 20	— 12	— 06
33	— 05	— 08	+ 03	— 09	+ 16
34	— 10	+ 05	— 07	+ 07	+ 05
35	— 08	— 21	— 10	— 19	+ 01
36	— 16	— 21	— 07	— 15	+ 03
37	— 25	+ 07	— 14	+ 03	— 06
38	— 21	+ 05	— 18	+ 05	— 12
39	+ 10	— 06	+ 10	— 03	+ 14
40	+ 13	— 03	+ 04	— 03	+ 06
41	+ 10	— 05	+ 11	— 03	+ 10
42	— 03	+ 04	— 10	+ 02	— 13
43	00	+ 04	— 05	— 01	— 10

The first column contains the ordinal numbers of each period of 10 days, the second and fourth columns contain the half sums of U_B and U_W in the two suppositions: that the correction of the constant of collimation mentioned in §§ 8 and 9 has, or has not been applied. The degree of precision of these numbers can be derived from the values of half the differences $\frac{1}{2}(U_B - U_W)$ which, if the observations are correct, must be equal to 0.

I have formed the means from each set of three successive values of $^1/_2\ (U_B + U_W)$ and have represented them graphically in Fig. 2.

Fig. 2.



If we try to represent the numbers $^1/_2\ (U_B + U_W)$ of the 4th column by an ordinary sinusoid and, assuming that this formula is exact for the middle-epoch, we reckon the time t in days from May 19th 1888, we obtain the following formula for the influence of the motion of the pole on the azimuth of the meridian marks at Leiden:

$$U = 0^s.0148\ Sin\left(\frac{360}{430,66}\,t + 19^\circ.0\right)$$

Column 6 of table III gives the differences of the results according to this formula and the observed quantities in column 4. According to this formula the influence of the motion of the pole on the azimuth is 0 on April 26th 1888; at that moment the latitude of Leiden resulting from that periodic variation attained its maximum.

The amplitude a of the circular motion of the pole, is found from the amplitude of the variation of azimuth $0^s.0148$ by means of the formula

$$a = 15 \times 0.0148\ Cos\ \varphi = 0.''136$$

in which φ represents the latitude of Leiden.

If from this formula for the variation of latitude for Leyden we want to deduce that for the variation of latitude for Greenwich, we must only take for the date of the maximum latitude 5 days earlier i. e. April 21st 1888, so that the co-ordinates of the motion of the pole for Greenwich are:

$$x = +\, 0''.136 \cos 2\,\pi \frac{t - 2410749}{430.66}$$

$$y = -\, 0''.136 \sin 2\,\pi \frac{t - 2410749}{430.66}$$

in which t represents the Julian date.

11. If we compare this epoch for the maximum latitude of Greenwich with that deduced by Dr. E. F. v. d. S. Bakhuyzen, we see a difference of 2181 and after subtracting 5 periods or 2153 days, it appears that the epoch found by me occurs 28 days later than that according to Dr. E. F. v. d. S. Bakhuyzen.

In the Astronomische Nachrichten n⁰ 3207 A. Sokoloff has given the results of an investigation of the motion of the pole in a period of 430 days by means of the meridian marks of the transit-instrument in the observatory at Pulkowa. In order to compare his results with those found by me, I here give the results deduced by Sokoloff from the observations from 1880—1887 made by Wagner, Wittram and Harzer.

a. From 476 transits of α Ursae-Minoris
Amplitude $= 0''.172$
Epoch for Greenwich = 2410743
i. e. 22 days later than according to E. F. v. d. S. B.

b. From 288 transits of δ Ursae-Minoris:
Amplitude $= 0''.195$
Epoch for Greenwich = 2410771
i. e. 50 days later than according to E. F. v. d. S. B.

c. From 226 transits of 51 H. Cephei:
Amplitude $= 0''.156$
Epoch for Greenwich = 2410759
i. e. 38 days later than according to E. F. v. d. S. B.

In Sokoloff's computation the time of the period is assumed to be 429.7 days. The epoch of maximum latitude of Greenwich given above, has been reduced by me from the year 1884 to the year 1888 by using a period of 430.66 days.