Citation:

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order, of which this evolute is a right section, meets the surface 1) moreover in a curve in space of the order 24.

6. The found image 1) can be useful in different researches about the system S of the circles of JOACHIMSTHAL. By determining the number of points common to this surface and a rectangular hyperbola, a parabola and a straight line, we find that the system S has the characterizing numbers 4,8,16; in other words, it contains four circles passing through two given points, eight circles passing through a given point and touching a given line, sixteen circles touching two given lines. In the same manner is proved that it contains sixteen circles touching two given circles, etc.

7. If we are given a parabola instead of an ellipse, all the circles passing through three conormal points pass also through the vertex of the parabola. Here the found surface of the eighth order is reduced to the right cone $x^2 + y^2 = z^2$, of which the vertex of the parabola $y^2 = 2px$ is the vertex. And the case of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $z = 0$ leads to the surface

$$\frac{4\,a^2\,x^2}{(u^2+a^2)^2} - \frac{4\,b^2\,y^2}{(u^2-b^2)^2} = 1$$

and is quite analogous to that of the ellipse.

Physics. — *„On maxima and minima of apparent brightness resulting from optical illusion."* By Dr. C. H. WIND. (Communicated by Prof. H. HAGA).

1. If we see on a surface two zones of different (real) brightness united by a transition-zone whose brightness decreases continuously from the brighter down to the darker zone, this transitionzone *seems* to be separated from the brighter zone by a still brighter line (maximum of brightness) and from the darker zone by a still darker line (minimum of brightness).

2. This phenomenon, which — as will be seen from what follows — presents itself under very different kinds of conditions was first observed by me in a drawing carefully and succesfully executed by Mr. VAN GRIEKEN, of the firm VAN DE WEYER at Groningen, by means of lithography. This drawing of which fig. 1 is a photographic reproduction (reduced to $1/4$ of its size), which unsatisfactory as it is, yet enables us to observe the phenomenon, consists of a great number of parallel lines of equal thickness drawn at intervals of 1 m.M. in two outer zones, at intervals of 0.4 m.M. in a middlezone,

at continuously increasing distances in the transitionzones between the lastnamed and the two firstnamed (The law of increase of the distances has been chosen so that the proximity of the stripes in the transition zones varies as a linear function).

3. This phenomenon may be produced even more obviously and in a simpler way by means of revolving discs, as are often used in physiological optics. If we paste a piece of white paper of suitable shape on a black disc and cause the disc to turn very fast in its plane we may get on the surface any distribution of light in which the brightness of the observed plane differs only for points situated at different distances from the centre of rotation and so we may compare it with the corresponding distribution of apparent brightness. We can also photograph the discs first in rest, afterwards in rotation, and then the first photograph gives an auxiliary figure, from which the real distribution of light on the figure given by the second photograph may be known.

Fig. 2, 3 and 4 show (reduced to ± $^1/_6$ of their real size) some of the discs used by me; the outlines of the white parts of the discs are partly radii pointing towards the centre of rotation, and partly arcs of spirals of Archimedes, for which, as is wellknown, the variation of the length of the radius-vector is proportional to that of its angle of rotation; at a rapid rotating of the discs represented by these figures we get in accordance with these curved outlines transitionzones in which the apparent brightness varies as a linear function.

The maxima and minima of brightness on the limits of the transitionzones as mentioned in 1., are clearly seen especially by direct observation of the discs while rotating, but also by examining the photographs taken of them. Fig. 5 and 6 are reproductions of such photographs; fig. 5, corresponding to fig. 3, shows pretty clearly the maxima and minima, alluded to above (being circles in this case); fig. 6, corresponding to fig. 4, shows them hardly at all. The reproductions of several of the photographs made by me proved so inadequate that I preferred not to have them inserted here at all; all those published, without an exception, show the phenomena in question far less distinctly than the originals [1]).

4. Now the question arises whether this optical illusion cannot be classed among phenomena already familiar to us. Although I dare not give a definite answer to this question I may be allowed to offer the following considerations.

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[1]) Before the assembly Prof. HAGA has demonstrated the phenomena described by me, as well by means of revolving discs as by photographs taken of them.

It is well known that two areas of different brightness, either adjoining each other, or at some distance of each other standing out against the same background, when viewed in one glance, influence each other's apparent brightness by contrast, so that the brighter area causes the brightness of the darker area apparently to decrease and conversely. Several psychologists LEHMANN [1]), EBBINGHAUS [2]), HESS and PRETORI [3]), KIRSCHMANN [4]) and others have made this phenomenon a subject of investigation and have brought to light some regularities in its occurring; the laws that govern this phenomenon have however not yet become completely known in spite of the important investigations of these scientists.

Now we should be inclined to suppose that the maxima and minima of brightness described above are to be reduced to the said effects of contrast. But then it would seem strange that there should be no intensifying whatever of the lines, whenever the transitionzone disappears between two areas of equal brightness; for we might expect that especially in this case there would be a prominent maximum on the one side of the border and a prominent minimum on the other side of it. But as it is, each of the two zones shows almost uniform brightness (fig. 7*a*, which shows the disc of fig. 7*b* while rotating) although it is not to be denied, that in both areas there can be detected, towards the limits a slight variation of brightness in the sense to be expected. Similar changes appear a little more distinct in a zone of uniform brightness bounded on the one side by a brighter zone, on the other side by a darker zone as represented in fig. 8*a* (corresponding to fig. 8*b*); but although in this case the contrasting influence of the neighbouring zones is more visible, yet the phenomenon observed shows quite another character than in the case of the presence of a transitionzone.

5. Presently we shall say a few words more about a possible explanation of the optical illusion; first however I'll give some further information about the conditions under which it appears and about the laws which seem to govern it:

1⁰. In the case of a transitionzone with constant gradient of decreasing or increasing intensity the maximum and minimum appear exactly on the borders of the transitionzone; at any rate the devia-

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1) LEHMANN, Wundt's Philosophische Studien 3, S. 497, 1886.

2) EBBINGHAUS, Berl. Sitzber. 1887, S. 995

3) HESS a. PRETORI, Von Graefe's Archiv f. Ophthalmologie 40, 4.

4) KIRSCHMANN, Wundt's Phylosophische Studien 6, S. 417, 1891.

tions, which might still exist, seem to fall within the limits of probable error in the determination of the place of those lines.

2°. If we examine successively several drawings in which a transitionzone occurs and which differ only in this regard that the transitionzone gets narrower and narrower, we see the bright and the black line growing finer and finer without there being any continuous increase in their distinctness; on the contrary the distinctness diminishes when the transitionzones are very narrow and it seems as if the narrowing transitionzone were a zone of almost constant brightness, with sharply defined limits towards the other zones. (Besides it seems to me that there is a general tendency to take the middle part of a transitionzone in the above sense, for a zone of uniform intermediate illumination). This gradual fading away of the lines at a narrowing of the transitionzone may easily be seen by slightly extending (for instance $^1/_2$ cM.) the opening in the middle of the rotating disc (fig. 7*b*), by means of which the disc is mounted on the axis, in the direction of one of the radial borders of the white paper and then causing the disc to rotate, it being mounted on the axis first as excentrically as possible afterwards with gradually decreasing excentricity. Indeed we get in this way a gradual narrowing transitionzone and at the same time the experiment shows very clearly the just mentioned fading away of the lines.

3°. Not only do the two lines appear when in the transitionzone the brightness varies in a direction perpendicular to the border of the zone with constant gradient, but also under quite different laws of intensity-variation as may be gathered from the fact that also the *photographs* of the rotating discs show the lines clearly; they are very prominent for instance when this gradient is infinite on the brighter border and becomes finite and diminishes gradually towards the other border, where it attains a minimum differing from zero.

4°. The lines can also be very conspicuous when the gradient continuously approaches zero on the borders of the transitionzone.

5°. Lastly the lines appear in some cases even where the zones of equal brightness on either side the transitionzone are replaced by zones of which the brightness gradually decreases (resp. increases) towards the transitionzone. It may easily be stated by means of a rotating disc that the brightness between the two transitionzones still may appear to have two maxima as in fig. 5, two minima as in fig. 6 the real brightness having even a faint maximum, resp. a faint minimum, in the middle between the two transitionzones.

6. To return to the explanation of the optical illusiou described above, it seems not impossible to me that we should have to look for it in the influences by contrast as mentioned in 4., starting for instance from the hypothesis that each element of the field in general influences the observed brightness of any definite element under consideration, an influence depending in a definite way on the distance between the two elements and on the brightness both of the „inducing" and the „reacting" element. But then we are not allowed, as we might be inclined to do at first sight, to assume that this influence increases as continually the distance between the inducing and the reacting element diminishes and as the difference in brightness between these two elements increases. For the peculiarity of the appearing of the lines mentioned in 5. sub 2°. would be incompatible with an influence acting in this manner. Moreover LEHMANN's [1]) investigations have already brought to light that the influence by contrast reaches its maximum at a definite value of the proportion between the brightness of the inducing and the reacting field. If we may as seems quite natural apply this law to the contrast between any two *elements* of the field and if at the same time further investigations might prove this „critical" proportion between inducing and reacting brightness to decrease as the distance between these two elements diminishes, it may be conceived that the optical illusion we have described and the ordinary effects of contrast will be found to obey the same laws. Accurate investigations will however be required [2]), especially concerning the influence of the distance between two contrasting fields on the amount of the influence, before we can arrive at exactly formulating these laws.

7. Among the methods which may be used to produce the optical illusion described I may still mention a very simple one: we send a beam of light through a not too narrow slit, so that it falls on a second slit parallel with the first, and receive the beam on a screen. If the second slit is *wider* than the first, a middle zone on the screen will be illuminated by all the elements of the first slit; on either side this middle zone transition-zones will be found illuminated by continuously decreasing parts of the first slit, and these zones will pass into other zones not at all illuminated by light through the slit.

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1) LEHMAN l. c. S. 525.

2) KIRSCHMANN announces in his treatise alluded to above the publication of his investigations already partly made on this influence of distance; but the publication seems not to have taken place yet.

The image on the screen will show very beautifully the bright and dark lines mentioned in 1. On photographs taken of this image, these lines are very clearly visible too; fig. 9, a reproduction of such a photograph also shows them, but much less well-defined.

If the second slit is sufficiently *narrower* than the first, the image on the screen will not show a part illuminated by the whole of the first slit, yet there will be again a middle band of maximum illumination across its whole extent; and for the rest the image on the screen will show as a whole the same characteristics as in the preceding case as to both real and apparent brightness. If the second slit is gradually narrowing towards the lower part we see an image projected on the screen in which two straight bright lines appear, these lines, parallel with the edges of the second slit, intersecting somewhere, but remaining clearly visible beyond the point of intersection.

If we cause the light emitted from the first slit to cast a shadow of a thin needle or thread, we get a silhouette with a middle zone having over the whole of its breadth uniform minimum brightness bounded by transition zones of uniformly increasing brightness which on their outsides again are bounded by areas of uniform maximum illumination. The maxima, and especially the minima alluded to in 1. are again very clearly visible here, even to such a degree that in some cases the appearing of the minima might lead one to speak of a doubling of the shadow cast by the thread.

In all these cases, if only the slits are wide enough, diffraction plays no perceptible part.

8. If we illuminate the first slit of 7. by a X-ray tube instead of by ordinary light, and if the rays are not caught on a screen but on a sensitive plate we get on developing, negatives of which the positives are exactly similar to the images described in 7. Fig. 10 shows a reproduction of such a positive, corresponding to a similar case as fig. 9.

Fig. 10 moreover shows a white rectangle, which covers part of the drawing. This effect was obtained by covering part of the negative, of which it is a reproduction, with a slip of paper during the copying. I did so in order to point out that the disappearing of the transition zone — which was effected on the spot in question, at least for the greater part, by this slip of paper — is sufficient to cause the line corresponding to it to vanish. Indeed the bright line which in the other parts of the image is still visible, however much it has lost of its clearness by repeated processes of reproduction is no longer to be traced where the transitionzone has been covered, which sufficiently proves the fictitious nature

Fig. 1.

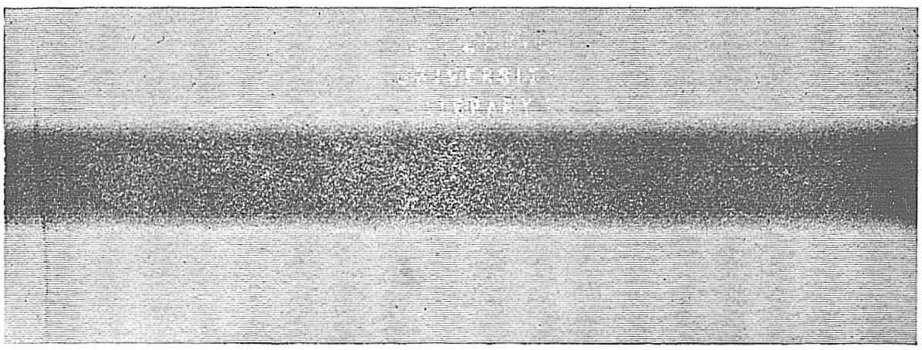


Fig. 2.

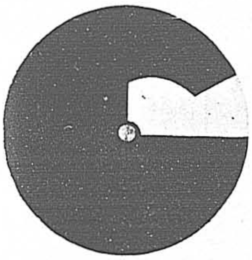


Fig. 3.

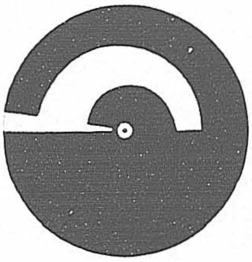


Fig. 4.

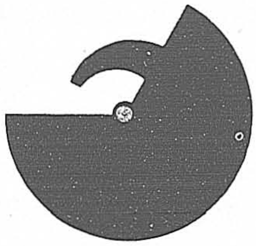


Fig. 5.

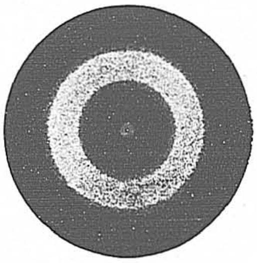


Fig. 6.

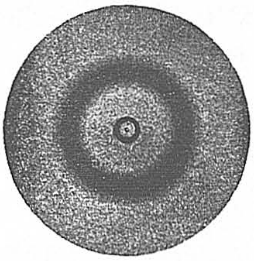


Fig. 7*a*.

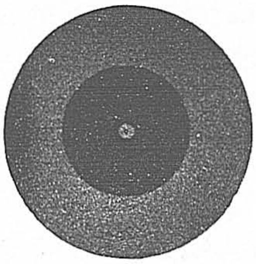


Fig. 7*b*.

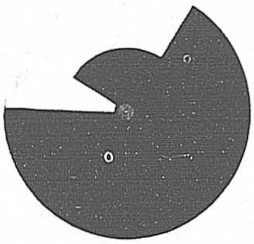


Fig. 8*a*.

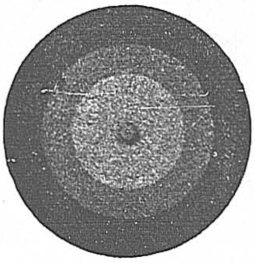


Fig. 8 *b*.

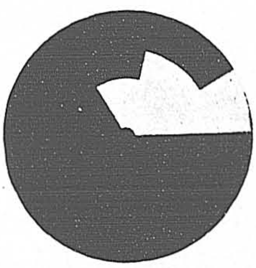


Fig. 9.

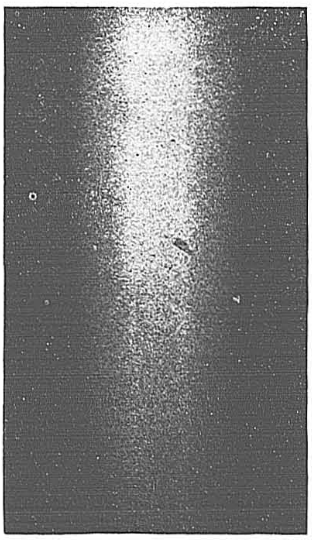
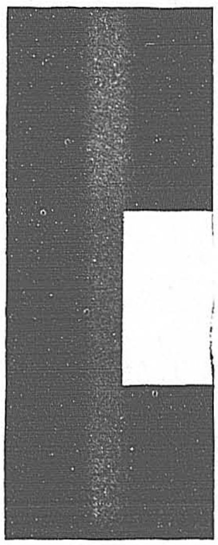


Fig. 10.



of the line. The same experiment may be made in the case of fig. 9 by covering somewhere the transitionzone (for the greater part) with a sharply ontlined piece of paper. That the disappearing of the line is really due to the disappearing of the transition-zone and not to ordinary contrast in consequence of the covering piece of paper will appear if we take first a dark piece of paper and afterwards a light one, both producing the same effect.

9. Although the optical illusion described above may be produced especially by the methods given here, yet it may be often observed in a simpler way. In nearly all cases where we have an umbra passing into a penumbra we may observe a dark and a bright line as borders of the penumbra, and the optical illusion appears in all the various forms described above and even more.

10. Attention has been drawn to the appearing of bright and dark lines conjointly with a penumbra, as described in 7 and 8 by SAGNAC (Journal de Physique VI p. 169, 1897). Formerly I did not conclude from what he said on this subject that he took those lines for the result of optical illusion only, although now it seems to me, if not probable, yet possible that this has been the case. That I feel justified to communicate my observations on this subject somewhat extensively is owing to the great importance that, in my opinion, must be attached to this optical illusion.

In the first place it has deluded many a physicist, who thought that he observed diffraction lines or other important lines in cases where now it is certain that no real maxima or minima of brightness of any importance existed; so it has especially caused bright and dark lines to be observed in *X*-rays-shadows the true nature of which some have tried to explain in various ways, but no one had yet sufficiently accounted for. (Another time I hope to return to this subject).

In the second place it seems to me that this optical illusion may under special conditions lead to the observation of the doubling of bright or dark lines or bands where in reality there is nothing but a broadening of these lines or bands together with the borders growing less sharply defined (compare what has been said in 5. sub 5^{0}), a thing that may occur f. i. when the optical system we use in observing is not accurately adjusted (resp. accomodated). It may be that in some cases this optical illusion must be held responsible for doublings which have been observed and which have not yet been explained in the right manner or not yet been explained at all.

In the third place the optical illusion may lead to errors in the estimation of the place of the maxima and minima of brightness in a system of bright and dark lines, as soon as the brightness in the neighbourhood of the lines is not perfectly symmetrical with respect to their centres.

At any rate we may conclude from what preceeds that we cannot be too critical while observing maxima and minima of brightness, and that in many cases we shall even have to convince ourselves of the existence or non-existence of real maxima and minima of brightness corresponding to the observed maxima and minima.

Bacteriology. — *On the relation of the obligatous anaërobics to free oxygen.* By Prof. M. W. Beijerinck.

The relation of the living cell to free oxygen is best to be judged from the influence of this gas on the *growth* and on the *mobility*. Of course, only the first method is of universal application.

As to the mobile microbes, some time ago I gave the name of „figures of respiration" [1]) to the peculiar groupings, which originate in preparations destined for the microscope, in consequence of the access of oxygen only along the edge of the examined drop under the cover-glass, the microbes being thereby enabled to seek that quantity of oxygen which is best adapted to their respiration. Three types may here be distinguished according as the microbes seek the highest tension of the oxygen along the edge, a middle tension at some distance of it, or the smallest tension in the centre of the preparation. These types I called the aërobic, the spirillous and the anaërobic type.

Further experience has shown that the anaërobic type, characterised by the accumulation of the moving microbes at that spot of the preparation where the oxygen tension is minimum, — commonly near the centre, — does not exist as a special type, but becomes visible only under particular circumstances, and further, that when the aëration of the preparation is sufficiently small, all anaërobics, examined till now, appear to belong to the spirillous type, that is to say, they not only don't fly those places in the preparation, where a small oxygen tension still exists, but they even seek them.

This tension, beneficient for the anaërobics, is however very slight, whence follows, that by using only a moderate number of microbes, consuming but very little oxygen, there may enter at the edge more oxygen than is wanted. In such a case the tension, most approaching

[1]) Centralblatt für Bacteriologie Bd. 14 pag. 837, 1893.